EUCLIDIS
ELEMENTORUM
Libri XV. breviter demonstrati.

Operâ

IS. BARROW, *Cantabrigiensis*,
Coll. TRIN. *Soc.*

Et prioribus mendis typographicis nunc demum purgati.

HIEROCL.

Καθαρμοὶ ψυχῆς λογικῆς εἰσιν αἱ μαθηματικαὶ ἐπιστῆμαι.

LONDINI,

Apud CAR. MEARNE, Bibliopolam Regium. MDCLXXXV.

A.K.

Nobiliſſimis & Generoſiſſimis

Adoleſcentibus,

no *EDOUARDO CECILIO,*

Illuſtriſſ. Comitis Sarisburienſis *Filio;*

no *JOHANNI KNATCHBUL,*

ET

FRANCIS. WILLOUGHBY,

ARMIGERIS.

UNicuique veſtrum (Optimi Adoleſcentes) tantum me debere reputo, quantum homo homini bere poteſt. Mea enim ſenten-, ultra ſincerum amorem non quod quiſpiam de alio bene

mereri poſſit. Hunc autem jamdiu eſt quo ex ſingulari veſtra bonitate mihi indultum experior; ejuſque ſenſus, intimis animi medullis inhærens, ipſi ardens ſtudium impreſſit quovis honeſto modo reciprocos affectus prodendi. Quandoquidem vero ea fortunarum mearum tenuitas, ea veſtrarum amplitudo, exiſtit, ut nec ego alia quam gratæ alicujus agnitionis ſignificatione uti queam, nec vos aliam admittere velitis; ea propter haud illibenter hanc occaſionem arripio, honoris & benevolentiæ, quibus vos proſequor, publicum hoc & durabile μνημόσυνον edendi. Etſi cum oblati anathematis exilitatem, & libellum veſtris nominibus conſecratum, quam is longe infra veſtrorum meritorum dignitatem ſubſidat, attentius conſidero, timor ſubinde aliquis & dubitatio animum inceſſant, ne hoc ſtudium erga vos meum vobis

bis dehoneſtamento ſit potius quam ornamento; ſcilicet memor cum ſim, ut malæ cauſæ, ſic & mali libri patrocinium in patroni contumeliam magis quam in gloriam cedere. Sed quum veſtrarum virtutum id robur, eam fore ſoliditatem, recognoſcerem, quæ veſtrum decus, meo quantumvis labefactato, inconcuſſum ſuſtinere poſſint; idcirco non dubitavi vos in aliquatenus commune mecum periculum induere. Virtutes illas intelligo, quibus nemo unquam in veſtra ætate aut in veſtro ordine, ſaltem me judice, majores deprehendit; quæ vos inſigniter gratos omnibus & amabiles reddunt; eximiam modeſtiam, ſobrietatem, benignitatem animi, morum comitatem, prudentiam, magnanimitatem, fidem, præclaram inſuper ingenii indolem, quæ vos ad omnem ingenuam ſcientiam non tantum excellenti

captu, ſed & appetitu forti ac ſincero, inſtruxit. Quas veſtras præclariſſimas dotes prout nemo eſt fortaſſis qui me melius novit, aut pro conſuetudine, quam jamdudum vobiſcum dulciſſimam coluiſſe ex veſtro favore mihi contigit, penitus introſpexit, ita nemo eſt qui impenſius miratur & ſuſpicit; aut qui ipſas libentius prædicare ac celebrare vellet, ſi non cum eloquii mei vires ſupergrederentur, tum etiam quæ in ſingulis vobis elucent, prolixi alicujus commentarii aut panegyricæ orationis libertatem, potius quam præſtitutas hujuſmodi ſalutationibus anguſtias, expoſcerent. Quin potius divinam clementiam imploro, ut vos earundem virtutum ſancto tramiti inſiſtere, atque hos egregios fructus vernæ veſtræ ætatis felicibus incrementis matureſcere concedat; vitamque vobis in hoc ſeculo ingenuam, innocentem, jucundam, & in futuro beatam

tam ac ſempiternam tranſigere largiatur. Minime autem dubito, ne pro conſueto veſtro in me candore hoc ultimum fortaſſis quod vobis præſtare potero, benevolentiæ erga vos & obſervantiæ teſtimonium, alacriter accepturi ſitis; quod vobis propenſiſſimo affectu offert

Veſtrî in æternum amantiſſimus,

& obſervantiſſimus,

I. B.

Benevolo LECTORI.

SI, quid in hac elementorum editione præstitum sit, scire desideras, amice Lector, accipe, pro genio operis, breviter. Ad duos præcipue fines conatus meos direxi. Primum, ut cum requisita perspicuitate summam demonstrationum brevitatem conjungerem, quo eam libello molem comparrarem, quæ commode absque molestia circumferri posset. Id quod assecutus videor, si absentem Typographi cura non frustretur. Concinnius enim quispiam meliori ingenio aut majori peritia excellens, at nemo forsan brevius pleraſque propositiones demonstraverit; præsertim cum in numero & ordine propositionum ipse nihil immutarim, nec licentiam mihi assumpserim quamcunque propositionem Euclideam procul ablegandi tanquam minus necessariam, aut quasdam faciliores in axiomatum censum referendi; quod nonnulli fecerunt: inter quos peritissimus Geometra Andr. Tacquetus, (quem ideo etiam nomino, quod quædam ex eo desumpta agnoscere honestū duco,) post cujus elegantissimam editionem, ipse nihil attentare

tare voluissem, si non visum fuisset doctissimo viro non nisi octo Euclidis libros suâ curâ adornatos publico communicare, reliquis septem, tanquam ad elementa Geometriæ minus spectantibus, omnino quasi spretis atque posthabitis. Mihi autem jam ab initio alia provincia demandata fuit, non elementa Geometriæ utcunque pro arbitrio conscribendi, verum Euclidem ipsum, eumque totum, quam possem brevissime, demonstrandi. Quod enim quatuor libros spectat, septimum, octavum, nonum, decimum, quamvis illi ad Geometriæ planæ & solidæ elementa, ut sex præcedentes & duo subsequentes, non tam prope pertineant; quod tamen ad res Geometricas admodum utiles sint, tam propter Arithmeticæ & Geometriæ valde propinquam cognationem, quam ob notitiam commensurabilium & incommensurabilium magnitudinum ad figurarum tam planarum quam solidarum intellectum apprime necessariam, nemo est è peritioribus Geometris qui ignorat. Quæ vero in tribus ultimis libris continetur, & corporum regularium nobilis contemplatio, illa non nisi injuria prætermitti potuit; quando nempe illius gratia noster στοιχειώτης, Platonicæ familiæ philosophus, hoc elementorum systema universum condidisse perhibetur;

uti

uti testis est * Proclus, *iis verbis,* Ὅθεν δὴ καὶ τῆς συμπάσης στοιχειώσεως τέλος προεστήσατο τὴν τῶν καλουμένων πλατωνικῶν σχημάτων σύστασιν. *Præterea facile in animum induxi ut opinarer, nemini harum scientiarum amanti non futurum esse cordi penes se habere integrum* Euclidæum *opus, quale passim ab omnibus citatur & celebratur. Quare nullum librum nullamque propositionem negligere volui earum quæ apud* P. Herigonium *habentur; cujus vestigiis presse insistere necesse habui, quoniam ejusce libri schematismis maxima ex parte uti statutum erat, quod præviderem mihi ad novas describendas tempus non suppetere; etsi nonnunquam id facere præoptassem. Eadem de causa nec alias plerasque quam* Euclidæas *demonstrationes adhibere volui, succinctiori forma expreßas, nisi forte in* 2, *&* 13, *& parce in* 7, 8, 9 *libris; ubi ab eo nonnihil deflectere operæ pretium videbatur. Bona igitur spes est saltem in hac parte cum nostris consiliis, tum studiosorum votis, aliquo modo satisfactum iri. Nam quæ adjecta sunt in Scholiis problemata quædam & theoremata, sive ob suum frequentem usum ad naturam elementarem accedentia, sive ad eorum quæ sequuntur expeditam demonstrationem conducentia, seu quæ regularum*

* lib. 2.

rum practicæ Geometriæ quarundam præcipuarum rationes innuunt ad suos fontes relatas, per ea, ut spero, libellus ultra destinatam molem magnopere non intumescet.

Alter scopus ad quem collineatum est, eorum desideriis consuluit qui demonstrationibus symbolicis potius quam verbalibus delectantur. In quo genere cum plerique apud nos Guilielmi Oughtredi *symbolis assueti sint, ea plerumque usurpare consultius duximus. Nam qui* Euclidem *hâc viâ tradere & interpretari aggressus sit, hactenus, quod ego sciam, præter unum* P. Herigonium, *repertus est nemo. Cujus viri longe doctissimi methodus, sane in multis egregia, ac ejus peculiari proposito admodũ accomodata, duplici tamen defectu laborare mihi visa est. Primo, quod cum Propositionũ ad unius alicujus theorematis aut problematis probationẽ adductarũ posterior à priori non semper dependeat; quando tamen illæ inter se cohærent, quando non, nec ex ordine singularũ, nec ullo alio modo, satis prompte innotescere potest: unde ob defectũ conjunctionũ & adjectivorũ (*ergo, rursus, *&c.) non raro difficultas & dubitandi occasio, præsertĩ minus exercitatis, inter legendũ oboriri solent. Deinde sæpenumero evenit, ut prædicta methodus supervacaneas repetitiones effugere nequeat, à quibus demonstrationes est quando prolixæ,*

xa, aliquando & magis intricata, evadunt. Quibus vitiis noster modus facile per verborum signorumq; arbitrariam mixturam medetur. Atque hæc de opella hujus intentione & methodo dicta sufficiant. Cæterum quæ in laudem Matheseos in genere, aut Geometriæ ipsius; & quæ de historia harum scientiarum, ideoque de Euclide *horum elementorum digestore, dici possent, & reliqua hujusmodi ἐξωτερικὰ, cui hæc placent, apud alios interpretes consulere potest. Neque nos angustias temporis quod huic operi impendi potuit, nec interpellationes negotiorum, nec adjumentorum ad hæc studia apud nos egestatem, & quædam alia, ut liceret non immerito, in excusationem obtendemus; metu scilicet inducti, ne hæc nostra omnibus minus satisfaciant. Verum quæ ingenui Lectoris usibus elaboravimus, eadem in solidum ipsius censuræ ac judicio submittimus; probanda si utilia sibi compererit; sin omnino secus, rejicienda.*

I. B.

Ad amiciſſimum Virum, *I. B.* de *EUCLIDE* contracto

Εὐφημισμός.

FActum bene! didicit Laconice loqui
Senex profundus, & aphoriſmos induit.
Immenſa dudum margo commentarii
Diagramma circuit minutum: utque Inſula
Problema breve natabat in vaſto mari.
Sed unda jam detumuit, & gloſſa arctior
Stringit Theoremata: minoris anguli
Lateribus ecce totus *Euclides* jacet,
Incluſus olim velut Homerus in nuce;
Pluteoque ſarcina modo qui incubuit, levis
En ſit manipulus. Pelle in exigua latet
Ingens Matheſis, matris ut in utero Hercules,
In glande quercus, vel Ithaca Eurus in pila.
Nec mole dum decreſcit, uſu fit minor;
Quin auctior jam evadit, & cumulatius
Contracta prodeſt erudita pagina.
Sic ubere magis liquor è preſſo effluit;
Sit pleniori vaſa inundat ſanguinis
Torrente cordis Syſtole; ſic fuſius
Procurrit æquor ex Abylæ anguſtiis.
Tantilli operis ars tanta referenda unice eſt
BAROVIANO nomini, ac ſolertiæ.
Sublimis euge mentis ingenium potens!
Cui invium nil, arduum eſſe nil ſolet.
Sic uſque pergas proſpero conamine,
Radiuſque multum debeat ac abacus tibi;
Sic creſcat indies feracior ſeges,
Simili colonum germine aſſiduo beans.
Specimen futuræ meſſis hic fiet labor,
Magnæque famæ illuſtria hæc præludia.
Juvenis dedit qui tanta, quid dabit ſenex?

Car. Robotham, *CANTAB.*
Coll. Trin. Sen. Soc.

In novam *Elementorum*

EUCLIDIS

Editionem à D. *IS. BARROW*,
Collegii SS. TRIN. *Socio*,
viro opt. & eruditiſſimo,
adornatam.

BEnigne Lector! ſi uſpiã auditũ eſt tibi,
Quantus tenella Nix Geometres ſiet;
Qua mille radiis, mille ludit angulis,
Totumque puro ducit Euclidem ſinu:
Amabis ultro candidiſſimum Virum,
Cui plena nivium eſt indoles, ſed quas tamẽ
Praclarus ardor mentis urget Enthea;
Et uſque blandis temperat caloribus:
Quo ſuavius nil vivit, & melius nihil.
Is, dum liquentes pectore excutit nives,
Et inde & inde ſpargit, en aliam tibi,
Lector benigne, è nivibus Geometriam!

G. C. *A. M. C. E. S.*

= æqualitatem.
⊏ majoritatem.
⊐ minoritatem.
+ plus, vel addendum esse.
— minus, vel subtrahendum esse.
—: differentiam vel excessum; item quantitates omnes, quæ sequuntur, subtrahendas esse, signis non mutatis.
× multiplicationem, vel ductum lateris rectanguli in aliud latus.
Idem denotat conjunctio literarum, ut AB = A × B.
√ Latus, vel radicem quadrati, vel cubi, &c.
Q. & q quadratum. C. & c cubum.
Q. Q. rationem quadrati numeri ad quadratum numerum.

} significat.

Reliquas, quæ ubicunque occurrunt, vocabulorum abbreviationes ipse Lector per se facile intelliget; exceptis iis, quas tanquam minus generalis usus. suis locis explicandas relinquimus.

LIB.

LIB. I.

Definitiones.

I. PUnctum eſt cujus pars nulla eſt.

II. Linea vero longitudo latitudinis expers.

III. Lineæ autem termini ſunt puncta.

IV. Recta linea eſt, quæ ex æquo ſua interjacet puncta.

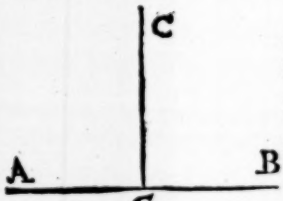
V. Superficies eſt, quæ longitudinem, latitudinemque tantum habet.

VI. Superficiei autem extrema ſunt lineæ.

VII. Plana ſuperficies eſt, quæ ex æquo ſuas interjacet lineas.

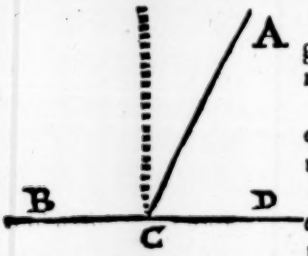
VIII. Planus vero angulus eſt, duarum linearum in plano ſe mutuo tangentium, & non in directum jacentium alterius ad alteram inclinatio.

IX. Cum autem quæ angulum continent, lineæ, rectæ fuerint, rectilineus ille angulus appellatur.



X. Cum vero recta linea C G ſuper rectam lineam A B conſiſtens, eos qui ſunt deinceps angulos C G A, C G B æquales inter ſe fecerit, rectus eſt uterque æqualium angulorum, & quæ inſiſtit recta linea C G, perpendicularis vocatur ejus (A B) cui inſiſtit.

Not. *Cum plures anguli ad unum punctum* : (ut ad G) *exſiſtunt, deſignatur quilibet angulus tribus literis, quarum media ad verticem eſt illius de quo agitur : ut angulus quem rectæ* C G, A G *efficiunt ad partes* A, *vocatur* C G A, *vel* A G C.



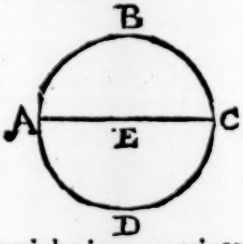
XI. Obtusus angulus est, qui recto major est, ut A C B.

XII. Acutus vero, qui minor est recto, ut A C D.

XIII. Terminus est, quod alicujus extremum est.

XIV. Figura est, quæ sub aliquo, vel aliquibus terminis comprehenditur.

XV. Circulus est figura plana, sub una linea comprehensa, quæ peripheria appellatur, ad quam ab uno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ inter se sunt æquales.



XVI. Hoc vero punctum centrum circuli appellatur.

XVII. Diameter autem circuli est recta quædam linea per centrum ducta, & ex utraque parte in circuli peripheriam terminata, quæ circulum bifariam secat.

XVIII. Semicirculus vero est figura, quæ continetur sub diametro, & sub ea linea, quæ de circuli peripheria aufertur.

In circulo EABCD. E *est centrum*, A C *diameter*, A B C *semicirculus.*

XIX. Rectilineæ figuræ sunt, quæ sub rectis lineis continentur.

XX. Trilateræ quidem, quæ sub tribus.

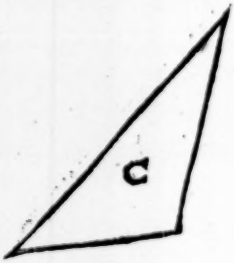
XXI. Quadrilateræ vero, quæ sub quatuor.

XXII. Multilateræ autem, quæ sub pluribus, quam quatuor rectis lineis comprehenduntur.

XXIII.

XXIII. Trilaterarum autem figurarum, æquilaterum est triangulum, quod tria latera habet æqualia, ut triangulum A.

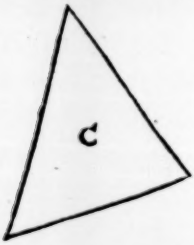
XXIV. Isosceles autem, quod duo tantum æqualia habet latera, ut triangulum B.



XXV. Scalenum vero, quod tria inæqualia habet latera, ut C.

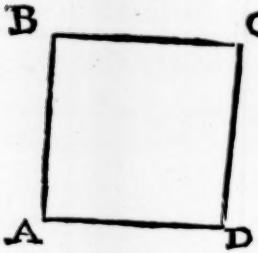
XXVI. Adhæc etiam trilaterarum figularum, rectangulum quidem triangulum est, quod rectum angulum habet, ut triangulum A.

XXVII. Amblygonium autem, quod obtusum angulum habet, ut B.

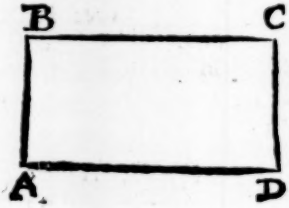


XXVIII. Oxygonium vero, quod tres habet acutos angulos, ut C.

Figura æquiangula est, cujus omnes anguli inter se æquales sunt. Duæ vero figuræ æquiangulæ sunt; si singuli anguli unius singulis angulis alterius sint æquales. Similiter de figuris æquilateris concipe.



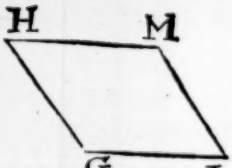
XXIX. Quadrilaterarum autem figurarum, quadratum quidem est, quod & æquilaterum, & rectangulum est, ut A B C D.



XXX. Altera vero parte longior figura est, quæ rectangula quidem, at æquilatera non est, ut A B C D.

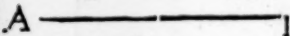
XXXI. Rhombus autem, quæ æquilatera, sed rectangula non est, ut A.

XXXII.

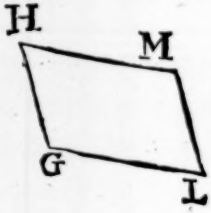


XXXII. Rhomboydes vero, quæ adversa & latera, & angulos habens inter se æquales, neque æquilatera est, neq; rectangula, ut G L M H.

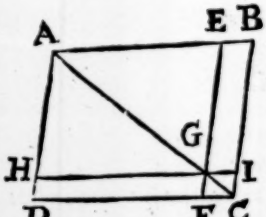
XXXIII. Præter has autem reliquæ quadrilateræ figuræ trapezia appellentur; ut G N D H.



XXXIV. Parallelæ rectæ lineæ sunt, quæ cum in eodem sint plano, & ex utraque parte in infinitum producantur, in neutram sibi mutuo incidunt, ut A, & B.



XXXV. Parallelogrammum est figura quadrilatera, cujus bina opposita latera sunt parallela, seu æquidistantia, ut G L H M.



XXXVI. Cum vero in parallelogrammo A B C D diameter A C ducta fuerit, duæque lineæ E F, H I, lateribus parallelæ secantes diametrum in uno eodemq; puncto G, ita ut parallelogrammum ab hisce

parallelis in quatuor distribuatur parallelogramma; appellantur duo illa D G, G B, per quæ diameter non transit, Complementa; duo vero reliqua H E, F I, per quæ diameter incedit, circa diametrum consistere dicuntur.

Problema est, *cum proponitur aliquid efficiendum.*

Theorema est, *cum proponitur aliquid demonstrandum.*

Corollarium est *consectarium, quod è facta demonstratione tanquam lucrum aliquod colligitur.*

Lemma est *demonstratio præmissæ alicujus, ut demonstratio quæsiti evadat brevior.*

Postulata.

1. POstuletur, ut à quovis puncto ad quodvis punctum rectam lineam ducere concedatur.

2. Et rectam lineam terminatam in continuum recta producere.

3. Item, quovis centro, & intervallo circulum describere.

Axiomata.

1. QUæ eidem æqualia, & inter se sunt æqualia.

ut A = B = C. ergo A = C, vel ergo omnes A, B, C, æquantur inter se.

Nota, *Cum plures quantitates hoc modo conjunctas invenias, concipe vi hujus axiomatis primam ultimæ & quamlibet earum cuilibet æquari. Quo in casu sæpe, brevitatis causa, ab hoc axiomate citando abstinemus; etsi vis consecutionis ab eo pendeat.*

2. Et si æqualibus æqualia adjecta sunt, tota sunt æqualia.

3. Et

3. Et ſi ab æqualibus æqualia ablata ſint, quæ relinquuntur ſunt æqualia.

4. Et ſi inæqualibus æqualia adjecta ſint, tota ſunt inæqualia.

5. Et ſi ab inæqualibus æqualia ablata ſint, reliqua ſunt inæqualia.

6. Et quæ ejuſdem vel æqualium ſunt duplicia, inter ſe ſunt æqualia. Idem puta de triplicibus, quadruplicibus, &c.

7. Et quæ ejuſdem, vel æqualium ſunt dimidia, inter ſe ſunt æqualia. Idem concipe de ſubtriplis, ſubquadruplis, &c.

8. Et quæ ſibi mutuo congruunt, ea inter ſe ſunt æqualia.

Hoc axioma in rectis lineis, & angulis valet converſum, ſed non in figuris, niſi illæ ſimiles fuerint.

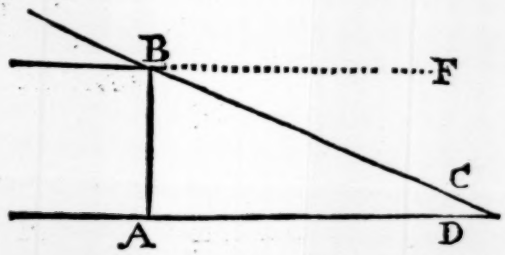
Cæterum, magnitudines congruere dicuntur, quarum partes applicatæ partibus, æqualem vel eundem locum occupant.

9. Et totum ſua parte majus eſt.

10. Duæ rectæ lineæ non habent unum & idem ſegmentem commune.

11. Duæ rectæ in uno puncto concurrentes, ſi producantur ambæ, neceſſario ſe mutuo in eo puncto interſecabunt.

12. Item omnes anguli recti ſunt inter ſe æquales.



13. Et ſi in duas rectas lineas A D, C B, in eodem plano jacentes altera recta B A incidens,

internos ad eaſdemque partes angulos B A D, A B C duobus rectis minores faciat, duæ illæ rectæ lineæ in infinitum productæ ſibi mutuo incident ad eas partes, ubi ſunt anguli duobus rectis minores.

14. Duæ rectæ lineæ ſpatium non comprehendunt.

15. Si æqualibus inæqualia adjiciantur, erit totorum exceſſus adjunctorum exceſſui æqualis.

16. Si inæqualibus æqualia adjungantur, erit totorum exceſſus exceſſui eorum, quæ à principio, æqualis.

17. Si ab æqualibus inæqualia demantur, erit reſiduorum exceſſus, exceſſui ablatorum æqualis.

18. Si ab inæqualibus æqualia demantur, erit reſiduorum exceſſus exceſſui totorum æqualis.

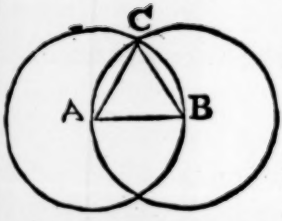
19. Omne totum æquale eſt omnibus ſuis partibus ſimul ſumptis.

20. Si totum totius eſt duplum, & ablatum ablati, erit & reliquum reliqui duplum. Idem de reliquis multiplicibus intellige.

Citationes intellige ſic. Cum duo numeri occurrunt, prior deſignat propoſitionem, poſterior librum. Ut per 4. 1. intelligitur quarta propoſitio primi libri, atque ita de reliquis. *Cæterum ax. axioma, poſt. poſtulatum, def. definitionem, ſch. ſcholium, cor. corollarium denotant, &c.*

LIB. I.

PROP. I.



Super data recta linea terminata A B *triangulum æquilaterum* A B C *constituere.*

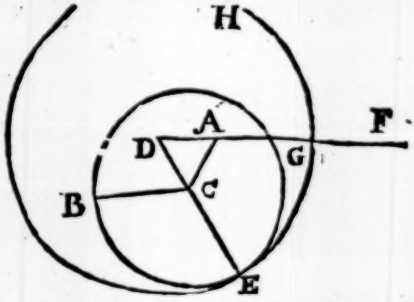
Centris A & B, eodem intervallo A B, vel B A *a* describe duos circulos se intersecantes in puncto C, ex quo *b* duc rectas C A, C B, Erit A C *c* = A B *c* = B C *d* = A C *e* Quare triangulum A C B est æquilaterum. Quod Erat Faciendum.

a 3. *post.*
b 1. *post.*
c 15. *def.*
d 1. *ax.*
e 23. *def.*

Scholium.

Eodem modo super A B describetur triangulum Isosceles, si intervalla æqualium circulorum majora sumantur, vel minora, quam A B.

PROP. II.



Ad datum punctum A *datæ rectæ lineæ* B C *æqualem rectam lineam* A G *ponere.*

Centro C, intervallo C B *a* describe circulum C B E. *b* Junge A C, super qua *c* fac triangulum æquilaterum A D C *d* produc D C ad E. cen-

a 3. *post.*
b 1. *post.*
c 1. 1.
d 2. *post.*

centro D, ſpatio DE, *a* deſcribe circulum DEH: cujus circumferentiæ occurrat D A e protracta ad G. Erit A G = C B.

e 2. *poſt.*
f 15. *def.*
g *conſtr.*

Nam D G *f* = D E, & D A *g* = DC. quare A G *h* = C E *k* = B C *l* = A G. Q.E.F

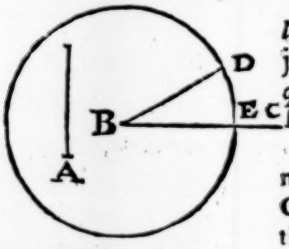
h 3 *ax.*
k 15. *def.*
l 1. *ax.*

Poſitio puncti A, intra vel extra datam B C, caſus variat, ſed ubique ſimilis eſt conſtructio, & demonſtratio.

Scholium.

Poterat A G circino ſumi, ſed hoc facere nulli poſtulato reſpondet, ut bene innuit Proclus.

PROP. III.



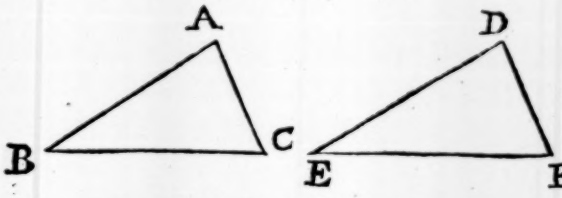
Duabus datis rectis lineis A, *&* BC, *de majore* BC *minori* A *æqualem rectam lineam* B E *detrahere.*

a 2. 1.

Ad punctum B *a* pone rectam B D = A. Circulus centro B, ſpatio B D deſcriptus auferet B E *b* = B D *c* = A *d* = B E, Q.E.F.

b 15. *def.*
c *conſtr.*
d 1. *ax.*

PROP. IV.



Si duo triangula B A C, E D F *duo latera* B A, A C *duobus lateribus* E D, D F *æqualia habeant, utrumque utrique* (hoc eſt B A = E D, *&* AC = D F) *habeant vero angulum* A, *angulo* D *æqualem,*

lem, ſub æqualibus rectis lineis contentum, & baſim B C *baſi* E F *æqualem habebunt; eritque triangulum* B A C *triangulo* E D F *æquale, ac reliqui anguli* B, C *reliquis angulis* E, F *æquales erunt, uterque utrique, ſub quibus æqualia latera ſubtendunt.*

Si punctum D puncto A applicetur, & recta D E rectæ A B ſuperponatur, cadet punctum E in B, quia D E *a* = A B. Item recta D F cadet in A C, quia ang. A *a* = D. Quinetiam punctum F puncto C coincidet, quia A C *a* = D F. Ergo rectæ E F, B C, cum eoſdem habeant terminos, *b* congruent, & proinde æquales ſunt. Quare triangula B A C, E D F; & anguli B, E; itemque anguli C, F etiam congruunt, & æquantur. Quod erat Demonſtrandum.

a *hyp.*

b 14 *ax.*

PROP. V.

Iſoſcelium triangulorum A B C *qui ad baſim ſunt anguli* A B C, A C B *inter ſe ſunt æquales. Et productis æqualibus rectis lineis* A B, A C *qui ſub baſe ſunt anguli* C B D, B C E *inter ſe æquales erunt.*

a Accipe A F = A D, & *b* junge C D, ac B F.

Quoniam in trianguls A C D, A B F, ſunt A B *c* = A C, & A F *d* = A D, anguluſque A communis, *e* erit ang. A B F = A C D; & ang. A F B *e* = A D C, & bas. B F *e* = D C; item F C *f* = D B. ergo in triangulis B F C, B D C *g* erit ang. F C B, = D B C. Q. E. D. Item ideo ang. F B C = D C B. atqui ang. A B F *h* = A C D. ergo ang. A B C *k* = A C B. Q. E. D.

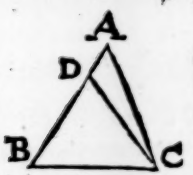
a 3. 1.
b 1 *poſt.*
c *hyp.*
d *conſtr.*
e 4. 1.
f 3 *ax.*
g 4. 1.
h *pr.*
k 3. *ax.*

Corollarium.

Hinc, Omne triangulum æquilaterum eſt quoque æquiangulum.

PROP.

PROP. VI.



Si trianguli A B C *duo anguli* A B C, A C B *æquales inter se fuerint, & sub æqualibus angulis subtensa latera* A B, A C *æqualia inter se erunt.*

Si fieri potest, sit utravis B A ⌐ C A, *a* Fac igitur B D = C A, & *b* duc C D.

a 3. 1.
b 1. *post.*
c *suppos.*
d *hyp.*
e 4. 1.
f 9. *ax.*

In triangulis D B C, A C B, quia B D *c* = C A, & latus B C commune est; atque ang. D B C *d* = A C B, *e* erunt triangula D B C, A C B æqualia inter se, pars & totum, *f* Quod Fieri Nequit.

Coroll.

Hinc, Omne triangulum æquiangulum est quoque æquilaterum.

PROP VII.

Super eadem rectâ lineâ A B *duabus eisdem rectis lineis* A C, B C, *aliæ duæ rectæ lineæ æquales* A D, B D, *utraque utrique* (*hoc est*, A D = A C, & B D = B C) *non constituentur ad aliud punctum* C, *atque aliud* D, *ad easdem partes* C, *eosdemque terminos* A, B *cum duabus initio ductis rectis lineis habentes.*

a 9. *ax.*

1. *Cas.* Si punctum D statuatur in A C *a* liquet non esse A D = A C.

2. *Cas.* Si punctum D dicatur intra triangulum A C B duc C D, & produc B D F, ac B C E.

b 5. 1.
c *suppos.*

Jam vis A D = A C, ergo ang. A D C *b* = A C D; item quia B D, *c* = B C, erit ang. F D C *b* = E C D

ergo

ergo ang. F D C *d* ⌐ A C D, id est ang. F D C d 9. *ax.*
⌐ A D C *d* Q. F. N.

3. *Cas.* Sin D cadat extra triangulum A C B jungatur C D.

Rursus, ang. A C D *e* = A D C, & B C D *e* = e 5. 1.
B D C *f* ergo ang. A C D ⌐ B D C, id est ang. f 9. *ax.*
A C D ⌐ B C D. Q. F. N,

PROP. VIII.

Si duo triangula A B C, D E F *habuerint duo latera* A B, A C *duobus lateribus* D E, D F, *utrumque utrique æqualia; habuerint vero & basim* B C, *basi* E F, *æqualem: angulum* A *sub æqualibus rectis lineis contentum angulo* D *æqualem habebunt.*

Quia B C *a* = E F, si basis B C superponatur basi E F, illæ *b* congruent. ergo, cum A B *c* = D E, & A C *c* = D F, cadet punctum A in D, (nam in aliud punctum cadere nequit, per præcedentem) *d* ergo angulorum A, & D latera coincidunt. *e* quare anguli illi pares sunt. Q. E. D.

a *hyp.*
b 8. *ax.*
c *hyp.*
d 14. *ax.*
e 8. *ax.*

Coroll.

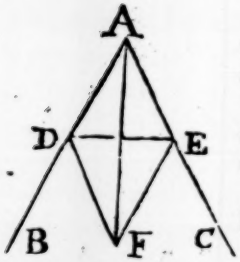
1. Hinc triangula sibi mutuo æquilatera, etiam mutuo x æquiangula sunt.

2. Triangula sibi mutuo æquilatera y æquentum inter se.

x 4 8.
y 4 1.

PROP.

PROP. IX.



Datum angulum rectilineum B A C *bifariam secare.*

a 3. 1. b 1. 1. *a* Sume A D = A E; duc D E, super qua *b* fac triang. æquilat. D F E. Ducta A F angulum B A C bisecabit.

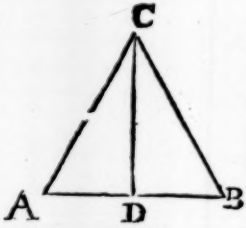
c *constr.* d 8. 1. Nam A D *c* = A E, & latus A F commune est, & bas. D F *c* = F E, *d* ergo ang. D A F = E A F. Q. E. F.

Coroll.

Hinc patet quomodo angulus secari possit in æquales partes 4, 8, 16, &c. Singulas nimirum partes iterum bisecando.

Methodus vero regula & circino angulos secandi in æquales quotcunque hactenus Geometras latuit.

PROP. X.



Datam rectam lineam A B *bifariam secare.*

a 1. 1. b 9. 1. Super data A B *a* fac triang. æquilat. A B C, ejus angulum C *b* bisecta recta C D. Eadem datam A B bisecabit.

c *constr.* d 4. 1. Nam A C *c* = B C, & latus C D est commune; & ang. A C D *c* = B C D, *d* ergo A D = B D. Q. E. F. Praxin hujus & præcedentis, constructio primæ hujus libri satis indicat.

PROP.

PROP. XI.

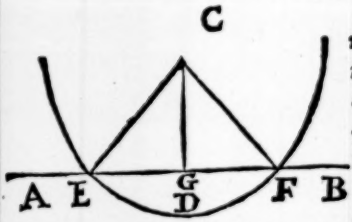
Data recta linea A B, *& puncto in ea dato* C, *rectam lineam* C F *ad angulos rectos excitare.*

a Accipe hinc inde C D = C E. Super D E *b* fac triang æquilat. D F E. Ducta F C perpendicularis eſt. a 3. 1. b 1. 1.

Nam triangula D F C, E F C ſibi mutuo *c* æquilatera ſunt. *d* ergo ang. D C F = E C F. *e* ergo F C perpendicularis eſt. Q. E. F. c *conſtr.* d 8. 1. e 10. *def.*

Praxis tam hujus, quam ſequentis expeditur facillime ope normæ.

PROP. XII.



Super datam rectam lineam infinitam A B, *à dato puncto* C *quod in ea non eſt perpendicularem rectam* C G *deducere.*

Centro C *a* deſcribe circulum, qui ſecet datam A B in punctis E & F *b* biſeca E F in G. ducta C G perpendicularis eſt. a 3. *poſt.* b 10. 1.

Ducantur enim C E, C F. Triangula E G C, F G C, ſibi mutuo *c* æquilatera ſunt. *d* ergo anguli E G C, F G C, æquales, & *e* proinde recti ſunt. Q. E. F. c *conſtr.* d 8. 1. e 10. *def.*

PROP. XIII.

Cum recta linea A B, *ſuper rectam lineam* C D *conſiſtens, facit angulos* A B C, A B D, *aut duos rectos, aut duobus rectis æquales efficiet.*

Si

a 10. *def.* b 11. 1. c 19 *ax.* d 3. *ax.* e 2. *ax.*

Si anguli A B C, A B D pares ſint *a* liquet illos rectos eſſe; ſin inæquales ſint, ex B *b* excitetur perpendicularis B E. Quoniam ang. A B C *c* = Rect. + A B E; & ang. A B D *d* = Rect. − A B E; erit A B C + A B D *e* = 2 Rect. + A B E − A B E = 2 Rect. Q. E. D.

Coroll.

1. Hinc, ſi unus ang. A B D rectus ſit, alter A B C etiam rectus erit; ſi hic acutus, ille obtuſus erit, & contra.

2. Si plures rectæ quam una ad idem punctum eidem rectæ inſiſtant, anguli fient duobus rectis æquales

3. Duæ rectæ invicem ſecantes efficiunt angulos quatuor rectis æquales.

4. Omnes anguli circa unum punctum conſtituti conficiunt quatuor rectos, patet ex Coroll. 2.

PROP. XIV.

Si ad aliquam rectam lineam A B, *atque ad ejus punctum* B *duæ rectæ lineæ* C B, B D *non ad eaſdem partes ductæ, eos qui ſunt deinceps angulos* A B C, A B D *duobus rectis æquales fecerint, in directum erunt inter ſe ipſæ rectæ lineæ* C B, B D.

Si negas, faciant C B, B E unam rectam, ergo ang. A B C + A B E *a* = 2 Rect. *b* = A B C + A B D. *c* Quod eſt abſurdum.

a 13. 1. b *hyp.* c 9. *ax.*

PROP. XV.

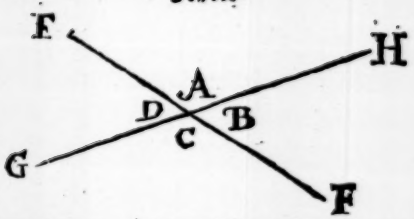
Si duæ rectæ lineæ A B, C D *ſe mutuo ſecuerint, angulos ad verticem* C E B, A E D *æquales inter ſe efficient.*

Nam ang. A E C + C E B *a* = 2 Rect. *a* = A E C + A E D. *b* Ergo C E B = A E D. Q. E. F.

a 13. 1. b 3. *ax.*

Schol.

Schol.



Si ad aliquam rectam lineam G H, atque ad ejus punctum, A duæ rectæ lineæ E A, A F non ad eaſdem partes ſumptæ, angulos ad verticem D, & B æquales fecerint, ipſæ rectæ lineæ E A, A F in directum ſibi invicem erunt.

Nam 2 Rect. = *a* D + A *a* = B + A *b* ergo E A, A F ſunt in directum ſibi invicem. Q.E.D.

a 13. 1.
b 14. 1.

Schol. 2.

Si quatuor rectæ lineæ E A, E B, E C, E D ab uno puncto E exeuntes, angulos oppoſitos ad verticem æquales inter ſe fecerint, erunt quælibet duæ lineæ A E, E B, & C E, E D in directum poſitæ.

Nam quia ang. A E C + A E D + C E B + D E B *a* = 4 Rect. erit A E C + A E D (= *b* C E B + D E B) = 2 Rect. *c* ergo C E D, & A E B ſunt rectæ lineæ. Q. E. D.

a 4. *Cor.* 13. 1.
b *Hyp.* 2 *ax.*
c 14. 1.

PROP. XVI.

Cujuſcunque Trianguli A B C *uno latere* B C *producto, externus angulus* A C D *utrolibet interno & oppoſito* C A B, C B A, *major eſt.*

Latera A C, B C *a* biſecent rectæ A H, B E, è quibus productis *b* cape E F = B E, *b* & H I = A H, Conjunganturque F C, I C, & producatur A C G.

a 10. 1. & 1. *poſt.*
b 3. 1.

c *constr.* Quoniam C E *c* = E A, & E F *c* = E B, &
d 15. 1. ang. F E C *d* = B E A, *e* erit ang. E C F = E A B.
e 4. 1. Simili argumento ang. I C H = A B H. ergo
f 15. 1. totus A C D (*f* B C G) *g* major est utrovis C A B,
g 9. *ax.* & A B C. Q. E. D.

PROP. XVII.

Cujuscunque trianguli A B C *duo anguli duobus rectis sunt minores, omnifariam sumpti.*

Producatur latus B C. Quoniam ang. A C D +
a 13. 1. A C B *a* = 2 Rect. & ang.
b 16. 1. A C D *b* ⊏ A, *c* erit A + A C B ⊐ 2 Rect. Eo-
c 4. *ax.* dem modo erit ang. B + A C B ⊐ 2 Rect. Denique producto latere A B, erit similiter ang. A + B ⊐ 2 Rect. Quæ E. D.

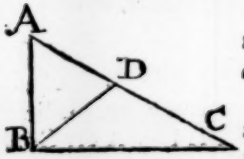
Coroll.

1. Hinc, in omni triangulo, cujus unus angulus fuerit rectus, vel obtusus, reliqui acuti sunt.

2. Si linea recta A E cum alia recta C D angulos inæquales faciat, unum A E D acutum, & alterum A E C obtusum, linea perpendicularis A D ex quovis ejus puncto A ad aliam illam C D demissa, cadet ad partes anguli acuti A E D. Nam si A C ad partes anguli obtusi ducta, dicatur perpendicularis, in triangulo A F C erit
x 17. 1. ang. A E C + A C E ⊏ 2 Rect. x Q. F. N.

3. Omnes anguli trianguli æquilateri, & duo anguli trianguli Isoscelis, supra basim, acuti sunt

PROP. XVIII.



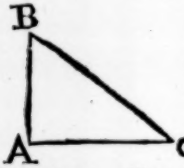
Omnis trianguli A B C *majus latus* A C *majorem angulum* A B C *subtendit.*

Ex A C *a* auser A D = A B, & junge D B. *b* ergo
a 3. 1. ang. A D B = A B D. Sed
b 5. 1. *c* A D B

c A D B ⊏ C. ergo A B D ⊏ C. *d* ergo totus ang. A B C ⊏ C. Eodem modo erit A B C ⊏ A. Q. E D.

c 16. 1.
d 9. *ax.*

PROP. XIX.

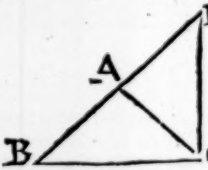


Omnis trianguli A B C *major angulus* A *majori lateri* B C *subtenditur.*

Nam si dicatur A B = B C, *a* erit ang. A = C. contra Hypoth. & si A B ⊏ B C, *b* erit ang. C ⊏ A, contra hyp. quare potius, B C ⊏ A B. & eodem modo B C ⊏ A C. Q. E. D.

a 5. 1.
b 18. 1.

PROP. XX.



Omnis trianguli A B C *duo latera* B A, A C *reliquo* B C *sunt majora quomodocunque sumpta.*

Ex B A producta *a* cape A D = A C, & duc D C. *b* ergo ang. D = A C D. *c* ergo totus B C D ⊏ D *d* ergo B D (*e* B A + A C) ⊏ B C. Q. E. D.

a 3. 1.
b 5. 1.
c 9. *ax.*
d 19. 1.
e *constr.* & 2. *ax.*

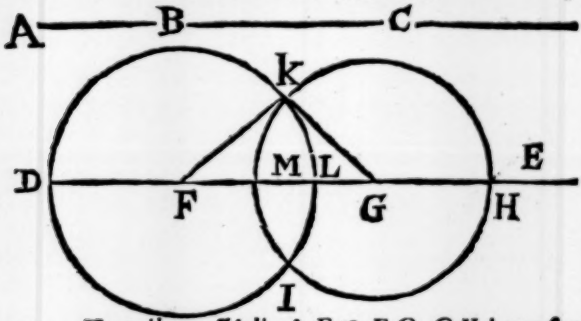
PROP. XXI.

Si super trianguli A B C *uno latere* B C, *ab extremitatibus duæ rectæ lineæ* B D, C D, *interius constitutæ fuerint, hæ constitutæ reliquis trianguli duobus lateribus* B A, C A *minores quidem erunt, majorem vero angulum* B D C *continebunt.*

Producatur B D in E. estque C E + E D *a* ⊏ C D adde commune B D, *b* erit B E + E C ⊏ B D + D C. Rursus B A + A E *a* ⊏ B E; *b* ergo B A + A C ⊏ B E + E C. quare B A + A C ⊏ B D + D C. Q. E. D. 2. Ang. B D C *c* ⊏ D E C *c* ⊏ A. ergo ang. B D C ⊏ A. Q. E. D.

a 20. 1.
b 4. *ax.*
c 16. 1.

PROP. XXII.



Ex tribus rectis lineis F K, F G, G K, *quæ sint tribus datis rectis lineis* A, B, C, *æquales, triangulum* F K G *constituere. Oportet autem duas reliqua esse majores omnifariam sumptas; quoniam uniuscujusque trianguli duo latera omnifariam sumpta reliquo sunt majora.*

3. 1. Et infinita D E *a* sume D F, F G, G H datis
3. *post.* A, B, C ordine æquales. Tum si *b* centris F, & G, intervallis F D, & G H ducantur circuli se intersecantes in K; junctis rectis K F, K G constituetur triangulum F K G, *c* cujus latera F K,
c 15. *def.* F G, G K tribus D F, F G, G H, *d* id est tribus
d 1. *ax.* datis A, B, C æquantur. Q. E. F.

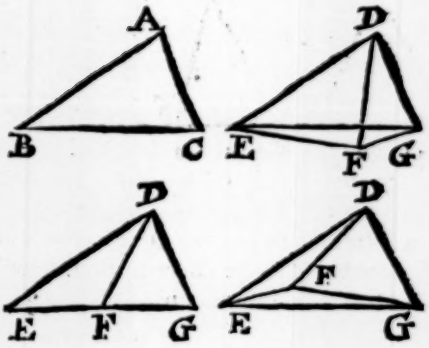
PROP. XXIII.

Ad datam rectam lineam A B, *datumque in ea punctum* A, *dato angulo rectilineo* D *æqualem angulum rectilineum* A *constituere.*

a 1. *post.* *a* Duc rectam C F secantem dati anguli latera utcunque, *b* Fac A G = C D. Super A G
b 3. 1. *c* constitute triangulum alteri C D F æquilaterum, ita ut A H = D F, & G H = C F; & ha-
c 22. 1. bebis ang. A *d* = D. Q. E. F.
d 8. 1.

PROP.

PROP. XXIV.



Si duo triangula A B C, D E F *duo latera* A B, A C *duobus lateribus* D E, D F *æqualia habuerint, utrumque utrique; angulum vero* A *angulo* E D F *majorem ſub æqualibus rectis lineis contentum, & baſim* B C, *baſi* E F, *majorem habebunt.*

a Fiat ang. E D G = A, & D G *b* = D F *c* = A C, connectanturque E G, F G.

1. *Caſ.* Si E G cadit ſupra E F. Quia A B *d* = D E, & A C = *e* D G, & ang. A *e* = E D G, *f* erit B C = E G. Quia vero D F *e* = D G, *g* erit ang. D F G = D G F. *h* ergo ang. D F G ⊏ E G F; *h* & proinde ang. E F G ⊏ E G F. *k* quare E G (B C) ⊏ E F. Q. E. D.

2. *Caſ.* Si baſis E F baſi E G coincidat, *l* liquet E G (B C) ⊏ E F.

3. Sin E G cadat infra E F. Quoniam D G + G E *m* ⊏ D F + F E, ſi hinc inde auferantur D G, D F, æquales, manet E G (B C) *n* ⊏ E F. Q. E. D.

a 23. 1.
b 3. 1.
c *hyp.*
d *hyp.*
e *conſtr*
f 4. 1.
g 5. 1.
h 9. *ax.*
k 19. 1.
l 9. *ax.*
m 21. 1.
n 5. *ax.*

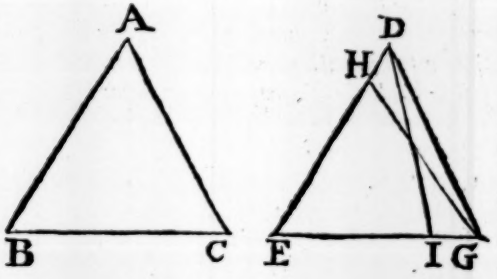
PROP. XXV.

Si duo triangula A B C, D E F *duo latera* A B, A C *duobus lateribus* D E, D F *æqualia habuerint, utrumque utrique, basim vero* B C *basi* E F *majorem; & angulum* A *sub æqualibus rectis lineis contentum angulo* D *majorem habebunt.*

a 4. 1. Nam si dicatur ang. A = D. *a* erit basis B C = E F, contra Hyp. Sin dicatur ang. A ⊐ D.

b 24. 1. *b* erit B C ⊐ E F, etiam contra Hyp. ergo B C ⌐ E F. Q. E. D.

PROP XXVI.



Si duo triangula B A C, E D G, *duos angulos* B, C, *duobus angulis* E, D G E, *æquales habuerint, utrumque utrique, unumque latus uni lateri æquale, sive quod æqualibus adjacet angulis, seu quod uni æqualium angulorum subtenditur: reliqua latera reliquis lateribus æqualia, utrumque utrique, & reliquum angulum reliquo angulo æqualem habebunt.*

1. *Hyp.* Sit B C = E G. Dico B A = E D, & A C = D G, & ang. A = E D G. Nam si dicatur

b 3. 1. E D ⌐ B A, *a* fiat E H = B A, ducaturq; G H.

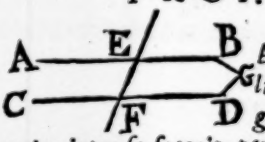
Quoniam

Quoniam A B *b* = H E, & B C *c* = E G, & ang. B *c* = E, erit ang. E G H *d* = C *e* = D G E. *f* Q. E. A. ergo A B = E D. Eodem modo A C = D G. *d* quare etiam ang. A = E D G.

2. *Hyp.* Sit A B = E D. Dico B C = E G, & A C = D G & ang. A = E D G. Nam si dicatur E G ⊏ B C, fiat E I = B C, & connectatur D I. Quia A B *g* = E D, & B C *h* = E I, & ang. B *g* = E, erit ang. E I D *k* = C *m* = E G D. *n* Q. E. A. ergo B C = E G. ergo ut prius, A C = D G, & ang. A = E D G. Q. E. D.

b *suppos.* c *hyp.* d 4. 1. e *hyp.* f 9. *ax.* g *hyp.* h *suppos.* k 4. 1. m *hyp.* n 16. 1.

PROP. XXVII.



Si in duas rectas lineas A B, C D *recta incidens linea* E F *alternatim angulos* A E F, D F E, *æquales inter se fecerit, parallelæ erunt inter se illæ rectæ lineæ* A B, C D.

Si A B, C D dicantur non esse parallelæ; conveniant productæ, nempe in G. quo posito angulus externus A E F interno D F E *a* major erit, cui tamen ponitur æqualis. Quæ repugnant.

a 16. 1.

PROP. XXVIII.

Si in duas rectas lineas A B, C D *recta incidens linea* E F *externum angulum* A G E *interno & opposito, & ad easdem partes* C H G *æqualem fecerit, aut internos & ad easdem partes* A G H, C H G *duobus rectis æquales; parallelæ erunt inter se ipsæ rectæ lineæ* A B, C D.

1. *Hyp.* Quia per hyp. ang A G E = C H G, *a* erit altern. B G H = C H G. *b* parallelæ igitur sunt A B, C D. Q. E. D.

2. *Hyp.* Quia ex hyp. Ang. A G H + C H G = 2 Rect. *a* = A G H + B G H, *b* erit C H G = B G H. Ergo *c* A B, C D parallelæ sunt. Q. E. D.

a 15. 1. b 17. 1. a 13. 1. b 3. *ax.* c 17. 1.

PROP. XXIX.

In parallelas rectas lineas A B, C D, *recta incidens linea* E F, *& alternatim angulos* D H G, A G H *æquales inter se efficit; & externum* B G E *interno, & opposito, & ad easdem partes* D H E *æqualem; & internos & ad easdem partes* A G H, C H G *duobus rectis æquales facit.*

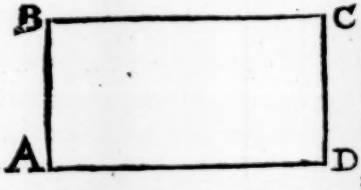
a 13. ax. Liquet A G H, + C H G = 2 Rect. *a* alias

b 13. 1. A B, C D non essent parallelæ, contra hyp. Sed

c 13. ax. & ang. D H G + C H G *b* = 2 Rect. ergo D H G

d 15. 1. *c* = A G H *d* = B G E. Q. E. D.

Coroll.



Hinc omne Parallelogrammum A C habens unum angulum rectum

a 29. 1. A, est rectangulum.

b 3. ax. Nam A + B *a* = 2 Rect. ergo cum A rectus sit, *b* etiam B rectus erit. Eodem argumento D, & C recti sunt.

PROP. XXX.

Quæ (A B, C D) *eidem rectæ lineæ* E F *parallelæ, & inter se sunt parallelæ.*

Tres rectas secet utcunque recta G I. Quo-

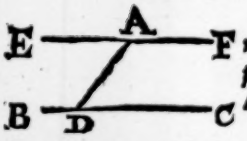
a 29. 1. niam A B, E F parallelæ sunt, *a* erit ang. A G I = E H I, Item propter C D, E F parallelas,

b 1. ax. *a* erit ang. E H I = D I G. *b* ergo ang. A G I =

c 27. 1. D I G, *c* quare A B, C D parallelæ sunt. Q. E. D.

PROP.

PROP. XXXI.

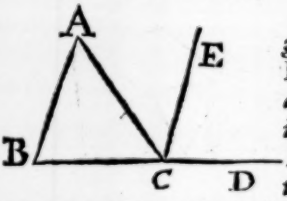


A *dato puncto* A *datæ rectæ lineæ* B C *ducere parallelam rectam lineam* A E.

Ex A ad datam B C duc rectam utcunque A D. ad quam, ejusque punctum A *a* fac ang. D A E = A D C. *b* erunt A E, B C parallelæ. Q. E. F.

a 23. I.
b 27. I.

PROP. XXXII.



Cujusçunque trianguli A B C *uno latero* B C *producto, externus angulus* A C D *duobus internis, & oppositis,* A, B *est æqualis. Et trianguli tres interni anguli,* A, B, A C B *duobus sunt rectis æquales.*

Per C *a* duc C E parall. B A. Ang. A *b* = A C E. & ang. B *b* = E C D. ergo A + B *c* = A C E + E C D *d* = A C D. Q. E. D. Porro A C D + A C B *e* = 2 Rect. *f* ergo A + B + A C B = 2 Rect. Q. E. D.

a 31. I.
b 29. I.
c 2. *ax.*
d 19. *ax.*
e 13. I.
f 1. *ax.*

Corollaria.

1. Tres simul anguli cujusvis trianguli æquales sunt tribus simul cujuscunque alterius. Unde

2. Si in uno triangulo duo anguli (aut singuli, aut simul) æquales sint duobus angulis (aut singulis, aut simul) in altero triangulo, etiam reliquus reliquo æqualis est. Item, si duo triangula unum angulum uni æqualem habeant, reliquorum summæ æquantur.

3. In triangulo si unus angulus rectus sit, reliqui unum rectum conficiunt. Item, angulus, qui duobus reliquis æquatur, rectus est.

4. Cum in Isoscele angulus æquis cruribus contentus rectus est, reliqui ad basim sunt semirecti.

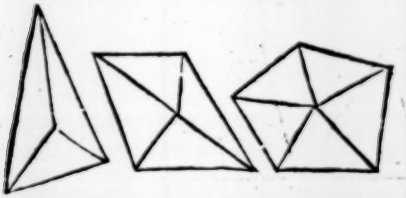
5. Tri-

5. Trianguli æquilateri angulus facit duas tertias unius recti, nam $\frac{1}{3}$ 2 Rect. $=\frac{2}{3}$ Rect.

Schol.

Hujus propositionis beneficio, cujuslibet figuræ rectilineæ tam interni quam externi anguli quot rectos conficiant, innotescet per duo sequentia theoremata.

THEOREMA 1.



Omnes simul anguli cujuscunque figuræ rectilineæ conficiunt bis tot rectos demptis quatuor, quot sunt latera figuræ.

Ex quovis puncto intra figuram ducantur ad omnes figuræ angulos rectæ, quæ figuram resolvent in tot triangula quot habet latera. Quare cum singula triangula conficiant duos rectos, omnia simul conficient bis tot rectos, quot sunt latera. Sed anguli circa dictum punctum conficiunt quatuor rectos Ergo, si ab omnium triangulorum angulis demas angulos circa id punctum, anguli reliqui qui componunt angulos figuræ conficient bis tot rectos demptis quatuor, quot sunt latera figuræ. Q. E. D.

Hinc *Coroll.* Omnes ejusdem speciei rectilineæ figuræ æquales habent angulorum summas.

THEOREMA 2.

Omnes simul externi anguli cujuscunque figuræ rectilineæ conficiunt quatuor rectos.

Nam singuli figuræ interni anguli cum singulis externis conficiunt duos rectos. Ergo interni

terni ſimul omnes, cum omnibus ſimul externis conficiunt bis tot rectos, quot ſunt latera figuræ. Sed (ut modo oſtenſum eſt,) interni ſimul omnes etiam cum quatuor rectis efficiunt bis tot rectos quot ſunt latera figuræ. Ergo externi anguli quatuor rectis æquantur. Q. E. D.

Coroll. Omnes cujuſcunque ſpeciei rectilineæ figuræ æquales habent externorum angulorum ſummas.

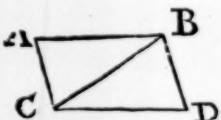
PROP. XXXIII.

Rectæ lineæ A C, B D, *quæ æquales & parallelas lineas* A B, C D, *ad partes eaſdem dem conjungunt, & ipſæ æquales ac parallelæ ſunt.*

Connectatur C B. Quoniam ob A B, C D parallelas, ang. A B C *a* = B C D, & per hyp. A B = C D, & latus C B commune eſt, *b* erit A C = B D, *b* & ang. A C B = D B C. *c* ergo A C, B D etiam parallelæ ſunt. Q. E. D.

a 29 1.
b 4. 1.
c 27. 1.

PROP. XXXIV.



Parallelogrammorum ſpatiorum A B D C *æqualia ſunt inter ſe quæ ex adverſo latera* A B, C D; ac A C, B D; *angulique* A, D, & A B D, A C D; *& illa biſariam ſecat diameter* C B.

Quoniam A B, C D *a* parallelæ ſunt, *b* erit ang. A B C = B C D. Item ob A C, D B *a* parallelas, *b* erit ang. A C B = C B D. *c* ergo toti anguli A C D, A B D æquantur. Similiter ang. A = D. Porro, cum communi lateri C B adjaceant anguli A B C, A C B, ipſi, B C D, C B D pares, *d* erunt A C = B D, *d* & A B = C D. adeoque etiam triang. A B C = C B D. Q. E. D.

a *hyp.*
b 29. 1.
c 2. *ax.*
d 26. 1.

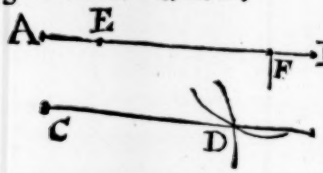
SCHOL.

SCHOL.

Omne quadrilaterum A B DC *habere latera oppoſita æqualia, eſt parallelogrammum.*

a 27. 1. Nam per 8. 1. ang. A B C = B C D. *a* ergo A B, C D parallelæ ſunt. Eadem ratione ang. B C A = C B D; *a* quare A C, B D etiam parallelæ ſunt. *b* Ergo A B D C eſt parallelogrammum. Q. E. D.

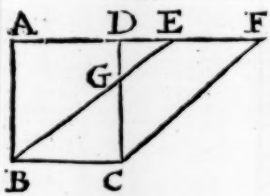
b 35. *def.* 1



Hinc expeditius per datum punctum C datæ rectæ A B ducetur parallela C D.

Sume in A B quodvis punctum E. centris E. & C ad quodvis intervallum duc æquales circulos E F, C D. centro vero F, ſpatio E C duc circulum F D, qui priorem C D ſecet in D. Erit ducta C D parall. A B. Nam ut modo demonſtratum eſt, C E F D eſt parallelogrammum.

PROP. XXXV.

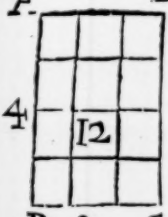


Parallelogramma B C D A, B C F E *ſuper eadem baſi* B C, *& in eiſdem parallelis* A F, B C *conſtituta, inter ſe ſunt æqualia.*

a 34. 1. Nam A D *a* = B C
b 2. *ax.* *a* = E F. adde communem D E, *b* erit A E =
c 29. 1. D F. Sed & A B *a* = D C, & ang. A *c* = C D F.
d 4. 1. *d* ergo triang. A B E = D C F. aufer commune
e 3. *ax.* D G E, *e* erit Trapez. A B G D = E G C F.
f 2. *ax.* adde commune B G C, *f* erit Pgr. A B C D = E B C F, Q. E. D. Reliquorum caſuum non diſſimilis, ſed ſimplicior & facilior eſt demonſtratio.

Scho-

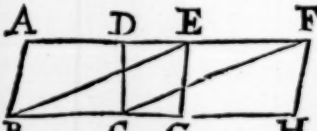
Scholium.



Si latus A B parallelogrammi rectanguli A B C D ferri intelligatur perpendiculariter per totam B C, aut B C per totam A B, producetur eo motu area rectanguli A B C D. Hinc rectangulum fieri dicitur ex ductu seu multiplicatione duorum laterum contiguorum. Sit exempl. gr. B C pedum 3, A B 4. Duc 3. in 4; proveniunt 12. pedes quadrati pro area rectanguli.

Hoc suppositio, ex hoc theoremate cujuscunq; parallelogrammi (* E B C F) habetur dimensio. Illius enim area producitur ex altitudine B A ducta in basim B C. Nam area rectanguli A C parallelogrammo E B C F æqualis, fit ex B A in B C, ergo, &c. * *v. fig. propos. 35.*

PROP. XXXVI.

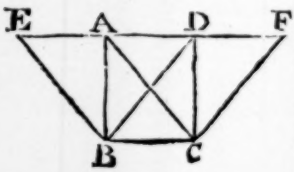


Parallelogramma B C D A, G H F E *super æqualibus basibus* B C G H, *& in eisdem parallelis* A F, B H *constituta, inter se sunt æqualia.*

Ducantur B E, C F. Quia B C *a* = G H *b* = E F, *c* erit B C F E parallelogrammum. ergo Pgr. B C D A *d* = B C F E *d* = G H F E. Q.E.D.

a *hyp.* b 34. 1. c 33. 1. d 35. 1.

PROP. XXXVII.



Triangula B C A, B C D *super eadem basi* B C *constituta, & in eisdem parallelis* B C, E F *inter se sunt æqualia.*

a Duc

a 31. 1. a Duc B E parall. C A, a & C F parall, B D;
b 34. 1. Erit triang. B C A b = ½ Pgr. B C A E = c ½
c 35. 1. & 7. ax. B D F C b = B C D. Q. E. D.

PROP. XXVIII.

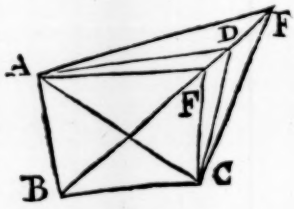
Triangula B C A, E F D *ſuper æqualibus baſibus* B C, E F *conſtitutæ, & in eiſdem parallelis* G H, B F, *inter ſe ſunt æqualia.*

Duc B G parall. C A. & F H parall. E D. erit triang. B C A a = ½ Pgr. B C A G b = ½
a 14. 1. b 36. 1 & 7. ax. E D H F c = E F D. Q. E. D.
c 34. 1.

Schol.

Si baſis B C ⊏ E F, liquet triang. B A C ⊏ E D F. & ſi B C ⊐ E F, erit B A C ⊐ E D F.

PROP. XXXIX.

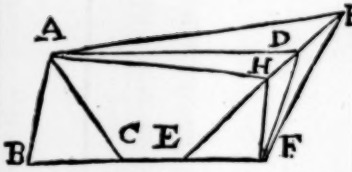


Triangula æqualia B C A, B C D, *ſuper eadem baſi* B C, *& ad eaſdem partes conſtituta, etiam in eiſdem ſunt parallelis* A D, B C.

Si negas, ſit altera A F parall. B C; & ducatur C F. ergo triang. C B F a = C B A b = C B D
a 37. 1. b *hyp.* c 9. *ax.* c Q. E. A.

PROP.

PROP. XL.

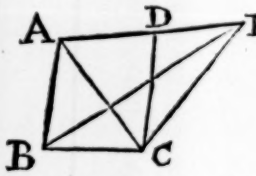


Triangula æqualia B C A, E F D *ſuper æqualibus baſibus* B C, E F, *& ad eaſdem partes conſtituta, & in eiſdem ſunt parallelis* A D. B F.

Si negas, ſit altera A H parall. B F. & ducatur F H. ergo triang. E F H *a* = B C A *b* = E F D. *c* Q. E. A.

a 38. 1.
b *hyp.*
c 9. *ax.*

PROP. XLI.



Si parallelogrammum A B C D *cum triangulo* B C E *eandem baſim* B C *habuerit, in eiſdemque fuerit parallelis* A E, B C, *duplum erit parallelogrammum* A B C D *ipſius trianguli* B C E.

Ducatur A C. Triang. B C A *a* = B C E. ergo Pgr. A B C D *b* = 2 B C A *c* = 2 B C E. Q. E. D.

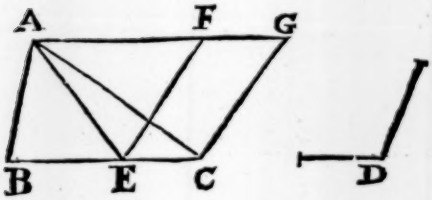
a 37. 1.
b 34. 1.
c 6. *ax.*

Scholium.

Hinc habetur area cujuſcunque trianguli B C E. Nam cum area parallelogrammi A B C D producatur ex altitudine in baſim ducta; producetur area trianguli ex dimidia altitudine in baſim ducta, vel ex dimidia baſi in altitudinem, ut ſi baſis B C ſit 8, & altitudo 7; erit trianguli B C E area, 28.

PROP.

PROP. XLII.

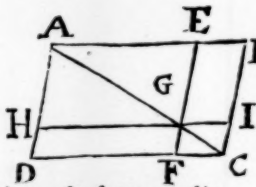


Dato triangulo A B C *æquale parallelogrammum* E C G F *conſtituere in dato angulo rectilineo* D.

a 31. 1. Per A *a* duc A G parall. B C. *b* fac ang. B C G
b 23. 1. = D. baſim B C *c* biſeca in E. *a* duc E F parall.
c 10. 1. C G. Dico factum.

Nam ducta A E. erit ex conſtr. ang. E C G
d 38. 1. = D, & triang. B A C *d* = ½ A E C *e* = ½ Pgr.
e 41. 1. E C G F. Q. E. F.

PROP. XLIII.

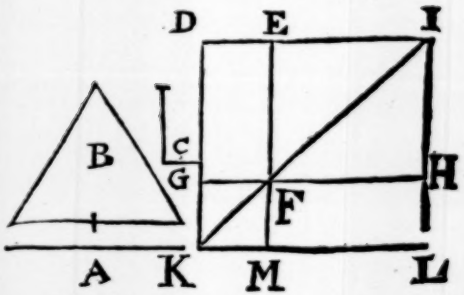


In omni parallelogrammo A B C D *complementa* D G, G B *eorum quæ circa diametrum* A C *ſunt parallelogrammorum* H E, F I *inter ſe ſunt æqualia.*

a 34. 1. Nam Triang. A C D, = *a* A C B, & triang.
A G H *a* = A G E. & triang. G C F *a* = G C I
b 3. *ax.* *b* ergo Pgr. D G = G B. Q. E. D.

PROP.

PROP. XLIV.



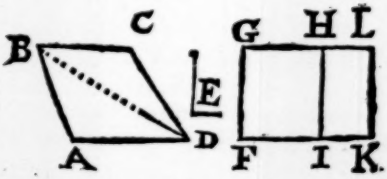
Ad datam rectam lineam A, *dato triangulo* B, *æquale parallelogrammum* F L *applicare in dato angulo rectilineo* C.

a Fac Pgr. F D = triang. B, ita ut ang. GFE = C. & pone lateri G F in directum F H = A. Per H *b* duc I L parall. EF; cui occurrat D E producta ad I. per I F ductæ rectæ occurrat D G protracta ad K. Per K *b* duc K L parall. G H; cui occurrant E F, & I H prolongatæ ad M, & L. Erit F L. Pgr. quæsitum.

Nam Pgr. F L *c* = F D = B *d* & ang. M F H = G F E = C. Q. E. F.

a 42. 1.
b 31. 1.
c 43. 1.
d 15. 1.

PROP. XLV.



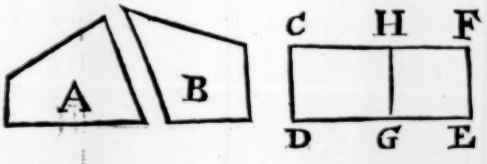
Ad datam rectam lineam F G *dato rectilineo* A B C D *æquale parallelogrammum* F L *constituere, in dato angulo rectilineo* E.

Datum rectilineum resolve in triangula B A D, B C D. *a* Fac. Pgr. F H = B A D ita ut ang F = E. producta F I, *a* fac (ad H I) Pgr.

a 44. 1.

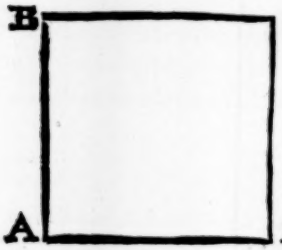
b 19. ax. c Constr. I L = B C D. erit Pgr. F L = *b* F H + I L *c* = ABCD. Q. E. F.

Schol.



Hinc facile invenitur exceſſus H E, quo rectilineum aliquod A ſuperat rectilineum minus B; nimirum ſi ad quamvis rectam CD applicentur Pgr. DF = A. & DH = B.

PROP. XLVI.



A data recta linea A D *quadratum* A C *deſcribere.*

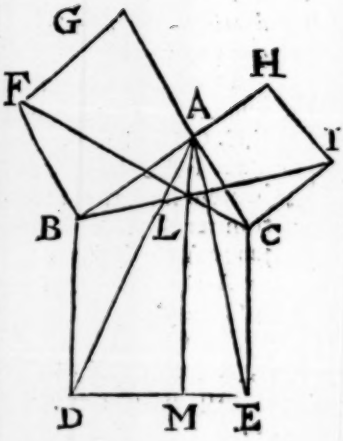
a 11. 1. b 3. 1. *a* Erige duas perpendiculares A B, D C *b* æquales datæ A D; & junge B C. dico factum.

c *conſtr.* d 28. 1. e *conſtr.* f 33. 1. g *Sch.* 29. 1 h 29. *def.* Cum enim ang. A + D *c* = 2 Rect. *d* erunt AB, DC parallelæ. Sunt vero etiam *e* æquales, *f* ergo AD, BC pares etiam ſunt, & parallelæ. ergo Figura A C eſt parallelogramma, & æquilatera. Anguli quoque omnes recti ſunt, *g* quoniam unus A eſt rectus. *h* ergo AC eſt quadratum. Q. E. F.

Eodem modo facile deſcribes rectangulum, quod ſub datis duabus rectis contineatur.

PROP.

PROP. XLVII.



In rectangulis triangulis BAC *quadratum* BE, *quod à latere* BC *rectum angulum* BAC *ſubtendente deſcribitur, æquale eſt eis*, BG, CH, *quæ à lateribus* AB, AC *rectum angulum continentibus deſcribuntur.*

Junge AE, AD; & duc AM. parall. CE.

Quoniam ang. DBC *a* = FBA; adde communem ABC, erit ang. ABD = FBC. Sed & AB *b* = FB, & BD *b* = BC. *c* ergo triang. ABD = FBC. atqui Pgr. BM. *d* = 2 ABD; & Pgr. BG *d* = 2 FBC (nam GAC eſt una recta per hyp. & 14. 1.) *e* ergo Pgr. BM = BG. Similli diſcurſu Pgr. CM = CH. Totum igitur BE = *f* BG + CH. Q. E. D.

a 12. *ax.*
b 29. *def.*
c 4. 1.
d 41. 1.
e 6. *ax.*
f 2. *ax.*

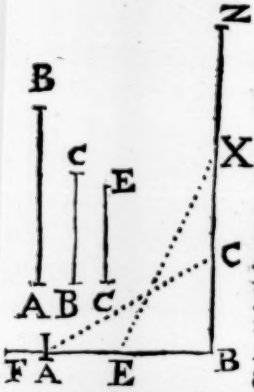
Schol.

Hoc nobiliſſimum, & utiliſſimum theorema ab inventore Pythagora, Pythagoricum dici meruit. Ejus beneficio quadratorum additio, & ſubſtractio perficitur; quò ſpectant duo ſequentia problemata.

PROBL. 1.

And. Tarq.

Datis quotcunque quadratis, unum omnibus æquale construere.

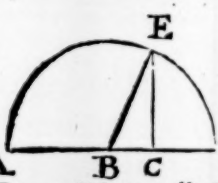


a 11. 1. Dentur quadrata tria, quorum latera sint AB, B C, C E. *a* Fac ang. rectum F B Z infinita habentem latera, in eaque transfer B A, & B C, &

b 47. 2. junge A C, *b* erit A Cq = A Bq + B Cq. Tum A C transfer ex B in X; & C E tertium latus datum transfer ex B in E, & junge E X, *b* erit

c 2. ax. EXq = EBq (CEq) + BXq (ACq) *c* = CEq + ABq + BCq. Q. E. F.

PROBL. 2.



Datis duabus rectis inæqualibus A B, B C, *exhibere quadratum, quo quadratum majoris* A B *excedit quadratum minoris* B C.

Centro B intervallo BA describe circulum. ex C erige perpendicularem C E occurrentem peripheriæ in E. & ducatur B E. *a* Erit B Eq

a 47. 1. (BAq) = BCq + CEq. *b* ergo BAq — BCq =

b 3. ax. CEq. Q. E. F.

PROBL.

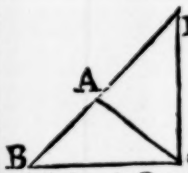
PROBL. 3.

Notis duobus quibuscunque lateribus trigoni rectanguli ABC, *reliquum invenire.*

Latera rectum angulum ambientia sint AC, AB, hoc 6. pedum, illud 8. ergo 47. 1. 8 cum AC q + AB q = 64 + 36 = 100 = BC q. erit BC = √ 100 = 10.

Nota sint deinde latera AB, BC, hoc 10. pedum, illud 6. ergo cum BC q — AB q = 100 — 36 = 64 47. 1. = AC q. erit AC q = √ 64 = 8.

PROP. XLVIII.



Si quadratum quod ab uno latere BC *trianguli describitur, æquale sit eis quæ à reliquis trianguli lateribus* AB, AC *describuntur quadratis, angulus* BAC *comprehensus sub* AB, AC *reliquis duobus trianguli lateribus, rectus est.*

Duc ad AC perpendicularem DA = AB, & junge CD.

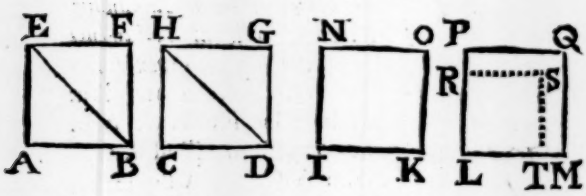
Jam CD q *a* = AD q + AC q = AB q + AC q = BC q. * ergo CD = BC. ergo trianguli CAB, CAD, sibi mutuo æquilatera sunt; quare ang. CAB *b* = CAD *c* = Rect. Q.E.D.

a 47. 1.
* *Vide seq. Theor.*
b 8. 1.
c *Hyp.*

Schol.

Assumpsimus exinde quod CD q. = BC q, sequi CD = BC. Hoc vero manifestum fiet ex sequenti theoremate.

THEOREMA.



Linearum æqualium AB, CD, *æqualia sunt quadrata* AF, CG; *& quadratorum æqualium* NK, PM *æqualia sunt latera* IK, LM.

Pro 1 Hyp. Duc diametros EB, HD. Li-
a 34. 1. quet AF = *a* 2 triang. EAB = *b* 2 triang.
b 4. 1. & HCD = *a* CG. Q.E.D.
6. ax.

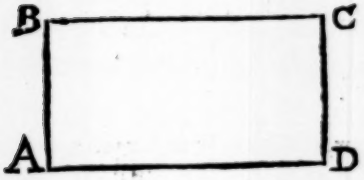
2. Hyp. Si fieri poteſt, ſit LM ⊏ IK. fac
a 46. 1. LT = IK; *a* ſitque LS = LTq. ergo LS
b 1. part. *b* = NK *c* = LQ. *d* Q.E.A. ergo LM = IK.
c hyp.
d 9. ax.

Coroll.

Eodem modo quælibet rectangula inter ſe æquilatera æqualia oſtendentur.

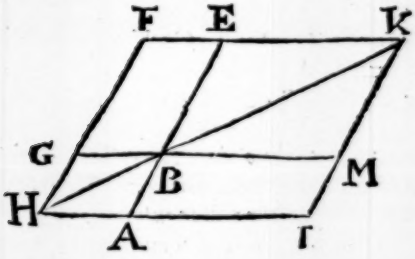
LIB. II.

Definitiones.



I. OMne parallelogrammum rectangulum ABCD contineri dicitur ſub rectis duabus AB, AD, quæ rectum comprehendunt angulum.

Quando igitur dicitur rectangulum ſub B A, A D, *vel brevitatis cauſa*; *rectangulum* B A D, *vel* BA × AD, (*vel* Z A *pro* Z × A;) *deſignatur rectangulum quod continetur ſub* B A, & A D *ad rectum angulum conſtitutis.*



II. In omni parallelogrammo ſpatio FHIK unumquodq; eorum, quæ circa diametrum illius ſunt, parallelogrammorum, cum duobus complementis Gnomon vocetur, *ut Pgr.* FB+BI+GA (EHM) *eſt Gnomon. item* Pgr. FB+BI+EM (GKA) *eſt Gnomon.*

PROP. I.

Si fuerint duæ rectæ lineæ AB, AF, *secetur quæ ipsarum altera* AB *in quotcunque segmenta* AD, DE, EB: *rectangulum comprehensum sub illis duabus rectis lineis* AB, AF, *æquale est eis, quæ sub insecta* AF, *& quolibet segmentorum* AD, DE, EB *comprehenduntur rectangulis.*

a 11. 1. *a* Statue AF, perpendicularem ad AB. *a* per F duc infinitam FG perpendicularem ad AF. *a* Ex D, E, B erige perpendiculares DH, EI, BG. erit AG rectangulum sub AF, AB, &
b 19. ax. 1. *b* est æquale rectangulis AH, DI, EG, hoc est
c 34. 1. (quia DH, EI, AF *c* pares sunt) rectangulis sub AF, AD; sub AF, DE; sub AF, EB. Q. E. D.

Schol.

Imo si fuerint duæ rectæ, secenturque ambæ in quotcunque partes, idem provenit ex ductu totius in totum, & partium in partes.

Nam sit Z=A+B+C. & Y=D+E; quia
a 1. 2. DZ *a*= DA+DB+DC, & EZ *a*=EA+EB
b 2. ax. +EC, & YZ *a* = DZ+EZ, *b* erit ZY=DA +DB+DC+EA+EB+EC. Q. E. D.

Hinc patet ratio ducendi rectas compositas in compositas. Nam omnia partium rectangula accipere oportet, & habetur rectangulum ex totis.

Sin linearum in se ducendarum signis + admisceantur signa —, etiam signorum ratio habenda est. Quippe ex + in — provenit —; at ex — in — provenit +. Nam sit + A ducenda in B — C. & quoniam + A non affirmatur de toto B, sed de ejus parte tantùm, qua superat C, debet AC m + nere negata, quare prodibit AB — AC. Vel sic; quia

quia B conſtat partibus C, & B—C, * erit AB =AC+A in B—C; aufer utrinque AC, erit AB —AC=A in B—C. Similiter ſi —A ducenda ſit in B—C, quoniam ex vi ſigni —, non negatur A de toto B, ſed de ejus ſolummodo exceſſu ſupra C, debet AC manere affirmata. proveniet ergo—AB+AC. Vel ſic; quia AB*=AC+A in B—C; tolle utrinque omnia, erit—AB=AC —A in B+C; adde AC utrinque, eritque —AB +AC=A in B—C. * 1. 2.

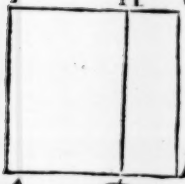
Atque ex his rite perſpectis, quæ ſubſequuntur 9 propoſitiones, aliæque ejuſmodi innumeræ, ex linearum in ſe ductarum comparatione emergentes (quas apud Vietam, & alios Analyſtas in numerato habes) nullo negotio demonſtrantur, rem plerumque quaſi ad ſimplicem calculum exigendo.

Porro, *liquet productum ex quapiam magnitudine in numeri cujuſlibet partes, æquari producto ex eadem in totum numerum. Ut 5 A+7 A=12 A & 4 A in 5 A+4A in 7 A=4 A in 12 A: quare quæ in hoc loco de rectarum in ſe ductu dicta ſunt, eadem de numerorum in ſe multiplicatione intelligi poſſunt. proinde etiam quæ in 9. ſequentibus theorematis de lineis affirmantur, eadem valent de numeris accepta; quippe cum illæ omnes ab hac prima immediate dependeant & deducantur. * 19. 4x.

Propoſitiones decem primæ hujus libri valent etiam in numeris. Reliquas quilibet tyro examinet. pro hac, ſit A F 6, & A B 12, ſectus in A D 5, D E 3, & E B 4. Eſtque 6x12 (A G) =72. 6x5 (A H) = 30 6 in 3 (D I) = 18. denique 6x4 (E G) = 24. Liquet vero 30+18+24=72.

PROP.

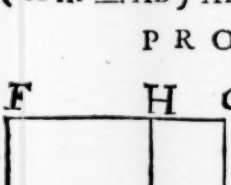
PROP. II.



Si recta linea AB *secta sit utcunque in* D, *rectangula quæ sub tota* AB *& quolibet segmentorum* AD, DB *comprehenduntur, æqualia sunt ei quod à tota* AB *fit quadrato.*

Erige AF perpendicularem & æqualem AB, & erunt
a 1. 2. *a* AF×AD+AF×DB = AF×AB; hoc est (ob AF = AB) AB×AD+AB×DB = ABq.

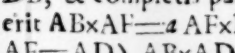
PROP. III.



Si recta linea AB *secta sit utcunque in* D, *rectangulum sub tota* AB *& uno segmentorum* AD *comprehensum, æquale est illi quod sub segmentis* AD, DB *comprehenditur rectangulo, & illi quod à prædicto segmento* AD *describitur quadrato.*

Nam erige AF perpendicularem & æqualem DB, & completis parallelogrammis FD, FB,
a 1. 2. erit AB×AF = *a* AF×DB+AF×AD, hoc est (ob AF = AD) AB×AD = AD×DB+ADq.

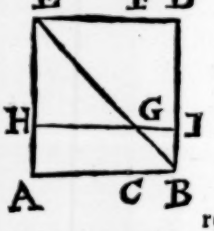
PROP. IV.



Si recta AB *secta sit utcunque in* D, *quadratum quod à tota* AB *describitur, æquale est illis quæ à segmentis* AD, DB *describuntur quadratis, & ei quod bis sub segmentis* AD, DB *comprehenditur rectangulo.*

a 2. 2. Nam ABq = *a* AB×AD+AB×DB. Cùm
b 3. 2. ergo *b* AB×AD = AD×DB+ADq & *b* AB×DB = AD

=ADxDB + DBq, erit *c* ABq=ADq+DBq +2 ADxDB. c 1. *ax.*



Aliter. Super AB fac quadratum AD, cujus diameter EB. per divisionis punctum C duc perpendicularem CF; & per G duc HI paralL. AB.

Quoniam ang. EHG=A rectus est, & AEB *d* semirectus, *e* erit reliquus HGE etiam semirectus. Ergo HE *f* = HG *g* = EF *g* = AC. *h* proinde HF quadratum est rectæ AC. eodem modo CI est CBq. ergo AG. GD rectangula sunt sub AC, CB. Quare totum quadratum AD *k* = ACq +CBq+2 ACB. Q.E.D.

d 4. *Cor.* 32. 1.
e 32. 1.
f 6. 1.
g 34. 1.
h 29. *def.* 1.
k 19. *ax.* 1.

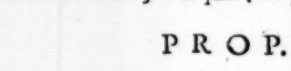
Coroll.

1. Hinc liquet parallelogramma circa diametrum quadrati esse quadrata.

2. Item diametrum cujusvis quadrati ejus angulos bisecare.

3. Si A = $\frac{1}{2}$ Z; erit Zq = 4 Aq, & Aq = $\frac{1}{4}$ Zq. item è contra, si Zq = 4 Aq. erit A = $\frac{1}{2}$ Z.

PROP. V.



Si recta linea AB *secetur in æqualia* AC *b* CB, *& non æqualia* AD, DB, *rectangulum sub inæqualibus segmentis* AD, DB *comprehensum una cum quadrato, quod fit ab intermedia sectionum* CD, *æquale est ei, quod à dimidia* CB *describitur, quadrato.*

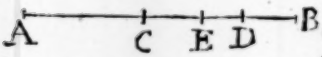
Dico CBq = ADB + CDq.

Æquantur

a 4. 2. Æquantur enim ista $\begin{cases} CBq. \\ a\ CDq+CDB+DBq+CDB \\ CDq+b\ CBD(c\ AC\times BD)+CDB \\ CDq+d\ ADB. \end{cases}$
b 3. 2.
c *hyp.*
d 1. 2. *Hoc Theorema paulo aliter effertur, & facilius demonstratur, sic; Rectangulum ex summa & differentia duarum rectarum* A, E, *æquatur differentiæ ex ipsis.*

* *sch.* 1. 2. Nam si A+E ducatur in A—E,* provenit Aq —AE+EA—Eq=Aq—Eq. Q.E.D.

Scholium.



Si A B aliter dividatur, propius scilicet puncto bisectionis, in E; dico AEB⊏ADB.

a 5. 2. & 3. *ax.* Nam AEB *a*=CBq—CEq. & ADB*a*=CBq —CDq. ergo quum CDq⊏CEq, erit AEB⊏ADB. Q.E.D.

Coroll.

Hinc ADq+DBq⊏AEq+EBq. Nam
b 4. 2. ADq+DBq+2 ADB*b*=ABq *b*=AEq+EBq +2 AEB ergo quum 2 AEB⊏2 ADB, erit ADq+DBq⊏AEq+EBq. Q.E.D.

c 3. *ax.* Unde 2. ADq+DBq—AEq *c*—EBq=2 AEB—2 ADB.

PROP. VI.

Si recta linea A *bifariam secetur, & illi recta quæpiam linea* E *in directum adjiciatur; rectangulum comprehensum sub tota cum adjecta* (sub. A+E,) *& adjecta* E, *una cum quadrato, quod à dimidia* ($\frac{1}{2}$ A,) *æquale est quadrato à linea, quæ tum ex dimidia, tum ex adjecta componitur, tanquam ab una* ($\frac{1}{2}$A+E) *descripto.*

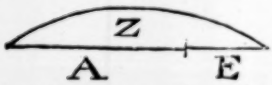
a 4. & 3. Dico $\frac{1}{4}$ Aq (*a* Q. $\frac{1}{2}$ A) +AE+Eq=Q. $\frac{1}{2}$ A
Cor. 4. 2. +E. *a* Nam Q. $\frac{1}{2}$ A+E=$\frac{1}{4}$Aq+Eq+AE.

Coroll.

Coroll.

Hinc ſi tres rectæ E, E + ½ A, E + A ſint in proportione Arithmetica, rectangulum ſub extremis E, E + A contentum, una cum quadrato exceſſus ½ A, æquale erit quadrato mediæ E + ½ A.

PROP. VII.



Si recta linea Z ſecetur utcunque; Quod à tota Z, quodque ab uno ſegmentorum E, *utraque ſimul quadrata, æqualia ſunt illi, quod bis ſub tota Z, & dicto ſegmento* E *comprehenditur, rectangulo, & illi, quod à reliquo ſegmento* A *fit, quadrato.*

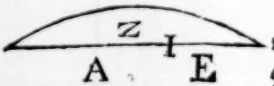
Dico Zq + Eq = 2 ZE + Aq. Nam Zq *a* = Aq + Eq + 2 AE. & 2 ZE *b* = 2 Eq + 2 AE. a 4. 2. b 3. 2.

Coroll.

Hinc, quadratum differentiæ duarum quarumcunque linearum Z, E, æquale eſt quadratis utriuſque minus duplo rectangulo ſub ipſis.

c Nam Zq + Eq — 2 ZE = Aq = Q. Z — E. c 7. 2 & 3. ax.

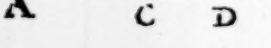
PROP. VIII.



Si recta linea Z ſecetur utcunque; rectangulum quater comprehenſum ſub tota Z & uno ſegmentorum E, *cum eo, quod à reliquo ſegmento* A *fit, quadrato, æquale eſt ei, quod à tota Z & dicto ſegmento* E, *tanquam ab una linea* Z + E *deſcribitur quadrato.*

Dico 4 ZE + Aq = Q. Z + E. Nam 2 ZE *a* = Zq + Eq — Aq ergo 4 ZE + Aq = Zq + Eq + 2 ZE *b* = Q. Z + E. Q.E.D. a 7. 2. & 3. ax. b 4. 2.

PROP IX.



Si recta linea AB *ſecetur in æqualia* AC,

AC, CB, *& non æqualia* AD, DB. *quadrata, quæ ab inæqualibus totius segmentis* AD, DB *fiunt, simul duplicia sunt, & ejus, quod à dimidia* AC, *& ejus, quod ab intermedia sectionum* CD *fit, quadrati.*

Dico ADq + DBq = 2 ACq + 2 CDq. Nam
a 4. 2. ADq + DBq *a* = ACq + CDq + 2 ACD + DBq.
b *hyp.* atqui 2 ACD (*b* 2 ECD) + DBq *c* = CBq
c 7. 2. (ACq) + CDq. *d* ergo ADq + DBq = 2 ACq
d 2. *ax.* + 2 CDq. Q.E.D.

Aliter effertur & facilius demonstratur, sic;

Aggregatum quadratorum ex summa, & differentia duarum rectarum A, E, *æquatur duplo quadratorum ex ipsis*

a 4 2. Nam Q. A + E *a* = Aq + Eq + 2 AE. & Q. A
b *Cor.* 7. 2. — E *b* = Aq + Eq — 2 AE. Hæc collecta faciunt 2 Aq + 2 Eq. Q.E.D.

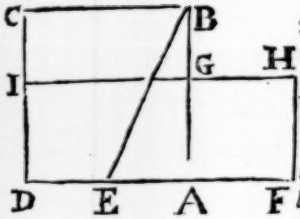
PROP. X.

Si recta linea A *secetur bifariam, adjiciatur autem ei in rectum quæpiam linea; Quod à tota* A *cum adjuncta* E, *& quod ab adjuncta* E *utraque simul quadrata, duplicia sunt & ejus, quod à dimidia* $\frac{1}{2}$ A; *& ejus, quod à composita ex dimidia, & adjuncta, tanquam ab una* $\frac{1}{2}$ A + E, *descriptum est, quadrati.*

a 4. 2. Dico Eq + Q. A + E, hoc est *a* Aq + 2 Eq + 2
b *Cor.* 4. 2. AE = 2 Q. $\frac{1}{2}$ A + 2 Q. $\frac{1}{2}$ A + E. Nam 2 Q. $\frac{1}{2}$ A *b*
c 4. 2. = $\frac{1}{2}$ Aq & 2 Q. $\frac{1}{2}$ A + E *c* = $\frac{1}{2}$ Aq + 2 Eq + 2 AE.

PROP.

PROP. XI.



Datam rectam lineam A B *secare in* G, *ut comprehensum sub tota* AB, *& altero segmentorum* BG *rectangulum, æquale sit ei quod à reliquo segmento* AG, *fit, quadrato.*

Super AB *a* describe quadratum A C. latus AD *b* bisseca in E duc EB. ex EA producta cape EF = EB ad AF *a* statue quadratum A H. Erit AH = AB x BG.

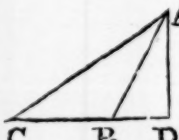
Nam protracta H G ad I; Rectang. D H + EAq *c* = EFq *d* = EBq *e* = BAq + EAq ergo DH *f* = BAq *d* = quad. AC. subtrahe commune AI; remanet quad. AH = GC; *d* id est AGq = ABx BG. Q. E. F.

a 46. 1.
b 10. 1.
c 6. 2.
d *constr.*
e 47 1.
f 3. *ax.*

Scholium.

Hæc Propositio numeris explicari nequit; *neque enim ullus numerus ita secari potest, ut productum ex toto in partem unam æquale sit quadrato partis reliquæ. *vid. 6. 13.

PROP. XII.



In amblygonio triangulo ABC *quadratum, quod fit à latere* A C *angulum obtusum* A B C *subtendente, majus est quadratis, quæ fiunt à lateribus* A B, B C *obtusum angulum* A B C *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* B C, *quæ sunt circa obtusum angulum* ABC, *in quod, cum protractum fuerit, cadit perpendicularis* AD, *& ab assumpta exterius linea* B D *sub perpendiculari* AD *prope angulum obtusum* A B C.

Dico

Dico $ACq = CBq + ABq + 2\,CB \times BD$.

Nam ista æqualia sunt inter se
$$\left\{\begin{array}{l} ACq. \\ a\ CDq + ADq. \\ b\ CBq + 2\,CBD + BDq + ADq \\ CBq + 2\,CBD\ c + ABq. \end{array}\right.$$

a 47. 1.
b 4. 2.
c 47. 1.

Schol.

Hinc, *cognitis lateribus trianguli obtusanguli* ABC, *facile invenientur tum segmentum* BD *inter perpendicularem* AD, *& obtusum angulum* ABC *interceptum, tum ipsa perpendicularis* AD.

Sic; Sit AC 10, AB 7, CB 5; unde ACq 100, ABq 49, CBq 25. Proinde $ABq + CBq = 74$. hunc deme ex 100, manet 26 pro 2 CBD. unde CBD erit 13. hunc divide per CB 5, provenit $2\frac{3}{5}$ pro BD. quare AD invenitur per 47.1.

PROP. XIII.

In oxygoniis triangulis ABC *quadratum à latere* AB *angulum acutum* ACB *subtendente, minus est quadratis, quæ fiunt à lateribus* AC, CB *acutum angulum* ACB *comprehendentibus, rectangulo bis comprehenso, & ab uno laterum* BC, *quæ sunt circa acutum angulum* ACB, *in quod perpendicularis* AD *cadit, & ab assumpta interius linea* DC *sub perpendiculari* AD, *prope angulum acutum* ACB.

Dico $ACq + BCq = ABq + 2\,BCD$.

Nam æquantur ista
$$\left\{\begin{array}{l} ACq + BCq. \\ a\ ADq + DCq + BCq. \\ b\ ADq + BDq + 2\,BCD. \\ c\ ABq + 2\,BCD. \end{array}\right.$$

a 47. 1.
b 7. 2.
c 47. 1.

Coroll.

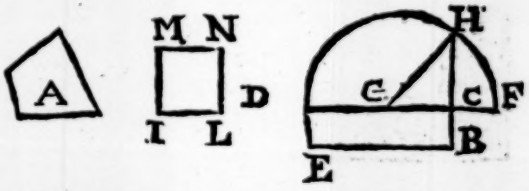
Hinc etiam *cognitis lateribus trianguli* ABC, *invenire est tam segmentum* DC *inter perpendicula-*

rem

rem AD, *& acutum angulum* ABC *interceptum, quam ipſam perpendicularem* AD.

Sit AB 13. AC 15. BC 14. Detrahe ABq (169) ex ACq + BCq hoc eſt ex 225 + 196 = 421; remanet 252 pro 2BCD; unde BCD erit 126. hunc divide per BC 14, provenit 9 pro DC, unde AD = √: 225 − 81 = 12.

PROP. XIV.



Dato rectilineo A *æquale quadratum* ML *invenire.*

a Fac rectangulum DB = A, cujus majus latus DC produc ad F, ita ut CF = CB. *b* Biſeca DF in G, quo centro ad intervallum GF deſcribe circulum FHD, producatur CB, donec occurrat circumferentiæ in H. Erit CHq = *ML = A.

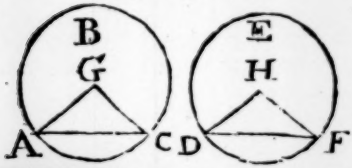
a 45. 1.
b 10. 2.
*46. 1.

Ducatur enim GH. Eſtque A *c* = DB *c* = CDF *d* = GFq − GCq *e* = HCq *e* = ML Q. E. F.

c *Conſtr.*
d 5. 2. & 3. 4x.
e 47. 1. & 3. 4x.

LIB. III.

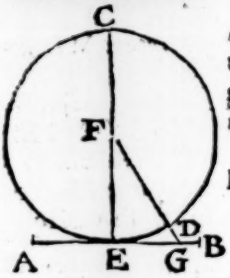
Definitiones.

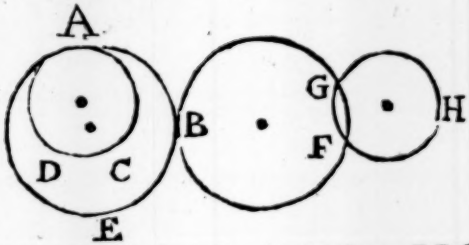


I. Quales circuli (GABC, HDEF) ſunt, quorum diametri ſunt æquales, vel quorum quæ ex centris rectæ lineæ GA, HD, ſunt æquales.

II. Recta linea AB circulum FED tangere dicitur, quæ cum circulum tangat, ſi producatur circulum non ſecat.

Recta FG ſecat circulum FED.



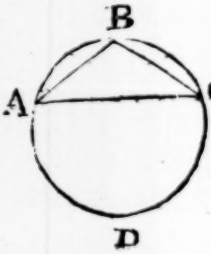


III. Circuli DAC, ABE (item FBG, ABE) ſe mutuo tangere dicuntur, qui ſe mutuo tangentes ſeſe mutuo non ſecant.

Circulus BFG ſecat circulum FGH.

IV. In

IV. In circulo G A B D æqualiter diſtare à centro dicuntur rectæ lineæ F E K L, cum perpendiculares G H, G N, quæ à centro G in ipſas ducuntur, ſunt æquales. Longius autem abeſſe illa B C dicitur, in quam major perpendicularis G I cadit.



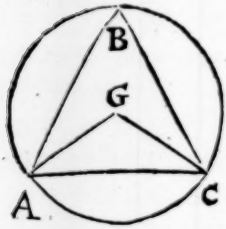
V. Segmentum circuli (A B C) eſt figura, quæ ſub recta linea A C, & circuli peripheria A B C comprehenditur.

VI. Segmenti autem angulus (C A B) eſt, qui ſub recta linea C A, & circuli peripheria A B comprehenditur.

VII. In ſegmento autem (A B C) angulus (A B C) eſt, cum in ſegmenti peripheria ſumptum fuerit quodpiam punctum B, & ab illo in terminos rectæ ejus lineæ A C, quæ ſegmenti baſis eſt, adjunctæ fuerint rectæ lineæ A B, C B, is inquam angulus A B C ab adjunctis illis lineis A B, C B comprehenſus.

VIII. Cum vero comprehendentes angulum A B C, rectæ lineæ A B, B C aliquam aſſumunt peripheriam A D C, illi angulus A B C inſiſtere dicitur.

XI. Sector autem circuli (ADB) est, cum ad ipsius circuli centrum D constitutus fuerit angulus ADB; comprehensa nimirum figura ADB, & à rectis lineis AD, BD angulum continentibus, & à peripheria AB ab illis assumpta.



X. Similia circuli segmenta (ABC, DEF) sunt, quæ angulos (ABC, DEF) capiunt æquales; aut in quibus anguli ABC, DEF inter se sunt æquales.

PROP. I.

Dati circuli ABC *centrum* F *reperire.*

Duc in circulo rectam A C utcunque, quam biseca in E. per E duc perpendicularem D B. hanc biseca in F. erit F centrũ.

Si negas, centrum esto G, extra rectam D B (nam in ea esse non potest, cum ubique extra F dividatur inæqualiter) ducanturque G A, G C, G E. Vis G centrum esse; *a* ergo GA=GC; & per constr. A E=E C, latus vero GE commune est; *b* ergo anguli GEA, GEC pares, & *c* proinde recti sunt. *d* ergo ang. GEC=FEC rect. *e* Q.E.A.

a 15. *def.* 1.
b 8. 1.
c 10. *def* 1.
d 12. *ax*
e 9. *ax.*

Coroll.

Coroll.

Hinc, ſi in circulo recta aliqua linea BD aliquam rectam lineam AC bifariam & ad angulos rectos ſecet, in ſecante BD erit centrum.

Facillime per normam invenitur centrum vertice Q ad circumferentiam applicato. Si enim recta DE jungens puncta D, & E, in quibus normæ latera QD, QE peripheriam ſecant, biſecetur in A, erit A centrum. Demonſtratio pendet ex 31. hujus. *And. Tarq.*

PROP. II.

Si in circuli CAB *peripheria duo quælibet puncta*, A, B *accepta fuerint, recta linea* AB, *quæ ad ipſa puncta adjungitur, intra circulum cadet.*

Accipe in recta AB quodvis punctum D, & ex centro C duc CA, CD, CB. & quoniam CA *a* = CB, *b* erit ang. A = B. Sed ang. C D B *c* ⊏ A; ergo ang. C D B ⊏ B. *d* ergo CB ⊏ C D. atqui C B tantum pertingit ex centro ad circumferentiam; ergo CD eo uſque non pertingit. ergo punctum D eſt intra circulum. Idemque oſtendetur de quovis alio puncto rectæ A B. Tota igitur A B cadit intra circulum. Q.E.D.

a 15. *def.* 1.
b 5. 1.
c 16. 1.
d 19. 1.

Coroll.

Hinc, recta circulum tangens, ita ut eum non secet, in unico puncto tangit.

PROP. III.

Si in circulo EABC *recta quædam linea* BD *per centrum extensa quandam* AC *non per centrum extensam bifariam secet,* (in F) *& ad angulos rectos ipsam secabit; & si ad angulos rectos eam secet, bifariam quoque eam secabit.*

Ex centro E ducantur EA, EC.

a *hyp.* b 15 *def.* 1. c 8. 1. d 10. *def.* 1. e *hyp. &* 12. *ax.* f 5. 1. g 26. 1.

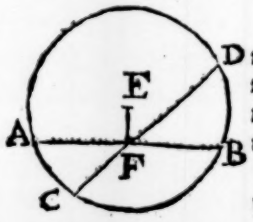
1. *Hyp.* Quoniam AF *a* = FC, & EA *b* = EC, latusque EF commune est, *c* erunt anguli EFA, EFC pares, & *d* consequenter recti. Q.E.D.

2. *Hyp.* Quoniam ang. EFA *e* = EFC, & ang. EAF *f* = ECF, latusque EF commune, *g* erit AF = FC. Bisecta est igitur AC. Q.E.D.

Coroll.

Hinc, in triangulo quovis æquilatero & Isoscele linea ab angulo verticis bilecans basim, perpendicularis est basi. & contra perpendicularis ab angulo verticis bisecat basim.

PROP. IV.



Si in circulo ACD *duæ rectæ lineæ* AB, CD *sese mutuo secent non per centrum* E *extensæ, sese mutuo bifariam non secabunt.*

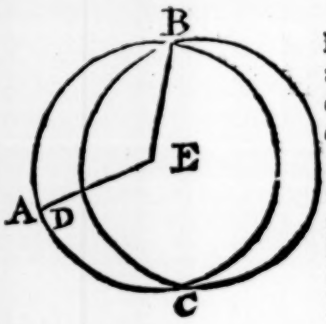
Nam si una per centrum transeat, patet hanc non

non bisecari ab altera, quæ ex hyp. per centrum non transit.

Si neutra per centrum transit, ex E centro duc E F. Si jam ambæ A B, C D forent bisectæ in F, anguli E F B, E F D *a* ambo essent recti, & proinde æquales. *b* Q.E.A.

a 3. 3.
b 9. *ax.*

PROP. V.

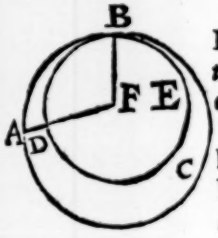


Si duo circuli B A C, B D C *sese mutuo secent, non erit illorum idem centrum* E.

Alias enim ductis ex communi centro E rectis E B, E D A, essent E D *a* = E B *a* = E A. *b* Q.E.A.

a 15. *def.* 1.
b 9. *ax.*

PROP. VI.



Si duo circuli B A C, B D E, *sese mutuo interius tangant* (in B) *eorum non erit idem centrum* F.

Alias ductis ex centro F rectis F B, F D A, essent F D *a* = F B *a* = F A. *b* Q.F.N.

a 15. *def.* 1.
b 9. *ax.*

PROP. VII.

Si in AB *diametro circuli quodpiam ſumatur punctum* G, *quod circuli centrum non ſit, ab eoque puncto in circulum quædam rectæ lineæ* GC, GD, GE *cadunt; maxima quidem erit ea* (GA) *in qua centrum* F, *minima vero reliqua* GB, *aliarum vero illi, quæ per centrum ducitur, propinquior* GC *remotiore* GD *ſemper major eſt. Duæ autem ſolum rectæ lineæ* GE GH *æquales ab eodem puncto in circulum cadunt, ad utraſque partes minimæ* GB, *vel maximæ* GA.

Ex centro F duc rectas FC, FD, FE; & a fac ang. BFH=BFE.

a 23. 1.

a 20. 1. 1. GF+FC (hoc eſt GA) a ⊏ GC. Q.E.D.

b 15. def. 1. 2. Latus FG commune eſt, & FC b = FD,

c 9. ax. atque ang. GFC c ⊏ GFD. d ergo baſ. GC

d 24. 1. ⊏ GD. Q.E.D.

e 20. 1. 3. FB (FE) e ⊐ GE+GF. ergo ablato

f 5. ax. communi FG f remanet BG ⊐ EG. Q.E.D.

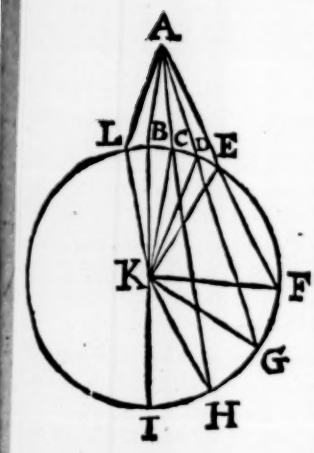
4. Latus FG commune eſt, & FE=FH; atq;

g conſtr. ang. BFH g = BFE. h ergo GE=GH. Quod

h 4. 1. vero nulla alia GD ex puncto G æquetur ipſi GE, vel GH, jamjam oſtenſum eſt. Q.E.D.

PROP.

PROP. VIII.



Si extra circulum ſumatur punctum quodpiam A, *ab eoq; puncto ad circulum deducantur quædam lineæ* AI, AH, AG, AF, *quarum una quidem* AI *per centrum* K *proten latur, reliquæ vero ut libet; in cavam peripheriam cadentium rectarum linearum maxima quidem eſt illa* AI, *quæ per centrum ducetur, aliarum autem ei quæ per centrum tranſit propinquior* AH *remotiore* AG *ſemper major eſt. In convexam vero peripheriam cadentium rectarum linearum minima quidem eſt illa* AB, *quæ inter punctum* A, *& diametrum* BI *interponitur; aliarum autem ea, quæ eſt minimæ propinquior* AC *remotiore* AD *ſemper minor eſt. Duæ autem tantum rectæ lineæ* AC, AL *æquales ab eo puncto in ipſum circulum cadunt, ad utraſque partes minimæ* AB, *vel maximæ* AI.

Ex centro K duc rectas KH, KG, KF; KC, KD, KE. & fac ang. AKL = AKC.

1. AI (AK + KH) *a* ⊐ AH. Q.E.D. a 20. 1.

2. Latus AK commune eſt; & KH = KG; atque ang. AKH ⊐ AKG. *b* ergo baſ. AH ⊐ AG. Q.E.D. b 24. 1.

3. KA *c* ⊐ KC + CA. aufer hinc inde æquales KC, KB, *d* erit AB ⊐ AC. c 20. 1. d 5. ax.

4. AC + CK *e* ⊐ AD + DK. aufer hinc inde æquales CK, DK, *f* erit AC ⊐ AD. Q.E.D. e 21. 1. f 5. ax.

Latus

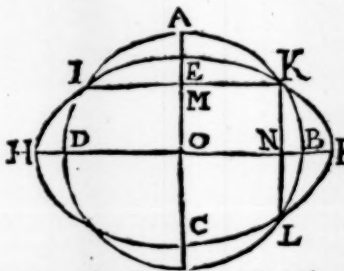
5. Latus KA est commune & KL = KC;
g constr. atque ang. AKL g = AKC; b ergo LA =
h 4. 1. CA. hisce vero nulla alia æquatur, ex modò ostensis. ergo, &c.

PROP. IX.

Si in circulo BCK *acceptum fuerit punctum aliquod* A, *& ab eo puncto ad circulum cadant plures, quam duæ rectæ lineæ æquales* AB, AC, AK, *acceptum punctum* A *centrum est ipsius circuli.*

a 7. 3. Nam *a* à nullo puncto extra centrum plures quam duæ rectæ lineæ æquales duci possunt ad circumferentiam. Ergo A est centrum. Q.E.D.

PROP. X.

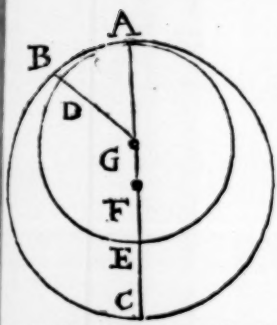


Circulus IAKBL *circulum* IEKFL *in pluribus quam duobus punctis non secat.*

Secet, si fieri potest, in tribus punctis I, K, L. Junctæ IK, KL bisecentur in M & N.
a Cor. 1. 3. *a* Ambo circuli centrum habent in singulis perpendicularibus MC, NH, & proinde in earum intersectione O. ergo secantes circuli idem cen-
b 5. 3. trum habent. *b* Q.F.N.

PROP.

PROP. XI.

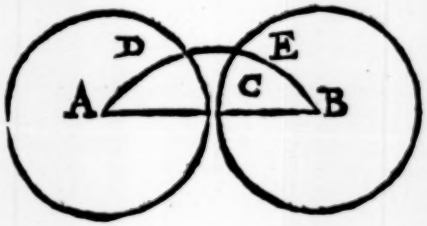


Si duo circuli GADE, FABC *ſeſe intus contingant, atque accepta fuerint eorum centra* G, F; *ad eorum centra adjuncta recta linea* FG, *& producta, in* A *contactum circulorum cadet.*

Si fieri poteſt, recta FG protracta ſecet circulos extra contactum A, ſic ut non FGA, ſed FGDB ſit recta linea. Ducatur GA. Et quia GD *a* = GA, & GB *b* ⌐ GA, (cum recta FGB tranſeat per F centrum majoris circuli) erit GB ⌐ GD. *c* Q.E.A.

a 15 *def* 1.
b 7. 3.
c 9. *ax.*

PROP. XII.



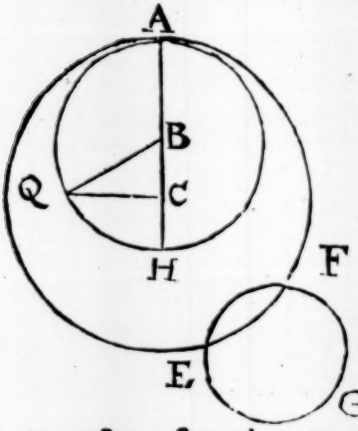
Si duo circuli ACD, BCE *ſeſe exterius contingant, linea recta* AB *quæ ad eorum centra* A, B *adjungitur, per contactum* C *tranſibit.*

Si fieri poteſt, ſit recta ADEB ſecans circulos extra contactum C in punctis D, E. Duc AC, CB. erit AD + EB (AC + CB) *a* ⌐ AD + EB. *b* Q.E.A.

a 20. 1.
b 9. *ax.*

PROP.

PROP. XIII.



Circulus CAF *circulum* BAH *non tangit in pluribus punctis, quam uno* A, *sive intus, sive extra tangat.*

1. Tangat si fieri potest, intus in punctis A, H. *a* ergo recta CB centra connectens, si producatur cadet tam in A, quam in H. Quoniam igitur CH *b* = CA, & BH ⊏ CH. erit BA (*c* BH) ⊏ CA. *d* Q.E.A.

a 11. 3.
b 15. *def.* 1.
c 15. *def.* 1.
d 9 *ax.*

2. Sin dicatur exterius contingere in punctis E & F, *e* ducta recta EF in utroque circulo erit. Circuli igitur se mutuo secant, quod non ponitur.

e 2. 3.

PROP. XIV.

In circulo EABC *æquales rectæ lineæ* AC, BD, *æqualiter distant à centro* E *& quæ* AC, BD *æqualiter distant à centro, æquales sunt inter se.*

Ex centro E duc perpendiculares EF, EG: *a* quæ bisecabunt AC, DB. connecte EA, EB.

a 3. 3.

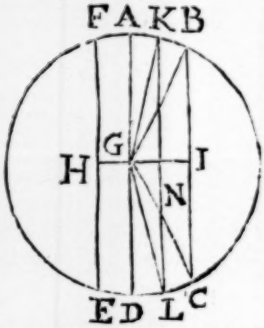
1. *Hyp.* AC = BD. ergo AF *b* = BG. sed & EA = EB, ergo FEq *c* = EAq — AFq = EBq

b 7. *ax.*

EBq — BGq *c* = EGq. *d* ergo FE = EG. Q.E.D.
2. *Hyp.* EF = EG. ergo AFq *c* = EAq — EFq = EBq — EGq *c* = GBq. ergo AF *d* = GB. *e* proinde AC = BD. Q.E.D.

c 47. 1. & 3. *ax.*
d *Schol.* 48. 1.
e 6. *ax.*

PROP. XV.

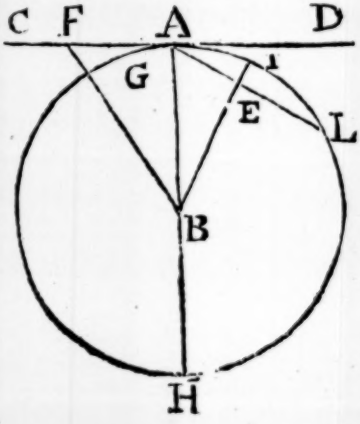


In circulo GABC *maxima quidem linea eſt diameter* AD; *aliarum autem centra* G *propinquior* FE *remotiore* BC *ſemper major eſt.*

1. Duc GB, GC. Diameter AD (*a*) a 15. *def.* 1. GB + GC) *b* ⊏ BC. b 20. 1. Q.E.D.

2. Sit diſtantia GI ⊏ GH. accipe GN = GH. per N duc KL perpend. GL. junge GK, GL. & quia GK = GB, & GL = GC; eſtque ang. KGL ⊏ BGC, *c* erit KL (FE) ⊏ BC. Q.E.D. c 24. 1.

PROP. XVI.



Quæ CD *ab extremitate diametri* HA *cujuſque circuli* BALH *ad angulos rectos ducitur, extra ipſum circulum cadet, & in locum inter ipſam rectam lineam, & peripheriam comprehen-*

prehensum altera recta linea A L *non cadet, & semicirculi quidem angulus* B A I *quovis angulo acuto rectilineo* B A L *major est; reliquus autem* D A I *minor.*

1. Ex centro B ad quodvis punctum F in recta A C duc rectam B F. Latus B F subtendens angulum rectum B A F *a* majus est latere B A, quod opponitur acuto B F A. ergo cum B A (B G) pertingat ad circumferentiam, B F ulterius porrigetur, adeoque punctum F; & eadem ratione quodvis aliud rectæ A C, extra circulum situm erit. Q.E.D.

a 19. 1.

2. Duc B E perpendic. A L. Latus B A oppositum recto angulo B E A *b* majus est latere B E, quod acutum B A E subtendit: ergo punctum E, adeoque tota E A cadit intra circulum. Q.E.D.

b 19. 1.

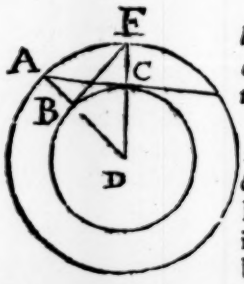
3. Hinc sequitur angulum quemvis acutum, nempe E A D angulo contactus D A I majorem esse. Idem angulum quemvis acutum B A L angulo semicirculi B A I minorem esse. Q.E.D.

Coroll.

Hinc, recta à diametri circuli extremitate ad angulos rectos ducta ipsum circulum tangit.

Ex hac propositione paradoxa consequuntur, & mirabilia bene multa, quæ vide apud interpretes.

PROP. XVII.



A dato puncto A *rectam lineam* A C *ducere, quæ datum circulum* D B C *tangat.*

Ex D dati circuli centro ad datum punctum A ducatur recta D A secans peripheriam in B. Centro D describ. per A alium circulum A E;

AE; & ex B duc perpendicularem ad AD, quæ occurrat circulo A E in E. duc E D occurrentem circulo BC in C. ex A ad C ducta recta tanget circulum D B C.

Nam DB *a* = DC, & DE *a* = DA, & ang. D communis est: *b* ergo ang. A C D = EBD, rect. *c* ergo AC tangit circulum C. Q.E.F.

a 15. *def.* 1.
b 4. 1.
c *Cor.* 16. 3.

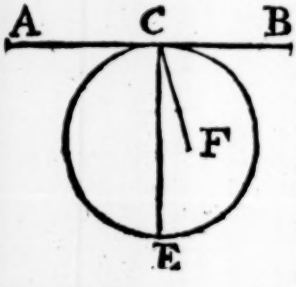
PROP. XVIII.

Si circulum F E D C *tangat recta quæpiam linea* A B, *à centro autem ad contactum* E *adjungatur recta quædam linea* F E; *quæ adjuncta fuerit* FE *ad ipsam contingentem* A B, *perpendicularis erit.*

Si negas, sit ex F centro alia quædam FG perpendicularis ad contingentem, *a* secabi ea circulum in D. Quum igitur ang. FGE rectus dicatur *b* erit ang. FEG acutus. *c* ergo FE (FD) ⊏ FG. *d* Q.E.A.

a 2. *def.* 3. & 16. 3.
b *Cor.* 17. 1.
c 19. 1.
d 9. *ax.*

PROP. XIX.



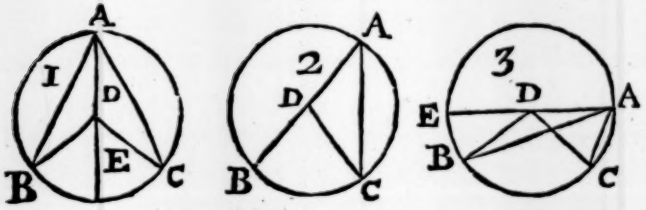
Si circulum tetigerit recta quæpiam linea A B, *à contactu autem* C *recta linea* C E *ad angulos rectos ipsi tangenti excitetur, in excitata* C E *erit centrum circuli.*

Si negas, sit centrum extra CE in F, & ab F ad contactum ducatur FC. Igitur ang. FCB * rectus est; & *a* proinde par angulo ECB recto per hypoth. *b* Q. E. A.

* 18 3.
a 11. *ax.*
b 9 *ax.*

PROP.

PROP. XX.

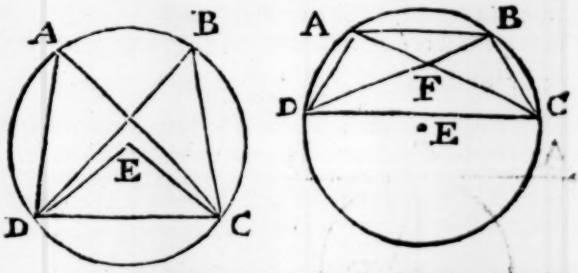


In circulo D A B C, *angulus* BDC *ad centrum duplex eſt anguli* BAC *ad peripheriam, cum fuerit eadem peripheria* B C *baſis angulorum.*

Duc diametrum ADE.

a 32. 1. Externus angulus BDE *a* = DAB + DBA *b* =
b 5. 1. 2 DAB. Similiter ang. EDC = 2 DAC. ergo
c 2. *ax.* in primo caſu *c* totus BDC = 2BAC; ſed in ter-
d 20. *ax.* tio caſu *d* reliquus angulus B D C = 2 B A C.
Q E. D.

PROP. XXI.



In circulo E D A C *qui in eodem ſegmento ſunt anguli,* DAC *&* DBC *ſunt inter ſe æquales.*

1. *Caſ.* Si ſegmentum DABC ſemicirculo ſit majus, ex centro E, duc ED, EC. Eritque 2 ang.
a 20. 3. A *a* = E *a* = 2 B. Q. E. D.

2. *Caſ.* Sin ſegmentum ſemicirculo majus non fuerit, ſumma angulorum trianguli ADF æquatur ſummæ angulorum in triangulo B C F. De-
b 15. 1. mantur hinc inde AFD *b* = BFC, & ADB *c* =
c per 1. *caſ.* ACB, remanent DAC = DBC. Q E. D.

PROP.

PROP. XXII.

Quadrilaterorum A B C D *in circulo deſcriptorum anguli* A D C, A B C, *qui ex adverſo, duobus rectis ſunt æquales.*

Duc A C, B D. Ang. A B C + B C A + B A C *a* = 2 Rect. Sed B D A *b* = B C A, & B D C *b* = B A C. *c* ergo A B C + A D C = 2 Rect. Q.E.D.

a 32. 1. *b* 21. 1. *c* 1. *ax.*

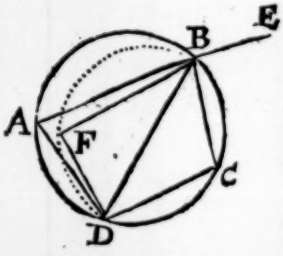
Coroll.

1. Hinc, ſi * A B unum latus quadrilateri in circulo deſcripti producatur, erit angulus externus E B C æqualis angulo interno A D C, qui opponitur ei A B C, qui eſt deinceps externo E B C ut patet ex 13. 1. & 3. ax.

**vide ſeq. diagram.*

2. Item circa Rhombum circulus deſcribi nequit; quia adverſi ejus anguli vel cedunt duobus rectis, vel eos excedunt.

SCHOL.

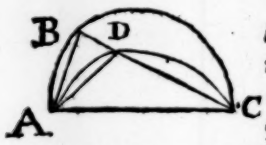


Si in quadrilatero A B C D *anguli* A, *&* C *qui ex adverſo duobus rectis æquantur, circa quadrilaterum circulus deſcribi poteſt.*

Nam circulus per quoſlibet tres angulos B, C, D tranſibit (ut patebit ex 5. 4.) dico eundem per A tranſire. Nam ſi neges, tranſeat per F. ergo ductis rectis

a 22. 3. BF, FD, BD; ang. C+F *a*=2 Rect. *b*= C+A
b *hyp.* *c* quare A=F. *d* Q.E.A.
c 3. *ax.*
d 21. 1.

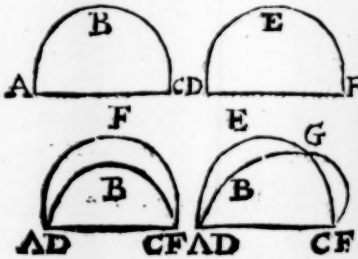
PROP. XXIII.



Super eadem recta linea A C *duo circulorum ſegmenta* A B C, ADC *ſimilia & inæqualia non conſtituentur ad eaſdem partes.*

Nam ſi dicantur ſimilia, duc C B ſecantem circumferentias in D, & B, & junge A D, ac
a 10 *def.* 3. AB. Quia ſegmenta ponuntur ſimilia, *a* erit ang.
b 16. 1. ADC=ABC. *b* Q.E.A.

PROP. XXIV.

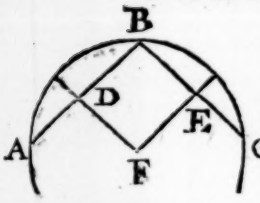


Super æqualibus rectis lineis AC, DF *ſimilia circulorum ſegmenta* ABC, DEF *ſunt inter ſe æqualia.*

Baſis A C ſuperpoſita baſi DF ei congruet, quia AC=DF. ergo ſegmentum ABC congruet ſegmento DEF
a 23. 3. (alias enim aut intra cadet, aut extra, *a* atque ita ſegmenta non erunt ſimilia, contra Hyp. aut ſaltem partim intra, partim extra, adeoque ip-
b 10. 3. ſum in tribus punctis ſecabit. *b* Q. E. A.) *c* pro-
c 8. *ax.* inde ſegmentum ABC=DEF. Q.E.D.

PROP.

PROP. XXV.



Circuli ſegmento ABC *dato, deſcribere circulum, cujus eſt ſegmentum.*

Subtendantur utcunque duæ rectæ AB, BC, quas biſeca in D, & E. Ex D, & E duc perpendiculares DF, EF occurrentes in puncto F. Hoc erit centrum circuli.

Nam centrum *a* tam in DF, quam in EF exſiſtit. ergo in communi puncto F. Q.E.F. *a Cor.* 1. 3.

PROP. XXVI.

In æqualibus circulis GABC, HDEF *æquales anguli æqualibus peripheriis* AC, DF *inſiſtunt, ſive ad centra* G, H, *ſive ad peripher.* B, E *conſtituti inſiſtant.*

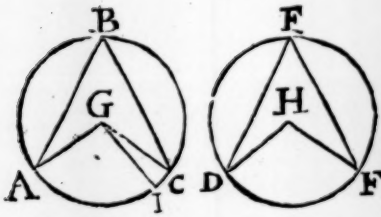
Ob circulorum æqualitatem, eſt GA = HD, & GC = HF, item per hyp. ang. G = H. *a* ergo AC = DF Sed & ang. B *b* = ½ G = *c* ½ H *b* = E. *d* ergo ſegmenta ABC, DEF ſimilia, *e* & proinde paria ſunt. *f* ergo etiam reliqua ſegmenta AC, DF æquantur. Q.E.D.

a 4. 1.
b 20. 3.
c *hyp.*
d 10. *def.* 3.
e 24. 3.
f 3. *ax.*

Scholium.

In circulo ABCD, ſit arcus AB par arcui DC; erit AD parall. BC. Nam ducta AC, *a* erit ang. ACB = CAD. quare per 27. 1. *a* 26. 3.

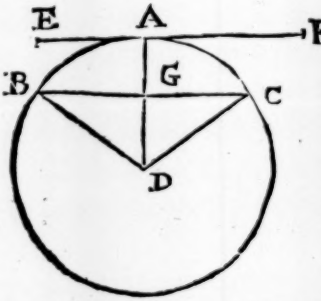
PROP. XXVII.



In æqualibus circulis, G A B C, H D E F, *anguli qui æqualibus peripheriis* A C, D F *insistunt, sunt inter se æquales, sive ad centra* G, H, *sive ad peripherias* B, E *constituti insistant.*

Nam si fieri potest, sit alter eorum A G C ⌐ D H F. fiatque A G I = D H F. ergo arcus A I
a 26. 3. *a* = D F *b* = A C. *c* Q.E.A.
b *hyp.*
c 9. *ax.*

SCHOL.



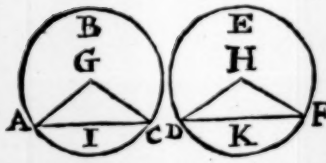
Linea recta E F, *quæ ducta ex* A *medio puncto peripheriæ alicujus* B C, *circulum tangit, parallela est rectæ lineæ* B C, *quæ peripheriam illam subtendit.*

i Duc è centro D ad contactum A rectam D A, & connecte D B, D C.

Latus D G commune est; & D B = D C, atque
a 27. 3. ang. B D A *a* = C D A (ob arcus B A, C A *b* æ-
b *hyp.* quales) *c* ergo anguli ad basim D G B, D G C
c 4. 1. æquales, & *d* proinde recti sunt. Sed interni an-
d 10. *def.* 1. guli G A E, G A F *e* etiam recti sunt. *f* ergo B C,
e *hyp.* E F sunt parallelæ. Q.E.D.
f 28. 1.

PROP.

PROP. XXVIII.



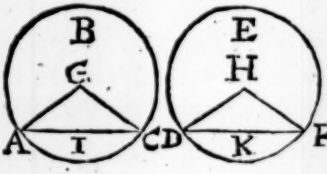
In æqualibus circulis GABC, HDEF, *æquales rectæ lineæ* AC, DF *æquales peripherias auferunt; majorem quidem* ABC *majori* DEF, *minorem autem* AIC *minori* DKF.

E centris G, H, duc GA, GC; & HD, HF. Quoniam GA = HD, & GC = HF, atque AC *a* = DF; *b* erit ang. G = H. *c* ergo arcus AIC = DKF. *d* proinde reliquus ABC = DEF. Q.E.D.

a *hyp.*
b 8. 1.
c 26. 3.
d 3. *ax.*

Quod si ſubtenſa AC ſit ⊏ vel ¬ DF, erit ſimili modo arcus AC ⊏ vel ¬ DF.

PROP. XXIX.



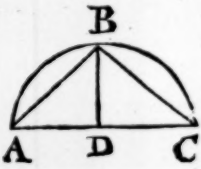
In æqualibus circulis GABC, HDEF, *æquales peripherias* ABC, DEF *æquales rectæ lineæ* AC, DF *ſubtendunt.*

Duc GA, GC; & HD, HF. Quia GA = HD; & GC = HF; & (ob arcus AC, DF *a* pares) etiam ang. G *b* = H; *c* erit baſ. AC = DF. Q.E.D.

a *hyp.*
b 27. 3.
c 4. 1.

Hæc & tres proxime præcedentes intelliguntur etiam de eodem circulo.

PROP. XXX.



Datam peripheriam ABC *biſariam ſecare.*

Duc AC; quam biſeca in D. ex D duc perpendicularem DB occurrentem arcui in B. Dico factum.

Jungantur enim A B, C B. Latus D B commune eſt; & A D *a* = D C; & ang. A D B *b* = C D B. *c* ergo A B = B C. *d* quare arcus A B = B C. Q. E. F.

a *conſt.*
b 12. *ax.*
c 4. 1.
d 28. 3.

PROP. XXXI.

In circulo angulus A B C, *qui in ſemicirculo, rectus eſt; qui autem in majore ſegmento* B A C, *minor recto; qui vero in minore ſegmento* B F C, *major eſt recto. Et inſuper angulus majoris ſegmenti recto quidem major eſt, minoris autem ſegmenti angulus, minor eſt recto.*

Ex centro D duc D B. Quia D B = D A, erit ang. A *a* = D B A. pariter ang. D C B *a* = D B C. *b* ergo ang. A B C = A + A C B *c* = E B C, *d* proinde A B C, & E B C recti ſunt. Q. E. D. *e* ergo B A C acutus eſt. Q. E. D. ergo cum B A C + B F C *f* = 2 Rect. erit B F C obtuſus. denique angulus ſub recta C B, & arcu B A C major eſt recto A B C. factus vero ſub C B, & B F C peripheria minoris ſegmenti, recto E B C *g* minor eſt. Q. E. D.

a 5. 1.
b 2. *ax.*
c 32. 1.
d 10. *def.* 1.
e *Cor.* 17. 1.
f 22. 3.
g 9. *ax.*

SCHOLIUM.

In triangulo rectangulo A B C, *ſi hypotenuſa* A C *biſecetur in* D, *circulus centro* D, *per* A *deſcriptus tranſibit per* B. ut facile ipſe demonſtrabis ex hac, & 21. 1.

PROP. XXXII.

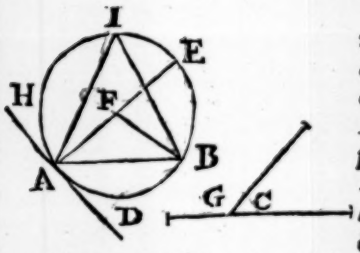
Si circulum tetigerit aliqua recta linea AB, *à contactu autem producatur quædam recta linea* CE *circulum secans: anguli* ECB, ECA, *quos ad contingentem facit, æquales sunt iis, qui in alternis circuli segmentis consistunt, angulis* EDC, EFC.

Sit CD latus anguli EDC perpendiculare ad AB (*a* perinde enim est) *b* ergo C D est diameter. *c* ergo ang. C E D in semicirculo rectus est. *d* ergo ang. D + DCE = Rect. *e* = ECB + DCE. *f* ergo ang. D = ECB. Q.E.D.

Cum igitur ang. ECB + ECA *g* = 2 Rect. *h* = D + F; auferatur hinc inde æquales ECB, & D, *k* remanent ECA = F. Q.E.D.

a 26. 3.
b 19. 3.
c 31. 3.
d 32. 1.
e *Constr.*
f 3. *ax.*
g 13. 1.
h 22. 3.
k 3. *ax.*

PROP. XXXIII.



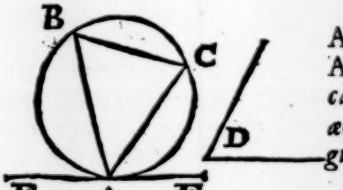
Super data recta linea AB *describere circuli segmentum* AIEB, *quod capiat angulum* AIB *æqualem dato angulo rectilineo* C.

a Fac ang. BAD = C. per A duc AE perpendicularem ad H D. ad alterum terminum datæ AB fac ang. ABF = BAF. cujus alterum latus secet AE in F. centro F per A describe circulum, quod transibit per B (quia ang. FBA *b* = FAB, *c* ideoque FB = FA;) segmentum AIB est id quod quæritur.

a 23. 1.
b *Constr.*
c 6. 1.

Nam quia HD diametro AE perpendicularis
d cor. 16. 3. est, *d* tangit HD circulum, quem secat AB. ergo
e 32. 3. ang. AIB *e*=BAD *f*=C. Q.E.F.
f Constr.

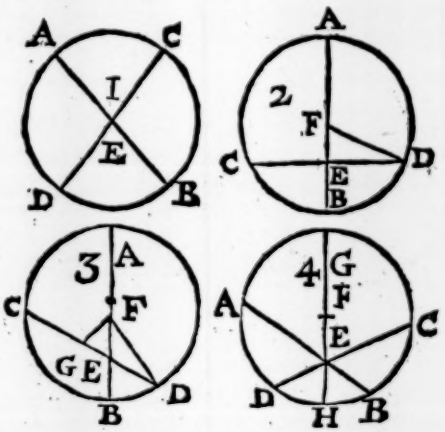
PROP. XXXIV.



A dato circulo ABC *segmentum* ABC *abscindere capiens angulum* B *æqualem dato angulo rectilineo* D.

a 17. 3. *a* Duc rectam EF, quæ tangat
b 23. 1. datum circulum in A. *b* ducatur item AC faciens
c 32. 3. ang. FAC=D. Hæc auferet segmentum ABC
d Constr. capiens angulum B *c*=CAF *d*=D. Q.E.F.

PROP. XXXV.



Si in circulo FBCA *duæ rectæ lineæ* AB, DC *sese mutuo secuerint, rectangulum comprehensum sub*

ſub ſegmentis AE, EB *unum, æquale eſt ei quod ſub ſegmentis* CE, ED *alterum comprehenditur, rectangulo.*

Caſ. 1. Si rectæ ſeſe in centro ſecent, res clara eſt.

2. Si una AB tranſeat per centrum F, & reliquam CD biſecet, duc FD. Eſtque Rectang. AEB + FEq *a* = FBq *b* = FDq *c* = EDq + FEq *d* = CED + FEq. *e* ergo Rectang. AEB = CED. Q.E.D.

a 5. 2.
b *ſch.* 48. 1.
c 47. 1.

3. Si una AB diameter ſit, alteramque CD ſecet inæqualiter, biſeca CD per FG perpendicularem ex centro.

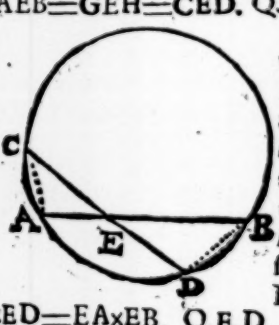
d *hyp.*
e 3. *ax.*

Æquantur iſta
- Rectang. AEB + FEq.
- *f* FBq (FDq)
- *g* FGq + GDq.
- FGq + *h* GEq + Rectang. CED.
- *k* FEq + CED.

f 5. 2.
g 47. 1.
h 5. 2.
k 47. 1.

l Ergo Rectang. AEB = CED. l 3. *ax.*

4 Si neutra rectarum AB, CD per centrum tranſeat, per interſectionis punctum E duc diametrum GH. Per modo demonſtrata Rectang. AEB = GEH = CED. Q.E.D.



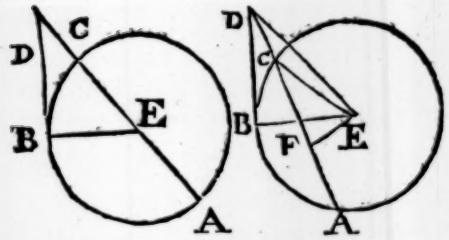
Facilius ſic, & univerſaliter; connecte AC & BD, atque ob angulos *a* CEA, DEB, *b* ipſoſque C, B (ſuper eodem arcu AD) pares; trigona CEA, BED, *c* æquiangula ſunt. *d* ergo CE. EA :: EB. ED. *e* proinde CE × ED = EA × EB. Q.E.D.

a 15. 1.
b 21. 3.
c *cor.* 32. 1.
d 4 6.
e 16. 6.

Quæ ex 6. lib. citantur, tam hic quam in ſeq. ab hac minime pendent; quare iis uti licuit.

PROP.

PROP. XXXVI.



Si extra circulum E B C *ſumatur punctum aliquod* D, *ab eoque puncto in circulum cadant duæ rectæ lineæ* D A, D B; *quarum altera* D A *circulum ſecet, altera vero* D B *tanget; Quod ſub tota ſecante* D A, *& exterius inter punctum* D, *& convexam peripheriam aſſumpta* D C *comprehenditur rectangulum, æquale erit ei, quod à tangente* D B *deſcribitur, quadrato.*

1. *Caſ.* Si ſecans A D tranſeat per centrum E, junge E B; *a* faciet hæc cum D B rectum angulum; quare D Bq + E Bq (E Cq) *b* = E Dq *c* = A D x D C + E Cq *d* ergo A D x D C = D Bq. Q.E.D.

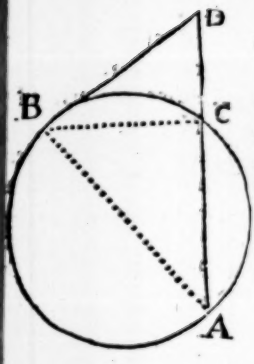
a 18. 3.
b 47. 1.
c 6. 2.
d 3. ax.

2. *Caſ.* Sin A D per centrum non tranſeat, duc E C, E B, E D; atque E F perpend. A D, quare *a* biſecta eſt A C in F.

Quoniam igitur B Dq + E Bq *b* = D Eq *b* = E Fq + F Dq *c* = E Fq + A D C + F Cq *d* = A D C + C Eq (E Bq;) *e* erit B Dq = A D C. Q.E.D.

a 3. 3.
b 47. 1.
c 6. 2.
d 47. 1.
e 3. ax.

Facilius



Facilius ac universalius sic;

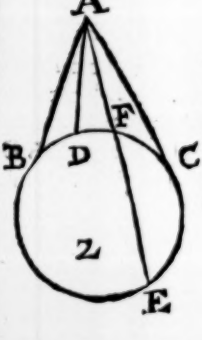
Duc AB, & BC. ac ob angulos A, DBC *a* pares, & D communem, triangula BDC, ADB *b* æquiangula sunt. *c* ergo AD. DB::DB. CD. *d* quare AD×DC = DBq. Q.E.D.

a 32. 3.
b 32. 1.
c 4. 6.
d 17. 6.

Coroll.

1. Hinc, si à puncto quovis A extra circulum assumpto, plurimæ lineæ rectæ AB, AC circulum secantes ducantur, rectangula comprehensa sub totis lineis AB, AC, & partibus externis AE, AF inter se sunt æqualia. Nam si ducatur tangens AD; erit CAF = ADq *a* = BAE.

a 36. 3.



2. Constat etiam duas rectas AB, AC ab eodem puncto A ductas, quæ circulum tangant, inter se æquales esse.

Nam si ducatur AE secans circulum; erit ABq *a* = EAF = ACq.

a 36. 3.

3. Per-

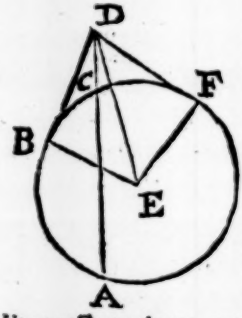
3. Perſpicuum quoque eſt ab eodem puncto A extra circulum aſſumpto, duci tantum poſſe duas lineas, AB, AC quæ circulum tangant.

Nam ſi tertia AD tangere dicatur, erit AD
c 2. cor. c = AB c = AC. d Q. F. N.
d 8. 3.

4. E contrà conſtat, ſi duæ rectæ æquales AB, AC ex puncto quopiam A in convexam peripheriam incidant, & earum una AB circulum tangat, alteram quoque circulum tangere.

Nam ſi fieri poteſt, non AC, ſed altera AD
e 2. cor. circulum tangat. ergo AD e = AC f = AB.
f hyp. g Q. E. A.
g 8. 3.

PROP. XXXVII.



Si extra circulum EBF *ſumatur punctum* D, *ab eoque in circulum cadant duæ rectæ lineæ* DA, DB; *quarum altera* DA *circulum ſecet, altera* DB *in eum incidat; ſit autem quod ſub tota ſecante* DA, *& exterius inter punctum, & convexam peripheriam aſſumpta* DC, *comprehenditur rectangulum, æquale ei, quod ab incidente* DB *deſcribitur quadrato, incidens ipſa* DB *circulum tanget.*

a 17. 3. Ex D *a* ducatur tangens DF; atque ex E centro duc E D, E B, E F. Quia DBq b = ADC
b hyp. c = DFq, d erit D B = D F. Sed E B = E F,
c 36. 3. & latus E D commune eſt; e ergo ang. E B D
d 1. ax & ſcb. 48. 1. = E F D. Sed E F D rectus eſt, f ergo E B D
e 8. 1. etiam rectus eſt. g ergo D B tangit circulum,
f 12. ax. Q. E. D.
g cor. 16. 3.

Coroll.

h 8. 1. Hinc, h ang. EDB = EDF.

LIB.

LIB. IV.

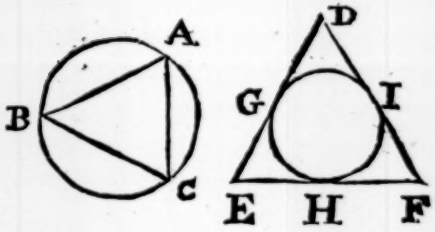
Definitiones.

I. FIgura rectilinea in figura rectilinea inſcribi dicitur, cum ſinguli ejus figuræ, quæ inſcribitur, anguli ſingula latera ejus in qua inſcribitur, tangunt.

Sic triangulum DEF *eſt inſcriptum in triangulo* ABC.

II. Similiter & figura circa figuram deſcribi dicitur, cum ſingula ejus, quæ circumſcribitur, latera ſingulos ejus figuræ angulos tetigerint, circa quam illa deſcribitur.

Ita triangulum A B C *eſt deſcriptum circa triangulum* DEF.



III. Figura rectilinea in circulo inſcribi dicitur, cum ſinguli ejus figuræ, quæ inſcribitur, anguli tetigerint circuli peripheriam.

IV. Figura vero rectilinea circa circulum deſcribi dicitur, cum ſingula latera ejus, quæ circumſcribitur, circuli peripheriam tangunt.

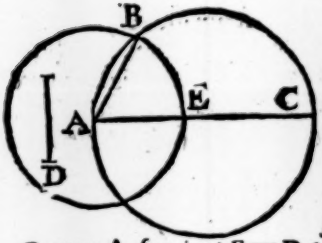
V. Similiter & circulus in figura rectilinea inſcribi dicitur, cum circuli peripheria ſingula latera tangit ejus figuræ, cui inſcribitur.

VI. Circulus autem circa figuram deſcribi di-

dicitur, cum circuli peripheria singulos tangit ejus figuræ, quam circumscribit, angulos.

VII. Recta linea in circulo accommodari, seu coaptari dicitur, cum ejus extrema in circuli peripheria fuerint; ut recta linea AB.

PROP. I. *Probl.* 1.

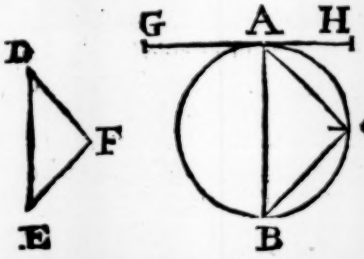


In dato circulo ABC *rectam lineam* AB *accommodare æqualem datæ rectæ lineæ* D, *quæ circuli diametro* AC *non sit major.*

a 3. post. & 3. 1. b 15. def. 1. c constr.

Centro A, spatio AE = D *a* describe circulum dato circulo occurrentem in B. Erit ducta AB *b* = AE *c* = D. Q.E.F.

PROP II. *Probl.* 2.



In dato circulo ABC *triangulum* ABC *describere dato triangulo* DEF *æquiangulum*

Recta GH circulum da-

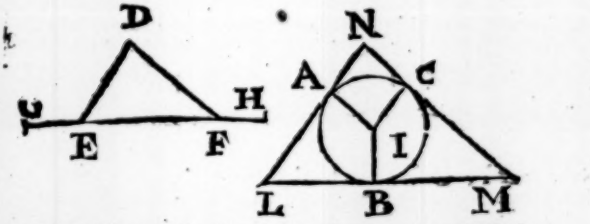
a 17. 3. b 23. 1.

tum *a* tangat in A. *b* Fac ang. HAC = E; *b* & ang. GAB = F, & junge BC. Dico factum.

Nam

Nam ang. B c = H A C d = E; & ang. C c 32. 3.
c = G A B d = F; *e* quare etiam ang. BAC = D. d *constr.*
ergo triang. B A C circulo inſcriptum triangulo e 32. 1.
DEF æquiangulum eſt. Q.E.F.

PROP. III. *Probl.* 3.



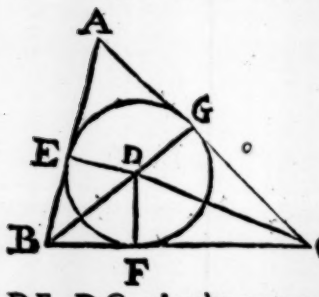
Circa datum circulum I A B C *triangulum* L N M *deſcribere, dato triangulo* D E F *æquiangulum.*

Produc latus E F utrinque. a Fac ad centrum a 23. 1.
I ang. A I B = D E G. & ang. B I C = D F H.
deinde in punctis A, B, C circulum b tangant b 17. 3.
tres rectæ L N, L M, M N. Dico factum.

Nam quod coibunt rectæ L N, L M, M N,
atque ita triangulum conſtituent, patet; c quia c 13. *ax.*
anguli L A I, L B I d recti ſunt, adeoque ducta d 18. 3.
A B angulos faciet L A B, L B A duobus rectis minores. Quoniam igitur ang. A I B + L e = 2 e *Schol.*
Rect. f = D E G + D E F; & A I B g = D E G, h erit 32. 1.
ang. L = D E F. Simili argumento ang. M = D F E. f 13. 1.
k ergo etiam ang. N = D. ergo triang. L N M g *constr.*
circulo circumſcriptum dato E D F eſt æquian- h 3. *ax.*
gulum. Q.E.F. k 32. 1.

PROP.

PROP. IV. *Probl.* 4.



In dato triangulo A B C *circulum* E F G *inscribere.*

a 9. 1. Duos angulos B, & C *a* bisecta rectis BD, CD coeuntibus in D.

b 12. 1. Ex D *b* duc perpendiculares DE, DF, DG. circulus centro D per E descriptus transibit per G, & F, tangetque tria latera trianguli.

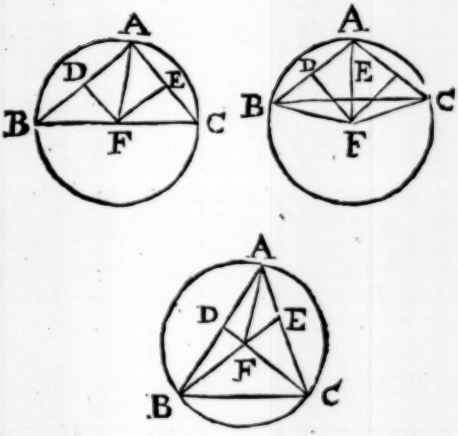
c *constr.* d 12. *ax.* e 26. 1. Nam ang. DBE *c* = DBF; & ang. DEB *d* = DFB; & latus DB commune est: *e* ergo DE = DF. Simili argumento DG = DF. Circulus igitur centro D descriptus transit per E, F, G; & cum anguli ad E, F, G sint recti, tangit omnia trianguli latera. Q. E. F.

Scholium.

Petr. Herig. Hinc, *cognitis lateribus trianguli, invenientur eorum segmenta, quæ fiunt à contactibus circuli inscripti.* Sic,

Sit A B 12, A C 18, B C 16. Erit A B + B C = 28. ex quo subduc 18 = AC = AE + FC, remanet 10 = BE + BF. ergo BE, vel BF = 5. proinde FC, vel C G = 11. quare G A, vel AE = 7.

PROP. V. *Probl.* 5.



Circa datum triangulum ABC *circulum* FABC *deſcribere.*

Latera quævis duo B A, AC *a* biſeca perpendicularibus DF, EF concurrentibus in F. Hoc erit centrum circuli. a 10. & 11. 1.

Nam ducantur rectæ FA, FB, FC. Quoniam AD *b* = DB ; & latus DF commune eſt ; & ang. FDA *c* = FDB, *d* erit FB = FA. eodem modo FC = F**A**. ergo circulus centro F per dati trianguli angulos B, A, **C** tranſibit. Q.E.F. b *conſtr.* c *conſtr.* & 12. *ax.* d 4. 1.

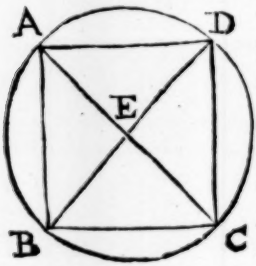
Coroll.

*Hinc, ſi triangulum fuerit acutangulum, centrum cadet intra triangulum; ſi rectangulum, in latus recto angulo oppoſitum ; ſi denique obtuſangulum, extra triangulum. * 31. 3.

Schol.

Eadem methodo deſcribetur circulus, qui tranſeat per data tria puncta, non in una recta linea exiſtentia.

PROP. IV. *Probl.* 6.



In dato circulo EABCD *quadratum* ABCD *inſcribere.*

a 11. 1. *a* Duc diametros AC, BD ſe mutuo ſecantes ad angulos rectos in centro E. junge harum terminos rectis AB, BC, CD, DA. Dico factum. Nam quia 4 anguli ad

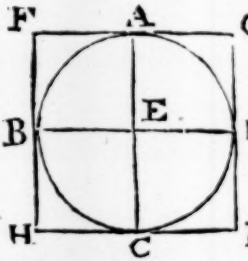
b 26. 3. E recti ſunt, *b* arcus, & *c* ſubtenſæ AB, BC,

c 29. 3. CD, DA pares ſunt. ergo ABCD æquilaterum eſt; ejuſque omnes anguli in ſemicirculis, adeo-

d 31. 3. que *d* recti ſunt. *e* ergo ABCD eſt quadratum,

e 29 *def.* 1. dato circulo inſcriptum. Q.E.F.

PROP. VII. *Probl.* 7.



Circa datum circulum EABCD *quadratum* FHIG *deſcribere.*

Duc diametros AC, BD ſe mutuo ſecantes perpendiculariter. per harum extrema *a* duc tangentes concurrentes in F, H, I, G. Dico

a 17. 3.

b 18. 3. factum. Nam ob angulos ad A, & C *b* rectos,

c 28. 1. *c* erit FG parall. HI. eodem modo FH parall. GI. ergo FHIG eſt parallelogrammum; & quidem rectangulum. ſed & æquilaterum, quia FG *d* = HI *d* = BD *e* = CA *d* = FH *d* = GI.

d 34. 1.

e 15. *def.* 1. quare FHIG eſt *f* quadratum, dato circulo cir-

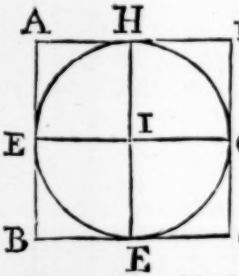
f 29. *def.* 1. cumſcriptum. Q.E.F.

SCHOL.

SCHOL.

Quadratum ABCD circulo circumſcriptum, duplum eſt quadrati EFGH circulo inſcripti. Nam rectang. HB = 2 HEF. & HD = 2 HGF. per 41. 1.

PROP. VIII. *Probl.* 8.



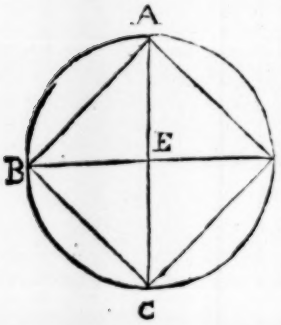
In dato quadrato A B C D *circulum* I E F G H *inſcribere.*

Latera quadrati biſeca in punctis H, E, F, G; junge HF, EG ſeſe ſecantes in I. circulus centro I per H deſcriptus quadrato inſcribetur.

Nam quia A H, B F *a* pares ac *b* parallelæ ſunt, *c* erit AB parall. HF parall. DC. eodem modo AD parall. E G parall. B C. ergo I A, I D, I B, I C ſunt parallelogramma. Ergo AH *d* = AE *e* = HI = EI = IF = IG. Circulus igitur centro I per H deſcriptus tranſibit per H, E, F, G, tangetque quadrati latera, cum anguli ad H, E, F, G ſint recti. Q.E.F.

a 7. *ax.*
b 28. 1.
c 33. 1.
d 7. *ax.*
e 34. 1.

PROP. IX. *Probl.* 9.



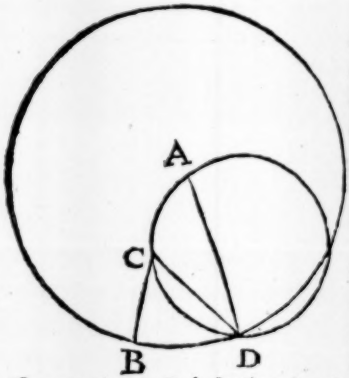
Circa datum quadratum ABCD *circulum* EABCD *describere.*

Duc diametros AC, BD secantes in E. centro E per A describe circulum. Is dato quadrato circumscriptus est.

Nam anguli ABD, & BAC *a* semirecti sunt; *b* ergo EA=EB. eodem modo EA=ED=EC. Circulus igitur centro E descriptus per A, B, C, D dati quadrati angulos transit. Q.E.F.

a 4. *cor.* 32. 1.
b 6. 1.

PROP. X. *Probl.* 10.



Isosceles triangulum ABD *constituere, quod habeat utrumque eorum quæ ad basim sunt angulorum* B *&* ADB *duplum reliqui* A.

Accipe quamvis rectam AB, quam *a* seca in C, ita ut AB × BC = AC q. Centro A per B describe circulum ABD; in hoc *b* accommoda BD=AC, & junge AD. erit triang. ABD quod quæritur.

a 11. 2.
b 1. 4.

Nam duc DC; & per CDA *c* describe circulum.

c 5. 4.

lum. Quoniam ABxBC=ACq. *d* liquet BD tangere circulum ACD, quem secat CD. *e* ergo ang. BDC=A. ergo ang. BDC+CDA *f*= A+CDA= *g* BCD. sed BDC+CDA= BDA *h*= CBD. ergo DC *l*=DB *m*=AC. *n* quare ang. CDA= A=BDC. ergo ADB = 2 A=ABD. Q. E. F.

d 37. 3.
e 32. 3.
f 2. *ax.*
g 32. 1.
h 5. 1.
k 1. *ax.*
l 6. 1.
m *constr.*
n 5. 1.

Hæc constructio Analytice indagatur sic; Factum sit; & angulum BDA bisecet recta DC. *a* ergo DA. DB :: CA. CB. item ob ang. CDA *b*= ½ ADB *c*=A, *d* est CA=DC. ac ob ang. DCB *e* = A+CDA= 2 A *e* = B, *d* erit DB=DC. *f* ergo DB=CA. proinde DA. (*e* BA.) CA :: CA. CB. *g* unde BAxCB=CAq.

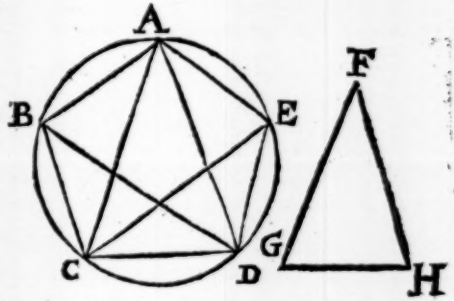
a 3. 6.
b *constr.*
c *hyp.*
d 6. 1.
e 32. 1.
f 1. *ax.*
g 17. 6.

Coroll.

Cum omnes anguli A, B, D *b* conficiant ⅔ 2 Rect. (2 Rect.) liquet A esse ⅓ 2 Rect.

h 32. 1.

PROP. XI. *Probl.* 11.



In dato circulo ABCDE *pentagonum æquilaterum & æquiangulum* ABCDE *inscribere.*

a 10. 4. *a* Describe triangulum Isosceles FGH, habens utrumque angulorum ad basim duplum anguli
b 2. 4. ad verticem. *b* Huic æquiangulum CAD inscribe circulo. Angulos ad basim ACD, & ADC
c 9. 1. *c* biseca rectis DB, CE occurrentibus circumferentiæ in B, & E connecte rectas CB, BA, AE, ED. Dico factum.

Nam ex constr. liquet quinque angulos CAD,
d 26. 3. CDB, BDA, DCE, ECA pares esse; quare *d*
e 29. 3. arcus *e* & subtensæ DC, CB, BA, AE, DE æquantur. Pentagonum igitur æquilaterum est.
f 27. 3. Est vero etiam æquiangulum, *f* quia ejus anguli
g 2. ax. BAE, AED, &c. insistunt arcubus *g* æqualibus BCDE, ABCD, &c.

Hujus problematis praxis facilior tradetur ad 10. 13.

Coroll.

Hinc, angulus pentagoni æquilateri & æquianguli æquatur $\frac{3}{5}$ 2 Rect. vel $\frac{6}{5}$ Rect.

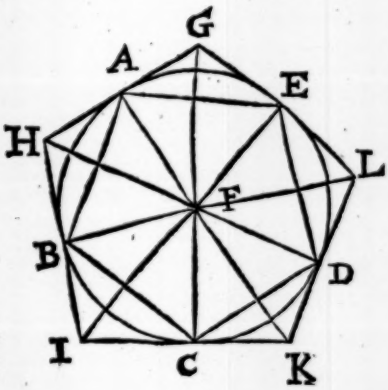
Schol.

Pet. Herig. *Universaliter figuræ imparium laterum inscribuntur circulo beneficio triangulorum Isoscelium, quorum anguli æquales ad basim multiplices sunt eorum, qui ad verticem sunt, angulorum; parium vero laterum figuræ in circulo inscribuntur ope Isoscelium triangulorum, quorum anguli ad basim multiplices sesquialteri sunt eorum, qui ad verticem sunt, angulorum.*

Ut in triangulo Isoscele CAB, si ang. A = 3 C = B; AB erit latus Heptagoni. Si A = 4 C; erit AB latus Enneagoni, &c. Sin vero A = $1\frac{1}{2}$ C, erit AB latus quadrati. Et si A = $2\frac{1}{2}$ C, subten-

ſubtendet A B ſextam partem circumferentiæ: pariterque ſi A = 3 ½ C; erit A B latus octagoni, &c.

PROP. XII. *Probl.* 12.



Circa datum circulum FABCDE *pentagonum æquilaterum & æquiangulum* HIKLG *deſcribere.*

a Inſcribe pentagonum ABCDE æquilaterum & æquiangulum; duc è centro rectas F A, F B, FC, FD, FE, iiſque totidem perpendiculares GAH, HBI, ICK, KDL, LEG concurrentes in punctis H, I, K, L, G. Dico factum. Nam quia GA, GE ex uno puncto G *b* tangunt circulum, *c* erit G A = G E. *d* ergo ang. G F A = G F E. ergo ang. AFE = 2 G F A. eodem modo ang. AFH = HFB; & proinde ang. AFB = 2 AFH. Sed ang. A F E *e* = AFB. *f* ergo ang. GFA = A F H. ſed & ang. FAH *g* = FAG; & latus FA eſt commune, *h* ergo HA = AG = G E = E L, &c. *k* ergo H G, G L, L K, K I, I H latera pentagoni æquantur: ſed & anguli etiam, utpote *l* æqualium AGF, AHF, &c. dupli; ergo, &c.

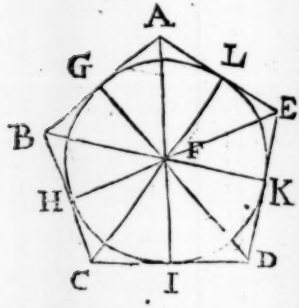
a 11. 4.
b *cor.* 16 3.
c 2. *cor.* 36. 3.
d 8. 1.
e 27. 3.
f 7. *ax.*
g 12. *ax.*
h 26. 1.
k 2 *ax.*
l 2. *cor.* 32. 1

Coroll.

Eodem pacto, si in circulo quæcunque figura æquilatera & æquiangula describatur, & ad extrema semidiametrorum ex centro ad angulos ductarum, excitentur lineæ perpendiculares, hæ perpendiculares constituent aliam figuram totidem laterum & angulorum æqualium circulo circumscriptam.

PROP. XIII. *Probl.* 13.

In dato pentagono æquilatero & æquiangulo A B C D E *circulum* F G H K *inscribere.*



a 9. 1. Duos pentagoni angulos A, & B a biseca rectis A F, B F concurrentibus in F. Ex F duc perpendiculares F G, FH, FI, FK, FL. Circulus centro F per G descriptus tanget omnia pentagoni latera.

b *hyp.* Duc FC, F D, F E. Quoniam BA *b* = BC;
c *constr.* & latus B F commune est; & ang. F B A *c* =
d 4. 1. FBC, *d* erit AF = FC; & ang. FAB = FCB.
e *hyp.* Sed ang. F A B *e* = ½ BAE *e* = ½ BCD. ergo ang. FCB = ½ BCD. eodem modo anguli totales C, D, E omnes bisecti sunt. Quum igitur
f 12. *ax.* ang. FGB *f* = FHB, & ang. FBH = FBG, &
g 26. 1. latus FB sit commune, *g* erit FG = FH. similiter omnes FH, FI, FK, FL, FG æquantur. Ergo circulus centro F per G descriptus transit per
h *cor.* 16. 3. H, I, K, L; *b* tangitque pentagoni latera, cum anguli ad ea puncta sint recti. Q.E.F.

Coroll.

Hinc, si duo anguli proximi figuræ æquilateræ & æquiangulæ bisecentur, & à puncto, in quo coeunt lineæ angulos bisecantes, ducantur rectæ

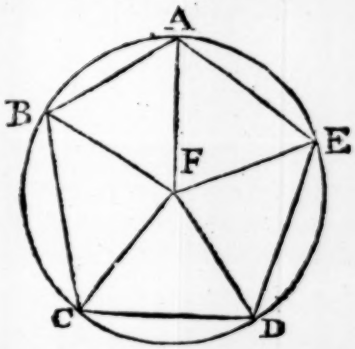
lineæ

lineæ ad reliquos figuræ angulos, omnes anguli figuræ erunt biſecti.

Schol.

Eadem methodo in qualibet figura æquilatera & æquiangula circulus deſcribetur.

PROP. XIV. *Probl.* 14.



Circa datum Pentagonum æquilaterum & æquiangulum ABCDE *circulum* FABCDE *deſcribere.*

Duos pentagoni angulos biſeca rectis AF, BF concurrentibus in F. Circulus centro F per A deſcriptus pentagono circumſcribitur.

Ducantur enim FC, FD, FE. *a* Biſecti itaque ſunt anguli C, D, E. *b* ergo FA, FB, FC, FD, FE æquantur, ergo circulus centro F deſcriptus, per A, B, C, D, E, pentagoni angulos tranſibit. Q. E. F.

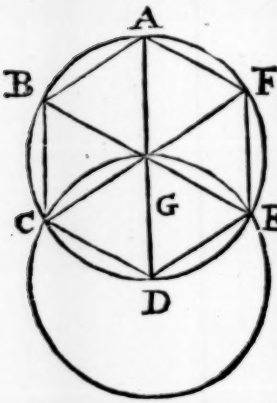
a cor. 13. 4.
b 6. 1.

Schol.

Eadem arte circa quamlibet figuram æquilateram & æquiangulam circulus deſcribetur.

PROP.

PROP. XV. *Probl.* 15.



In dato circulo GABCDEF *hexagonum, & æquilaterum & æquiangulum* ABCDEF *inscribere.*

Duc diametrum AD; centro D per centrum G describe circulum, qui datum secet in C, & E. duc diametros CF, EB. junge AB, BC, CD, DE, EF, FA. Dico factum.

a 32. 1. b 15. 1. c *cor.* 13. 1. d 26. 3. e 29. 3. f 27. 3.

Nam ang. CGD $a = \frac{1}{3}$ 2 Rect. a = DGE b = AGF b = AGB. c ergo BGC = $\frac{1}{3}$ 2 Rect. = FGE. d ergo arcus e & subtensæ AB, BC, CD, DE, EF æquantur. Hexagonum igitur æquilaterum est: sed & æquiangulum f quia singuli ejus anguli arcubus insistunt æqualibus. Q.E.F.

Coroll.

1. Hinc latus Hexagoni circulo inscripti semidiametro æquale est.

2. Hinc facile triangulum æquilaterum ACE in circulo describetur.

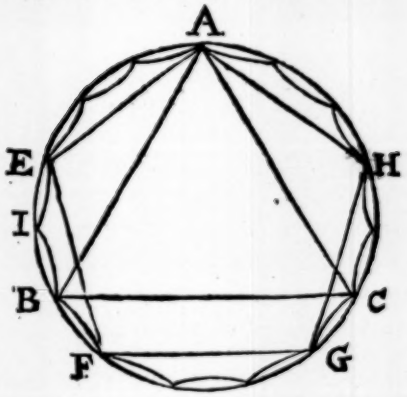
Schol. Probl.

And. Terq. a 1. 1.

Hexagonum ordinatum super data recta CD *ita construes.* a Fac triangulum CGD æquilaterum super data CD. centro G per C, & D describe circulum. Is capiet Hexagonum super data CD.

PROP.

PROP. XVI. *Probl.* 16.



In dato circulo AEBC *quindecagonum æquilaterum & æquiangulum inſcribere.*

Dato circulo *a* inſcribe pentagonum æquilaterum AEFGH; *b* itemque triangulum æquilaterum ABC. erit BF latus quindecagoni quæſiti. a 11. 4. b 3. 4.

Nam arcus AB *c* eſt $\frac{1}{3}$, vel $\frac{5}{15}$ peripheriæ, cujus AF eſt $\frac{2}{5}$ vel $\frac{6}{15}$ ergo reliquus BF $= \frac{1}{15}$ periph. c *conſtr.*

ergo quindecagonum, cujus latus BF, æquilaterum eſt; ſed & æquiangulum, *d* cum ſinguli ejus anguli arcubus inſiſtant æqualibus, quorum unuſquiſque eſt $\frac{1}{15}$ totius circumferentiæ. ergo, &c. d 27. 3.

Schol.

Circulus dividitur Geometrice in partes
- 4,8,16,&c. per 6,4,& 9,1.
- 3,6,12,&c. per 15,4,& 9,1.
- 5,10,20,&c. per 11,4,& 9,1.
- 15,30,60,&c. per 16,4,& 9,1.

Cæterum diviſio circumferentiæ in partes datas etiamnum deſideratur; quare pro figurarum quarumcunq; ordinatarum conſtructionibus ſæpe ad mechanica artificia recurrendum eſt, propter quæ Geometræ practici conſulendi ſunt.

LIB.

LIB. V.

Definitiones.

I. PArs eſt magnitudo magnitudinis, minor majoris, cum minor metitur majorem.

II. Multiplex autem eſt major minoris, cum minor metitur majorem.

III. Ratio eſt duarum magnitudinum ejuſdem generis mutua quædam ſecundum quantitatem habitudo.

In omni ratione ea quantitas, quæ ad aliam refertur, dicitur antecedens rationis; ea vero, ad quam alia refertur, conſequens rationis dici ſolet. ut in ratione 6 ad 4; antecedens eſt 6, & conſequens 4.

Nota.

Cujuſque rationis quantitas innoteſcit dividendo antecedentem per conſequentem. ut ratio 12 ad 5 effertur per $\frac{12}{5}$ item quantitas rationis A ad B eſt $\frac{A}{B}$. Quare non raro brevitatis cauſa, quantitates rationum ſic deſignamus, $\frac{A}{B} \sqsubset$, vel $=$, vel $\sqsupset \frac{C}{D}$; hoc eſt, ratio A ad B major eſt ratione C ad D, vel ei æqualis, vel minor. Quod probe animadvertat, quiſquis hæc legere volet.

Rationis, ſive proportionis ſpecies, ac diviſiones vide apud interpretes.

IV. Proportio vero eſt rationum ſimilitudo.

Rectius quæ hic vertitur proportio, proportionalitas, ſive analogia dicitur; nam proportio idem denotat quod ratio, ut pleriſque placet.

V. Rationem habere inter ſe magnitudines dicuntur, quæ poſſunt multiplicatæ ſe mutuo ſuperare.

VI. In

E, 12. | A, 4. B, 6. | G, 24. VI. In eadem ratione magnitudines
F, 30. | C, 10. D, 15. | H, 60.

dicuntur esse, prima A ad secundam B, & tertia C ad quartam D, cum primæ A, & tertiæ C æquemultiplicia E, & F à secundæ B, & quartæ D æquemultiplicibus G, & H, qualiscunque sit hæc multiplicatio, utrumque E, F ab utroque G, H, vel una deficiunt, vel una æqualia sunt, vel una excedunt, si ea sumantur E, G; & F, H quæ inter se respondent.

Hujus nota est ::. ut A. B :: C. D. *hoc est* A *ad* B, *&* C *ad* D *in eadem sunt ratione. aliquando sic scribimus* $\frac{A}{B} = \frac{C}{D}$ *id est*, A. B :: C. D.

VII. Eandem autem habentes rationem (A. B :: C. D) proportionales vocentur.

E, 30. | A, 6. B, 4. | G, 28. VIII. Cum vero æquemultiplicium, E multiplex primæ magnitudinis A excesserit G multiplicem secundæ B; at F multiplex tertiæ C non excesserit H multiplicem quartæ D; tunc prima A ad secundam B majorem rationem habere dicetur, quam tertia C ad quartam D.
F, 60. | C, 12. D, 9. | H, 63.

Si $\frac{A}{B} \sqsubset \frac{C}{D}$, *necessarium non est ex hac definitione, ut* E *semper excedat* G; *quum* F *minor est quam* H; *sed conceditur hoc fieri posse.*

IX. Proportio autem in tribus terminis paucissimis consistit. *Quorum secundus est instar duorum.*

X. Cum autem tres magnitudines A, B, C proportionales fuerint, prima A ad tertiam C duplicatam rationem habere dicetur ejus, quam habet ad secundam B: at quum quatuor magnitudines A, B, C, D, proportionales fuerint, prima A ad quartam D triplicatam rationem habere

dicetur

dicetur ejus, quam habet ad ſecundam B; & ſemper deinceps uno amplius, quamdiu proportio extiterit.

Duplicata ratio exprimitur ſic $\frac{A}{C} = \frac{A}{B}$ *bis. Hoc eſt, ratio* A *ad* C *duplicata eſt rationis* A *ad* B. *Triplicata autem ſic* $\frac{A}{D} = \frac{A}{B}$ *ter.* id eſt, *ratio* A *ad* D *triplicata eſt rationis* A *ad* B.

$\div$ *denotat continue proportionales.* ut A, B, C, D; *item* 2,6,18,54 ſunt $\div$

XI. Homologæ, ſeu ſimiles ratione, magnitudines dicuntur, antecedentes quidem antecedentibus, conſequentes vero conſequentibus.

Ut ſi A.B :: C.D; *tam* A *&* C, *quam* B *&* D *homologæ magnitudines dicuntur.*

XII. Alterna ratio, eſt ſumptio antecedentis ad antecedentem, & conſequentis ad conſequentem.

Ut ſit A. B :: C. D. *ergo alterne, vel permutando, vel viciſſim,* A.C :: B.D. *per* 16.5.

In hac definitione, & 5. ſequentibus imponuntur nomina ſex modis argumentandi, quibus mathematici frequenter utuntur; quarum illationum vis innititur propoſitionibus hujus libri, quæ in explicationibus citantur.

XIII. Inverſa ratio, eſt ſumptio conſequentis ceu antecedentis, ad antecedentem velut ad conſequentem.

Ut A. B :: C. D. *ergo inverſe,* B. A :: D. C. *per cor.* 4, 5.

XIV. Compoſitio rationis, eſt ſumptio antecedentis cum conſequente, ceu unius, ad ipſam conſequentem.

Ut A.B :: C. D. *ergo componendo,* A + B. B :: C + D.D *per* 18.5.

XV. Diviſio rationis, eſt ſumptio exceſſus, quo conſequentem ſuperat antecedens, ad ipſam conſequentem.

Ut A. B :: C. D. *ergo dividendo,* A-B. B :: C- D. D. *per* 17.5.

XVI. Converſio rationis, eſt ſumptio antecedentis ad exceſſum, quo ſuperat antecedens ipſam conſequentem.

Ut A. B :: C. D. *ergo per converſam rationem,* A.A-B :: C. C- D. *per* cor. 19.5.

XVII. Ex æqualitate ratio eſt, ſi plures duabus ſint magnitudines, & his aliæ multitudine pares, quæ binæ ſumantur, & in eadem ratione; cum ut in primis magnitudinibus prima ad ultimam, ſic & in ſecundis magnitudinibus prima ad ultimam ſeſe habuerit. Vel aliter; ſumptio extremorum, per ſubductionem mediorum.

XVIII. Ordinata proportio eſt, cum fuerit quemadmodum antecedens ad conſequentem, ita antecedens ad conſequentem: fuerit etiam ut conſequens ad aliud quidpiam, ita conſequens ad aliud quidpiam.

Ut ſi A. B :: D. E. *item* B. C :: E. F. *erit ex æquo* A. C :: D. F. *per* 22.5.

XIX. Perturbata autem proportio eſt; cum tribus poſitis magnitudinibus, & aliis, quæ ſint his multitudine pares, ut in primis quidem magnitudinibus ſe habet antecedens ad conſequentem, ita in ſecundis magnitudinibus antecedens ad conſequentem: ut autem in primis magnitudinibus conſequens ad aliud quidpiam, ſic in ſecundis magnitudinibus aliud quidpiam ad antecedentem.

Ut ſi A. B :: F. G. *item* B. C :: E. F. *erit ex æquo perturbate* A. C :: E. G. *per* 23. 5.

XX. Quotlibet magnitudinibus ordine poſitis, proportio primæ ad ultimam componitur ex proportionibus primæ ad ſecundam, & ſecundæ ad tertiam, & tertiæ ad quartam, & ita deinceps, donec extiterit proportio.

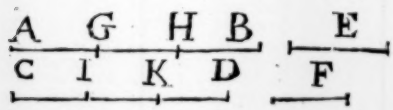
Sint

Sint quotcunque A, B, C, D; ex hac def. $\frac{A}{D}=\frac{A}{D}+\frac{B}{C}+\frac{C}{D}$.

Axioma.

Æquemultiplices eidem multiplici, sunt quoq; inter se æquemultiplices.

PROP. I.



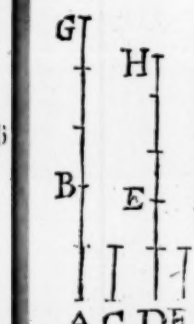
Si sint quotcunque magnitudines AB, CD, *quotcunque magnitudinum* E, F *æqualium numero, singulæ singularum, æquemultiplices; quam multiplex est unius* E *una magnitudo* AB, *tam multiplices erunt & omnes* AB+CD *omnium* E+F.

Sint AG, GH, HB partes quantitatis AB ipsi E æquales. item CI, IK, KD partes quantitatis CD ipsi F pares. Harum numerus illarum numero æqualis ponitur. Quum igitur AG+CI =E+F; & GH+IK=E+F; & HB+KD=E+F, liquet AB+CD æque multoties continere E+F, ac una AB unam E continet. Q.E.D.

a 2. ax.

PROP.

PROP. II.



Si prima A B *ſecundæ* C *æque fuerit multiplex, atque tertia* D E *quartæ* F; *fuerit autem & quinta* B G *ſecundæ* C *æque multiplex, atque ſexta* E H *quartæ* F, *erit & compoſita prima cum quinta* (A G) *ſecundæ* C *æque multiplex, atque tertia cum ſexta* (D H) *quartæ* F.

Numerus partium in A B ipſi C æqualium æqualis ponitur numero partium in D E ipſi F æqualium. Item numerus partium in B G ponitur æqualis numero partium in E H. *a* ergo numerus partium in A B + B G æquatur numero partium in D E + E H. hoc eſt tota A G æquemultiplex eſt ipſius C, atque tota G H ipſius F. Q. E. D. *a* 2. *ax.* *b*

PROP. III.

Sit prima A *ſecundæ* B *æquemultiplex, atque tertia* C *quartæ* D; *ſumantur autem* E I, F M *æquemultiplices primæ & tertiæ*; *erit & ex æquo, ſumptarum utraque utriuſque æquemultiplex: altera quidem* E I *ſecundæ* B, *altera autem* F M *quartæ* D.

Sint E G, G H, H I partes multiplicis E I ipſi A pares; item F K, K L, L M partes multiplicis F M ipſi C æquales. Harum numerus illarum numero æquatur. Porro A, id eſt E G, vel G H, vel G I ipſius B ponitur æquemultiplex atque C, vel F K, &c. ipſius D. *a* *hyp.*

G *b* ergo

b 2. 5. *b* ergo E G + G H æquemultiplex eſt ſecundæ
c 2. 5. B, atque F K + K L quartæ D. *c* Simili argumento E I (E H + H I) tam multiplex eſt ipſius B, quam F M (F L + L N) ipſius D. Q. E. D.

PROP. IV.

Si prima A *ad ſecundam* B *eandem habuerit rationem, & tertia* C *ad quartam* D; *etiam* E *&* F *æquemultiplices primæ* A, *& tertiæ* C *ad* G, *&* H *æquemultiplices ſecundæ* B, *& quartæ* D, *juxta quamvis multiplicationem, eandem habebunt rationem, ſi prout inter ſe reſpondent, ita ſumptæ fuerint.* (E. G :: F. H.)

Sume I, & K ipſarum E, & F; item L & M ipſarum G, & H æ-
a 3. 5. quemultiplices. *a* Erit I ipſius A æquemultiplex atque K ipſius C; *a* pariterque L tam multiplex ipſius B quam M ipſius D. Itaque
b *hyp.* cum ſit A. B *b* :: C. D; juxta 6 *def.* ſi I ⊏, =, ⊐ L; conſequenter pari modo K ⊏, =, ⊐ M. ergo cum I, & K ipſarum E, & F ſumptæ ſint æquemultiplices, atque L, & M ipſarum G & H; erit juxta 7. *def.* E. G :: F. H. Q. E. D.

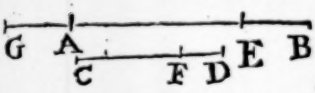
Coroll.

Hinc demonſtrari ſolet inverſa ratio.

Nam quoniam A. B :: C. D, ſi E ⊏, =, ⊐
c 6. *def* 5. G, *c* erit ſimiliter F ⊏, =, ⊐ H. ergo liquet, quod ſi G ⊏, =, ⊐ E, eſſe H ⊏, =, ⊐ F.
d 6. *def.* 5. *d* ergo B, A :: D. C. Q. E. D.

PROP.

PROP. V.

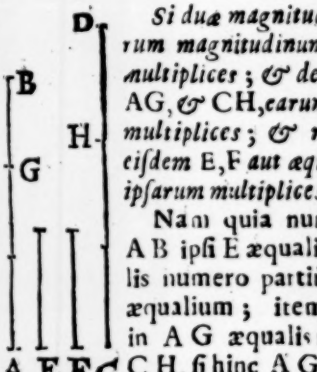


Si magnitudo AB *magnitudinis* CD *æque fuerit multiplex, atque ablata* AE *ablata* CF; *etiam reliqua* EB *reliquæ* FD *ita multiplex erit, ut tota* AB *totius* CD.

Accipe aliam quandam GA, quæ reliquæ FD ita sit multiplex, atque tota AB totius CD, vel ablata AE ablatæ CF. *a* ergo tota GA + AE totius CF + FD æquemultiplex est, ac una AE unius CF, hoc est, ac AB ipsius CD. *b* ergo GE = AB. *c* proinde, ablata communi AE, manet GA = EB. ergo, &c.

a 1. 5.
b 6. ax.
c 3. ax.

PROP. VI.

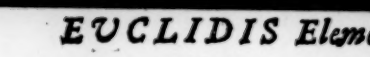


Si duæ magnitudines AB, CD *duarum magnitudinum* E, F *sint æquemultiplices; & detractæ quædam sint,* AG, *&* CH, *earundem* E, *&* F *æquemultiplices; & reliquæ* GB, HD *eisdem* E, F *aut æquales sunt, aut æque ipsarum multiplices.*

Nam quia numerus partium in AB ipsi E æqualium ponitur æqualis numero partium in CD ipsi F æqualium; item numerus partium in AG æqualis numero partium in CH. si hinc AG, inde CH detrahatur, *a* remanet numerus partium in reliqua GB æqualis numero partium in HD. ergo si GB sit E semel, erit HD etiam C semel. si GB sit E aliquoties, erit HD etiam C toties accepta. Q.E.D.

a 3. ax.

PROP. VII.



Æquales A & B *ad eandem* C *eandem habent rationem; & eadem* C *ad æquales* A & B.

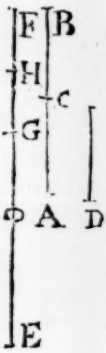
Sumantur D & E æqualium A & B æquemultiplices, & F utcunque multiplex ipsius C; a erit D = E. quare si D ⊏, =, ⊐ F, erit similiter E ⊏, =, ⊐ F. b ergo A. C :: B. C. inverse igitur C.A c :: C.B. Q.E.D.

a 6. ax.
b 6. def. 5.
c cor. 4. 5.

Schol.

Si loco multiplicis F sumantur duæ æquemultiplices, eodem modo ostendetur æquales magnitudines ad alias inter se æquales eandem habere rationem.

PROP. VIII.



Inæqualium magnitudinum AB, AC, *major* AB *ad eandem* D *majorem rationem habet, quam minor* AC. *Et eadem* D *ad minorem* AC *majorem rationem habet, quam ad majorem* AB.

Sume EF, EG, ipsarum AB, AC, æquemultiplices, ita ut E H ipsius D multiplex, major sit quam E G, at minor quam E F. (Quod facile continget, si utraque E G, G F majores accipiantur ipsa D.) Liquet juxta 8. def. 5. fore $\frac{AB}{D} \sqsubset \frac{AC}{D}$; ac $\frac{D}{AB} \sqsupset \frac{D}{A}$. Quæ E.D.

Rursus quia I K ⊏ H G, at I K ⊐ H F (ut d 8. def. 5. prius dictum) d erit $\frac{D}{C} \sqsubset \frac{D}{AB}$. Q.E.D.

d 8. def. 5.

PROP.

PROP. IX.

Quæ ad eandem eandem habent rationem, æquales ſunt inter ſe. Et ad quas eadem eandem habet rationem, eæ quoque ſunt inter ſe æquales.

1. *Hyp.* Sit A. C :: B. C. dico A = B. Nam ſit A ⊏, vel ⊐ B, *a* erit ideo $\frac{A}{C}$ ⊏, vel ⊐ $\frac{B}{C}$. contra Hyp. a 8. 5.

2. *Hyp.* Sit C. B :: C. A, dico A = B, nam ſit A ⊏ B. *b* ergo $\frac{C}{B}$ ⊏ $\frac{C}{A}$. contra Hyp. b 8. 5.

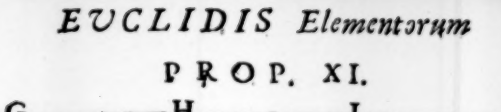
PROP. X.

Ad eandem magnitudinem rationem habentium, quæ majorem rationem habet, illa major eſt: ad quam vero eadem majorem rationem habet, illa minor eſt.

1. *Hyp.* Sit $\frac{A}{C}$ ⊏ $\frac{B}{C}$. Dico A ⊏ B. Nam ſi dicatur A = B, *a* erit A. C :: B. C. contra Hyp. a 7. 5.
Sin A ⊐ B, *b* erit $\frac{A}{C}$ ⊐ $\frac{B}{C}$ etiam contra Hyp. b 8. 5.

2. *Hyp.* Sit $\frac{C}{B}$ ⊏ $\frac{C}{A}$. Dico B ⊐ A. Nam dic B = A, *c* ergo C. B :: C. A. contra Hyp. vel dic B ⊏ A. *d* ergo $\frac{C}{A}$ ⊏ $\frac{C}{B}$ etiam contra Hyp. c 7. 5. d 8. 5.

PROP. XI.



Quæ eidem sunt eædem rationes, & inter se sunt eædem.

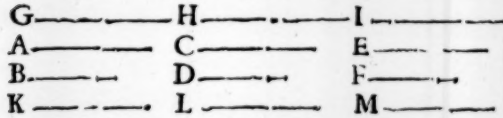
Sit A. B :: E. F. item C. D :: E. F. dico A. B :: C. D. sume ipsarum A, C, E æquemultiplices G, H, I; atque ipsarum B, D, F æquemultiplices K, L, M Et quoniam *a* A. B :: E. F. si G ⊏, =, ⊐ K, *b* erit pari modo I ⊏, =, ⊐ M. pariterque quia *a* E. F :: C. D. si I ⊏, =, ⊐ M, *b* erit H similiter ⊏, =, ⊐ L. ergo si G ⊏, =, ⊐ K, erit similiter H ⊏, =, ⊐ L. *c* quare A. B :: C. D. Q. E. D.

a *hyp.*
b *6. def. 5.*
c *6. def. 5.*

Schol.

Quæ eisdem rationibus sunt eædem rationes sunt quoque inter se eædem.

PROP. XII.



Si sint magnitudines quotcunque A, *&* B; C, D; E, *&* F *proportionales; quemadmodum se habuerit una antecedentium* A *ad unam consequentium* B, *ita se habebunt omnes antecedentes,* A, C, E *ad omnes consequentes,* B, D, F.

Sume antecedentium æquemultiplices G, H, I; & consequentium K, L, M. Quoniam quare multiplex est una G unius A, *a* tam multiplices sunt omnes G, H, I omnium A, C, E; pariterque quam multiplex est una K unius B, *a* tam multiplices sunt omnes K, L, M omnium B, D, F. Porro ob *b* A. B :: C. D :: E. F. si G ⊏, =, ⊐ K, erit similiter

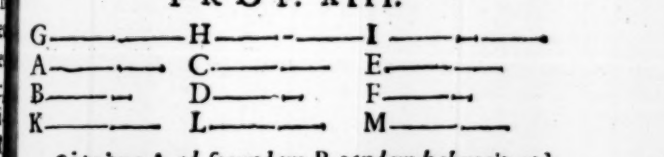
a *1. 5.*
b *hyp.*

H ⊏,=,⊐ L, & I ⊏,=,⊐ M, & proinde si G ⊏,=,⊐ K, erit simili modo G+H+I ⊏,=,⊐ K+L+M. *c* quare A. B :: A+C+E. B+D+F. Q E.D. c 6. *def.* 5.

Coroll.

Hinc, si similia proportionalia similibus proportionalibus addantur, tota erunt proportionalia.

PROP. XIII.



Si prima A *ad secundam* B *eandem habuerit rationem quam tertia* C *ad quartam* D; *tertia vero* C *ad quartam* D *majorem habuerit rationem, quam quinta* E *ad sextam* F; *prima quoque* A *ad secundam* B *majorem rationem habebit, quam quinta* E *ad sextam* F.

Sume ipsarum A, C, E æquemultiplices G, H, I : ipsarumque B, D, F æquemultiplices K, L, M. Quia A. B :: C. D; si H ⊏ L, *a* erit G ⊏ K. Sed quia $\frac{C}{D} \sqsubset \frac{E}{D}$, *b* fieri potest ut sit H ⊏ L, & I non ⊏ M. ergo fieri potest ut G ⊏ K, & I non ⊏ M *c* ergo $\frac{A}{B} \sqsubset \frac{E}{F}$. Q E.D. a 6. *def.* 5. b 8. *def.* 5. c 8. *def.* 5.

SCHOL.

Quod si $\frac{C}{D} \sqsupset \frac{E}{F}$, erit quoque $\frac{A}{B} \sqsupset \frac{E}{F}$. Item si $\frac{A}{B} \sqsubset \frac{C}{D} \sqsubset \frac{E}{F}$, erit $\frac{A}{B} \sqsubset \frac{E}{F}$. & si $\frac{A}{B} \sqsupset \frac{C}{D} \sqsupset \frac{E}{F}$, erit $\frac{A}{B} \sqsupset \frac{E}{F}$.

PROP. XIV.

Si prima A *ad secundam* B *eandem habuerit rationem, quam tertia* C *ad quartam* D; *prima vero* A, *quam tertia* C *major fuerit; erit & secunda* B *major quam quarta* D. *Quod si prima* A *fuerit æqualis tertiæ* C, *erit & secunda* B *æqualis quartæ* D; *si vero* A *minor, &* B *minor erit.*

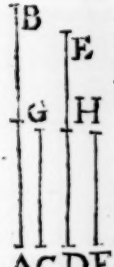
a 8. 5. Sit A ⊏ C. *a* ergo $\frac{A}{B} \sqsubset \frac{C}{B}$. *b* sed
b *hyp.*
c 13. 5. $\frac{A}{B} = \frac{C}{D}$. *c* ergo $\frac{C}{D} \sqsubset \frac{C}{B}$. *d* ergo B ⊏ D.
d 10. 5. Simili argumento si A ⊐ C, *d* erit B ⊐ D. Sin
e 7. 5. ponatur A = C; ergo C. B *e* :: A. B *f* :: C. D.
f *hyp.*
g 11. & 9. 5. *g* ergo B = D. Quæ E.D.

SCHOL.

A fortiori, si $\frac{A}{B} \sqsupset \frac{C}{D}$, atque A ⊏ C, erit B ⊏ D. Item si A = B, erit C = D. Et si A ⊏, vel ⊐ B, erit pariter C ⊏, vel ⊐ D.

PROP XV.

Partes C *&* F *cum pariter multiplicibus* AB, *&* DE *in eadem sunt ratione, si prout sibi mutuo respondent, ita sumantur.* (AB. DE :: C. F.)



Sint A G, G B partes multiplicis A B ipsi C æquales: item D H, H E partes multiplicis D E ipsi F æquales. *a* Harum numerus illarum numero æquatur. ergo quum *b* A G. D H
a *hyp.*
b 7. 5.
c 12, 5. :: C. F :: GB. HE. *c* erit A G + G B (AB.) D H + H E (DE) :: C. F. Q.E.D.

PROP.

PROP. XVI.

Si quatuor magnitudines A, B, C, D *proportionales fuerint; & viciſſim proportionales erunt.* (A.C :: B.D.)

Accipe E & F æquemultiplices ipſarum A & B. ipſarumque C & D æquemultiplices G & H. Itaque E.F *a* :: A.B. *b* :: C.D *a* :: G.H. Quare ſi E ⌐, =, ¬ G, *c* erit ſimiliter F ⌐, =, ¬ H. *d* ergo A.C :: B.D. Q.E.D.

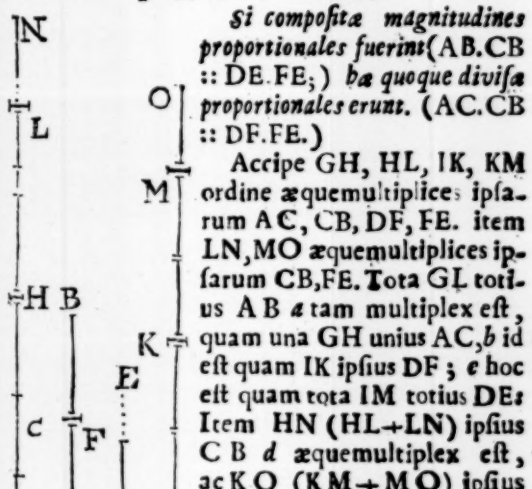
a 15. 5.
b *hyp.*
c 11. 5. & 14. 5.

SCHOL.

Alterna ratio locum tantum habet, quando quantitates ejuſdem ſunt generis. Nam Heterogeneæ quantitates non comparantur.

d *6. def. 5.*

PROP. XVII.



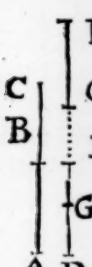
Si compoſitæ magnitudines proportionales fuerint (AB.CB :: DE.FE;) *hæ quoque diviſæ proportionales erunt.* (AC.CB :: DF.FE.)

Accipe GH, HL, IK, KM ordine æquemultiplices ipſarum AC, CB, DF, FE. item LN, MO æquemultiplices ipſarum CB, FE. Tota GL totius AB *a* tam multiplex eſt, quam una GH unius AC, *b* id eſt quam IK ipſius DF; *c* hoc eſt quam tota IM totius DE: Item HN (HL + LN) ipſius CB *d* æquemultiplex eſt, ac KO (KM + MO) ipſius FE. Quum igitur per hyp. AB. BC :: DE. EF. ſi GL ⌐, =, ¬ HN, etiam ſimiliter

a 1. 5.
b *conſtr.*
c 1. 5.
d 2. 5.

e 6. def. 5. militer *e* erit IM ⊏,=,⊐ KO. Itaque ablatis hinc inde communibus HL, KM. si reliqua GH
f 5. ax. ⊏,=,⊐ LN, *f* erit similiter IK ⊏,=,⊐ MO.
g 6. def. 5. *g* unde AC.CB :: DF.FE. Q.E.D.

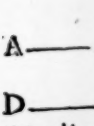
PROP. XVIII.



Si diviſæ magnitudines ſint proportionales (AB.BC :: DE. EF,) *hæ quoque compoſitæ proportionales erunt* (AC CB :: DF.FE.)

Nam ſi fieri poteſt, ſit AC. CB :: DF. FG ⊐ FE. *a* ergo erit diviſim
a 17. 5.
b hyp.& 11. 5. AB. BC :: DG. GF. *b* hoc eſt DG. GF :: DE. EF. ergo cum DG ⊏ DE,
c 14. 5. *c* erit GF ⊏ EF. Q. E. A. Simile
d 9. ax. abſurdum *d* ſequetur, ſi dicatur AC. CB :: DF. GF ⊏ FE.

PROP. XIX.



Si quemadmodum totum AB *ad totum* DE, *ita ablatum* AC *ſe habuerit ad ablatum* DF, *& reliquum* CB *ad reliquum* FE, *ut totum* AB *ad totum* DE, *ſe habebit.*

a hyp. Quoniam *a* AB.DE :: AC.DF, *b* erit permu-
b 16. 5. tando AB. AC :: DE. DF. *c* ergo diviſim AC.
c 17. 5. CB :: DF. FE. quare rurſus *b* permutando AC.
d hyp.& 11. 5. DF::CB.FE; *d* hoc eſt AB.DE::CB.FE. Q.E.D.

Coroll.

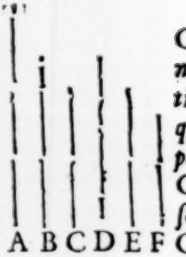
1. Hinc, ſi ſimilia proportionalia ſimilibus proportionalibus ſubducantur, reſidua erunt proportionalia.

2. *Hinc demonſtrabitur converſa ratio.*

Sit AB. CB :: DE. FE. Dico AB. AC :: DE.
a 16. 5. DF. Nam *a* permutando AB.DE :: CB.FE. *b* er-
b 19. 5. go AB.DE :: AC DF. quare iterum permutando, AB. AC :: DE.DF. Q.E.D.

PROP.

PROP. XX.



Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur* (A. B :: D. E. *atque* B. C :: E. F;) *ex æquo autem prima* A *major fuerit, quam tertia* C; *erit & quarta* D *major quam sexta* F. *Quod si prima* A *tertiæ* C *fuerit æqualis; erit & quarta* D *æqualis sextæ* F. *Sin illa minor, hæc quoque minor erit.*

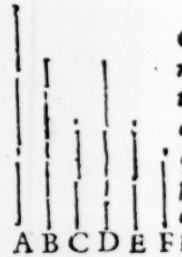
Hyp. Si A ⊏ C. quoniam *a* E. F :: B. C. *b* erit inverse F. E :: C. B. *c* Sed $\frac{C}{B} \sqcap \frac{A}{B}$ *d* ergo $\frac{F}{E} \sqcap \frac{A}{B}$ vel $\frac{D}{E}$. *e* ergo D ⊏ F. Q. E D.

2. *Hyp.* Simili argumento, si A ⊐ C, ostenditur D ⊐ F.

3. *Hyp.* Si A = C. quoniam F. E :: C. B :: *f* A. B :: D. E *g* erit D = F. Q. E D.

a *hyp.*
b *cor.* 4. 5.
c *hyp. &* 8. 5.
d *schol.* 13. 5.
e 10. 5.
f 7. 5.
g 11. 5. *&* 9. 5.

PROP. XXI.



Si sint tres magnitudines A, B, C; *& aliæ* D, E, F *ipsis æquales numero, quæ binæ & in eadem ratione sumantur, fueritq; perturbata earum proportio,* (A. B :: E F. *atque* B. C :: D. E;) *ex æquo autem prima* A *quam tertia* C *major fuerit; erit & quarta* D *quam sexta* F *major. Quod si prima fuerit tertiæ æqualis, erit & quarta æqualis sextæ: sin illa minor, hæc quoque minor erit.*

1. *Hyp.* A ⊏ C. Quoniam *a* D. E :: B. C, invertendo erit E. D :: C. B. atqui $\frac{C}{B}$ *b* $\sqcap \frac{A}{B}$. *c* ergo

a *hyp.*
b 8. 5.

e *schol.* 13. 5. *c* ergo $\frac{E}{D} \sqsupset \frac{A}{B}$, hoc est $\frac{E}{F}$. *d* ergo D ⊏ F Q.E.D.

d 10. 5. 2. *Hyp.* Similiter, si A ⊐ C, erit D ⊐ F.

3. *Hyp.* Si A = C. quoniam E. D *e* :: C. B :: e A. B :: *f* E. F. *g* erit D = F. Q.E.D.

f *hyp.*

g 9. 5.

PROP. XXII.

Si sint quotcunque magnitudines A, B, C; *& aliæ ipsis æquales numero* D, E, F, *quæ binæ & in eadem ratione sumantur* (A. B :: D. E. & B. C :: E. F;) *& ex æqualitate in eadem ratione erunt* (A C :: D. F.)

Accipe G, H ipsarum A, D; & I, K ipsarum B, E; item L, M ipsarum E, F æquemultiplices.

a *hyp.*

b 4. 5. Quoniam *a* A. B :: D. E. *b* erit G. I :: H. K. eodem modo, erit I. L :: K. M. ergo si G ⊏, =, ⊐ L, *c* erit H ⊏, =, ⊐ M; *d* ergo A. C :: D. F. Eodem pacto si ulterius C. N :: F. O, erit ex æquali A. N :: D. O. Q.E.D.

c 20. 5.

d 6. *def.* 5.

PROP.

PROP. XXIII.

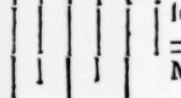
Si ſint tres magnitudines A, B, C, *aliæq;* D, E, F *ipſis æquales numero, quæ binæ in eadem ratione ſumantur; fuerit autem perturbata earum proportio.* (A. B :: E. F. & B. C :: D. E.) *etiam ex æqualitate in eadem ratione erunt* A. C :: D. F. Sume G, H, I, ipſarum A, B, D; item K, L, M ipſarum C, E, F æquemultiplices. erit G. H *a* :: A. B *b* :: E. F *a* :: L. M. porro quia *a* 15. 5. *b* B. C :: D. E. erit *c* H. I :: K. L. *b hyp.* ergo G, H, K; & I, L, M habent *c* 4. 5. ſe juxta 21. 5. quare ſi G ⊏, =, ⊐ K, erit ſimiliter I ⊏, =, ⊐ M. *d* proinde A. C :: D. F. Q. E. D. *d* 6. *def.* 5.

Eodem modo ſi plures fuerint magnitudinibus tribus, &c.

Corolll.

Ex* his ſequitur, rationes ex iiſdem rationibus compoſitas eſſe inter ſe eaſdem. item, earundem rationum eaſdē partes inter ſe eaſdem eſſe. *def.* 5. *22. & 23. 5. & 20.

PROP. XXIV.



Si prima A B *ad ſecundam* C *eandem habuerit rationem quam tertia* DE *ad quartam* F; *habuerit autem & quinta* BG *ad ſecundam* C *eandem rationem, quam ſexta* EH *ad quartam* F; *etiam compoſita prima cum quinta* (AG) *ad ſecundam* C *eandem habebit rationem, quam tertia cum ſexta* (DH) *ad quartam* F.

Nam quia *a* A B. C :: D E. F. atque ex hyp. & inverſe C. BG :: F. EH, erit *b* ex æquali AB. BG :: DE. EH. *c* item BG. C :: EH. F. *b* ergo rurſus ex æquo, AG. C :: DH. F. Q. E. D. *a hyp.* *b* 22. 5. *c hyp.*

PROP.

PROP. XXV.

Si quatuor magnitudines proportionales fuerint (AB. CD :: E.F.) *maxima* AB *& minima* F *reliquis* CD *&* E *majores erunt.*

Fiant AG=E; & GH=F. Quoniam AB.CD *a* :: E.F *b* :: AG.CH. *c* erit AB. C D :: G B. HD. *d* sed AB⊐CD. *e* ergo GB ⊐ HD. atqui AG+F= E + CH. ergo AG+F+GB⊐E+CH+HD, hoc est AB+F⊐E+CD. Q.E D.

a *hyp.*
b 7. 5.
c 19. 5.
d *hyp.*
e *schol.* 14. 5.

Quæ sequuntur propositiones non sunt Euclidis; sed ex aliis desumptæ, ob frequentem earum usum Euclidæis subjungi solent.

PROP. XXVI.

A——·——C——·— *Si prima ad secundam*
B——·— D—— *habuerit majorem pro-*
E——·—— *portionem, quam tertia ad quartam; habebit convertendo, secunda ad primam minorem proportionem, quam quarta ad tertiam.*

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{B}{A} \sqsupset \frac{D}{C}$. Nam concipe $\frac{C}{D} = \frac{E}{B}$ *a* ergo $\frac{A}{B} \sqsubset \frac{E}{B}$ *b* quare A⊏E. *c* ergo $\frac{B}{A} \sqsupset \frac{B}{E}$, *d* vel $\frac{D}{C}$. Q.E.D.

a 13. 5.
b 10 5.
c 8. 5.
d *cor.* 4. 5.

PROP. XXVII.

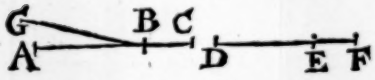
A——·——C——·— *Si prima ad secundam*
B——·— D—— *habuerit majorem pro-*
E——·—— *portionem, quam tertia ad quartam; habebit quoque vicissim prima ad tertiam majorem proportionem, quam secunda ad quartam.*

Sit $\frac{A}{B} \sqsubset \frac{C}{D}$. Dico $\frac{A}{C} \sqsubset \frac{B}{D}$. Nam puta $\frac{E}{B} = \frac{C}{D}$ *a* ergo A⊏E. *b* ergo $\frac{A}{C} \sqsubset \frac{E}{C}$, *c* vel $\frac{B}{D}$. Q.E.D.

a 10. 5.
b 8. 5.
c 16. 5.

PROP.

PROP. XXVIII.

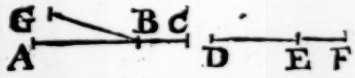


Si prima ad ſecundam habuerit majorem proportionem, quam tertia ad quartam; habebit quoque compoſita prima cum ſecunda ad ſecundam majorem proportionem, quam compoſita tertiæ cum quarta ad quartam.

Sit $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Dico $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Nam cogita $\frac{GB}{BC} = \frac{DE}{EF}$. *a* ergo AB ⊏ GB. adde utrinque BC, *b* erit AC ⊏ GC. *c* ergo $\frac{AC}{BC} \sqsubset \frac{GC}{BC}$. *d* hoc eſt $\frac{DF}{EF}$. Q.E.D.

a 10. 5.
b 4. *ax.*
c 8. 5.
d 18. 5.

PROP. XXIX.



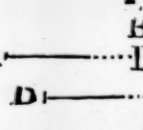
Si compoſita prima cum ſecunda ad ſecundam majorem habuerit proportionem, quam compoſita tertia cum quarta ad quartam; habebit quoque dividendo prima ad ſecundam majorem proportionem quam tertia ad quartam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. Intellige $\frac{GC}{BC} = \frac{DF}{EF}$. *a* ergo AC ⊏ GC. aufer commune BC, *b* erit AB ⊏ GB. *c* ergo $\frac{AB}{BC} \sqsubset \frac{GB}{BC}$ *d* vel $\frac{DE}{EF}$. Q.E.D.

a 10. 5.
b 5. *ax.*
c 8. 5.
d 17 5.

PROP.

PROP. XXX.



Si composita prima cum secunda ad secundam habuerit majorem proportionem, quam composita tertia cum quarta ad quartam; habebit, per conversionem rationis, prima cum secunda ad primam minorem rationem, quam tertia cum quarta ad tertiam.

Sit $\frac{AC}{BC} \sqsubset \frac{DF}{EF}$. Dico $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Nam quia
a *hyp.* $\frac{AC}{BC}$ *a* $\sqsubset \frac{DF}{FE}$. *b* erit dividendo $\frac{AB}{BC} \sqsubset \frac{DE}{EF}$. *c* conver-
b 29. 5.
c 26. 5. tendo igitur $\frac{BC}{AB} \sqsupset \frac{EF}{DE}$. *d* ergo componendo
d 28. 5. $\frac{AC}{AB} \sqsupset \frac{DF}{DE}$. Q.E.D.

PROP. XXXI.

A——— D———
B——— E———
C——— F———
G———
H———

Si sint tres magnitudines A, B, C, *& aliæ ipsis æquales numero* D, E, F; *sitque major proportio primæ priorum ad secundam, quam primæ posteriorum ad secundam* ($\frac{A}{B} \sqsubset \frac{D}{E}$;) *item secundæ priorum ad tertiam major, quam secundæ posteriorum ad tertiam* ($\frac{B}{C} \sqsubset \frac{E}{F}$;) *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ posteriorum ad tertiam* ($\frac{A}{C} \sqsubset \frac{D}{F}$.)

a 10. 5. Concipe $\frac{G}{C} = \frac{E}{F}$. *a* ergo B $\sqsubset$ G. *b* ergo $\frac{A}{G} \sqsubset \frac{A}{B}$.
b 8. 5.
c 13. 5. Rursus puta $\frac{H}{G} = \frac{D}{E}$. *c* ergo $\frac{H}{G} \sqsupset \frac{A}{B}$; *d* ergo for-
d 10. 5. tius $\frac{H}{G} \sqsupset \frac{A}{G}$. *d* quare A $\sqsubset$ H. *e* proinde $\frac{A}{C} \sqsubset \frac{H}{C}$,
e 8. 5.
f 22. 5. *f* vel $\frac{D}{F}$. Q.E.D.

PROP.

PROP. XXXII.

Si sint tres magnitudines A, B, C, *& aliæ ipsis numero æquales* D, E, F; *sitque major proportio primæ priorum ad secundam, quam secundæ posteriorum ad tertiam* ($\frac{A}{B} \sqsubset \frac{E}{F}$;) *item secundæ priorum ad tertiam major, quam primæ posteriorum ad secundam* ($\frac{B}{C} \sqsubset \frac{D}{E}$;) *erit quoque ex æqualitate major proportio primæ priorum ad tertiam, quam primæ posteriorum ad tertiam* ($\frac{A}{C} \sqsubset \frac{D}{F}$.)

Intellige $\frac{G}{C} = DE$. *a* ergo $B \sqsubset G$. *b* ergo $\frac{A}{G} \sqsubset \frac{A}{B}$. Rursus concipe $\frac{H}{G} = \frac{E}{F}$. *c* ergo $\frac{H}{G} \sqsupset \frac{A}{G}$. *a* quare $A \sqsubset H$. *b* proinde $\frac{A}{C} \sqsubset \frac{H}{C}$ *d* vel $\frac{D}{F}$. Q. E. D.

a 10. 5.
b 8. 5.
c *sch.* 13. 5.
d 13. 5.

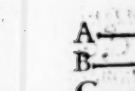
PROP. XXXIII.

Si fuerit major proportio totius AB *ad totum* CD, *quam ablati* AE *ad ablatum* CF; *erit & reliqui* EB *ad reliquum* FD *major proportio, quam totius* AB *ad totum* CD.

Quoniam $\frac{AB}{CD}$ *a* $\sqsubset \frac{AE}{CF}$, *b* erit permutando $\frac{AB}{AE} \sqsubset \frac{CD}{CF}$. *c* ergo per conversionem rationis $\frac{AB}{EB} \sqsupset \frac{CD}{FD}$. permutando igitur $\frac{AB}{CD} \sqsupset \frac{EB}{FD}$. Q. E. D.

a *hyp.*
b 27. 5.
c 30. 5.

PROP. XXXIV.



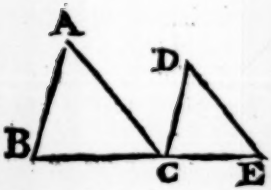
Si sint quotcunque magnitudines, & aliæ ipsis æquales numero, sitque major proportio primæ priorum ad primam posteriorum, quam secundæ ad secundam; & hæc major quam tertiæ ad tertiam, & sic deinceps: habebunt omnes priores simul ad omnes posteriores simul, majorem proportionem, quam omnes priores, relicta prima, ad omnes posteriores, relicta quoque prima; minorem autem, quam prima priorum ad primam posteriorum; majorem denique etiam, quam ultima priorum ad ultimam posteriorum.

Horum demonstratio est penes interpretes; qui adeat, qui eam desiderat. nos omisimus, brevitatis studio; & quia illorum nullus usus in his elementis.

LIB.

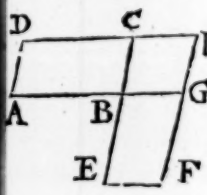
LIB. VI.

Definitiones.

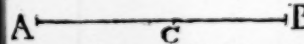


I. Similes figuræ rectilineæ ſunt (ABC, DCE,) quæ & angulos ſingulos ſingulis æquales habent; atque etiam latera, quæ circum angulos æquales, proportionalia.

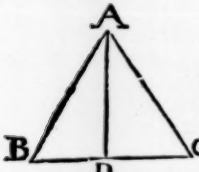
Ang. B = DCE; *&* AB. BC :: DC. CE *item ang.* A = D; *atque* BA. AC :: CD. DE *denique ang.* ACB = E. *atque* BC. CA :: CE. ED.



II. Reciprocæ autem ſunt (BD, BF,) cum in utraque figura antecedentes, & conſequentes rationum termini fuerint, (hoc eſt, AB. BG :: EB. BC.)



III. Secundum extremam & mediam rationem recta linea AB ſecta eſſe dicitur, cum ut tota AB ad majus ſegmentum AC, ita majus ſegmentum AC ad minus CB ſe habuerit. (AB. AC :: AC. CB.)



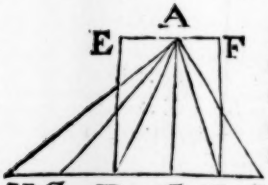
IV. Altitudo cujuſque figuræ ABC eſt linea perpendicularis AD, à vertice A ad baſim BC deducta.

V. Ratio ex rationibus componi dicitur, cum rationum quantitates inter ſe multiplicatæ, aliquam effecerint rationem.

Ut ratio A *ad* C, *componitur ex rationibus* A *ad* B, *&* B *ad* C. *nam* $\frac{A}{B} + \frac{B}{C} a = \frac{A}{C} b = \frac{AB}{BC}$.

a 20. *def.* 5.
b 15. 5.

PROP. I.



Triangula ABC, ACD, *& parallelogramma* BCAE, CDFA, *quorum eadem fuerit altitudo, ita ſe habent inter ſe, ut baſes* BC, CD.

a 3. 1. *a* Accipe quotvis BG, HG, ipſi BC æquales; item DI = CD. & connecte AG, AH, AI.

b 38. 1. *b* Triangula ACB, ABG, AGH æquantur; *b* item triang. ACD = ADI. ergo triangulum ACH tam multiplex eſt trianguli ACB, quam baſis HC baſis BC. & æquemultiplex eſt triang. ACI trianguli ACD, ac baſis CI baſis CD.

c *ſch.* 38. 1. Verum ſi HC ⊏, =, ⊐ CI, *c* erit ſimiliter
d 6. *def.* 5. triang. AHC ⊏, =, ⊐ ACI. *d* ideoque BC,
e 41. 1. & 15. 5. CD :: triang. ABC, ACD :: *e* Pgr. CE. CF. Q. E. D.

Schol.

schol.

Hinc, *triangula* ABC, DEF, *& parallelogramma* AGBC, DEFH, *quorum æquales sunt baſes* BC, EF, *ita ſe habent ut altitudines* AI, DK.

a Sume IL = CB; & KM = EF; ac junge LA, LG, MD, MH. liquet eſſe triang. ABC. DEF :: *b* ALI. DKM :: *c* AI. DK :: *d* Pgr. AGBC. DEFH. Q.E.D.

a 3. 1.
b 7. 5.
c 1. 6.
d 41. 1. & 15. 5.

PROP. II.

Si ad unum trianguli ABC *latus* BC, *parallela ducta fuerit recta quædam linea* DE, *hæc proportionaliter ſecabit ipſius trianguli latera* (AD. BD :: AE. EC.) *Et ſi trianguli latera proportionaliter ſecta fuerint* (AD. BD :: AE. EC) *quæ ad ſectiones* D, E *adjuncta fuerit recta linea* DE, *erit ad reliquum ipſius trianguli latus* BC *parallela.* Ducantur CD, BE.

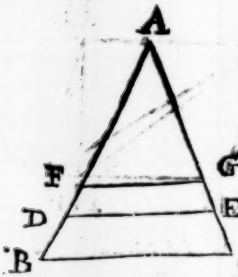
1. *Hyp.* Quia triang. DEB *a* = DEC; *b* erit triang. ADE. DBE :: ADE. ECD. atqui triang. ADE. DBE *c* :: AD. DB. & triang. ADE. DEC *c* :: AE. EC. *d* ergo AD. DB :: AE. EC.

a 37. 1.
b 7. 5.
c 1. 6.
d 11. 5.

2. *Hyp.* Quia AD. DB :: AE. EC. *e* hoc eſt triang. ADE. DBE :: ADE. ECD; *f* erit triang. DBE = ECD. *g* ergo DE, BC ſunt parallelæ, Q. E. D.

e 1. 6.
f 9. 5.
g 39. 1.

Schol.



Imo si plures DE, FG, *ad unum latus* BC *parallelæ fuerint, erunt omnia laterum segmenta proportionalia.*

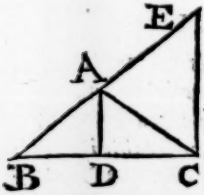
Nam DF. FA *a* :: EG. GA; & componendo, invertendoque FA. DA :: GA. EA; *a* ac DA. DB :: EA. EC. ergo ex æquo DF. DB :: EG. EC. Q.E.D.

a 2. 6.

Coroll.

Si DF. DB :: EG. EC; *a* erunt BC, DE, FG parallelæ.

PROP. III.



Si trianguli BAC *angulus* BAC *bifariam sectus sit, secans autem angulum recta linea* AD *secuerit & basim, basis segmenta eandem habebunt rationem quam reliqua ipsius trianguli latera* (BD. DC :: AB. AC.) *Et si basis segmenta eandem habeant rationem quam reliqua ipsius trianguli latera* (BD. DC :: AB. AC) *recta linea* AD *quæ à vertice* A *ad sectionem* D *ducitur, bifariam secat trianguli ipsius angulum* BAC.

Produc BA; & fac AE = AC. & junge CE.

1. *Hyp.* Quoniam AE = AC, erit ang. ACE *a* = E *b* = $\frac{1}{2}$ BAC *c* = DAC. *d* ergo DA, CE parallelæ sunt. *e* quare BA. AE (AC) :: BD. DC. Q.E.D.

a 5. 1.
b 32. 1.
c *hyp.*
d 27. 1.
e 2. 6.

2. *Hyp.* Quoniam BA. AC. (AE) :: BD. DC. *f* erunt DA, CE parallelæ: *g* ergo ang. BAD = E; & ang. DAC *g* = ACE *h* = E. *k* erg. ang. BAD = DAC, bisectus igitur est ang. BAC Q.E.D.

f 2. 6.
g 29. 1.
h 5. 1.
k 1. *ax.*

PROP.

PROP. IV.

Æquiangulorum triangulorum ABC, DCE *proportionalia ſunt latera, quæ circum æquales angulos* B, DCE (AB. BC :: DC. CE, &c.) *& homologa ſunt latera* AB, DC, &c. *quæ æqualibus angulis* ACB, E, &c. *ſubtenduntur.*

Statue latus BC in directum lateri CE, & produc BA, ac ED donec *a* occurrant. a 32. 1. &

Quoniam ang. B *b* = ECD. *c* ſunt BF, CD b 13. *ax.* parallelæ. Item quia ang. BCA *b* = CED, *c* ſunt b *hyp.* CA, EF parallelæ. Figura igitur CAFD eſt c 28. 1. parallelogramma. *d* ergo AF = CD; *d* & AC = FD. Liquet igitur AB. AF (CD) *e* :: BC. d 34. 1. CE. *f* permutando igitur AB. BC :: CD. CE. e 2. 6. *e* item BC. CE :: FD. (AC) DE. *f* ergo per- f 16. 5. mutando BC. AC :: CE. DE. quare etiam *g* ex æquo AB. AC :: CD. DE. ergo, &c. g 22. 5.

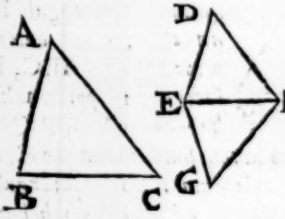
Coroll.

Hinc AB. DC :: BC. CE :: AC. DE.

Schol.

Hinc ſi in triangulo FBE ducatur uni lateri FE parallela AC; erit triangulum ABC ſimile toti FBE.

PROP. V.

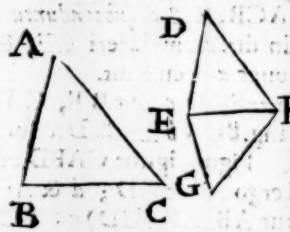


Si duo triangula ABC, DEF *latera proportionalia habeant* (AB. BC :: DE. EF. *&* AC. BC :: DF. EF. item AB. AC :: DE. DF) *æquiangula erunt triangula, & æquales habebunt eos angulos, ſub quibus homologa latera ſubtenduntur.*

Ad latus EF *a* fac ang. FEG = B; *a* & ang. a 23. 1.

b 32. 1. EFG = C, *b* quare etiam ang. G = A. ergo
c 4. 6. GE. EF. *c* :: AB. BC :: *d* DE. EF. *e* ergo
d *byp.* GE = DE. Item GF. FE *c* :: AC. CB *d* ::
e 11. 5. DF. FE. *e* ergo GF = DF. Triangula igitur
& 9. 5. DEF, GEF ſibi mutuo æquilatera ſunt, *f* ergo
f 8. 1. ang. D = G = A. *f* & ang. FED = FEG = B.
g 32. 1. *g* proinde & ang. DFE = C. ergo, &c.

PROP. VI.

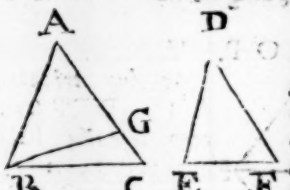


Si duo triangula ABC, DEF *unum angulum* B *uni angulo* DEF *æqualem, & circum æquales angulos* B, DEF *latera proportionalia habuerint* (AB. BC :: DE. EF;) *æquiangula erunt triangula* ABC, DEF; *æqualeſque habebunt angulos, ſub quibus homologa latera ſubtenduntur.*

Ad latus EF fac ang. FEG = B, & ang. EFG
a 32. 1. = C. *a* unde & ang. G = A. ergo GE. EF *b* ::
b 4. 6. AB. BC *c* :: DE. EF. *d* ergo DE = GE. atqui
c *byp.* ang. DEF *e* = B *f* = GEF. *g* ergo ang. D = G
d 9. 5. = A. *h* proinde etiam ang. EFD = C. Q. E. D.
e *byp.*

PROP. VII.

f *conſtr.*
g 4. 1.
h 32. 1.



Si duo triangula ABC, DEF *unum angulum* A *uni angulo* D *æqualem, circa autem alios angulos* ABC, E *latera proportionalia habeant* (AB. BC :: DE. EF;) *reliquorum autem ſimul utrunque* C, F *aut minorem aut non minorem recto, æquiangula erunt triangula* ABC, DEF, *& æquales habebunt eos angulos circum quos proportionalia ſunt latera.*

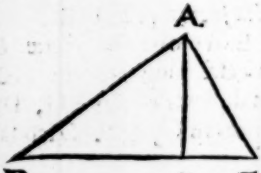
Nam ſi fieri poteſt, ſit ang. ABG = E; ergo cum ang. A =
a *byp.* D, igitur

b erit

b erit etiam ang. AGB = F. ergo AB. BG *c* :: DE. EF :: AB. BC. *e* ergo BG = BC. *f* ergo ang. BGC = BCG. *g* ergo ang. BGC. vel C minor eſt recto; *b* proinde ang. AGB, vel F recto major eſt. ergo anguli C & F non ſunt ejuſdem ſpeciei, contra Hyp.

b 32. 1.
c 4. 6.
d *hyp.*
e 9. 5.
f 5. 1.
g *cor.* 17. 5
h *cor.* 13. 1

PROP. VIII.



Si in triangulo rectangulo ABC, *ab angulo recto* BAC *in baſin* BC *perpendicularis* AD *ducta eſt; quæ ad perpendicularem triangula* ADB, ADC, *tum toti triangulo* ABC, *tum ipſa inter ſe, ſimilia ſunt.*

Nam ob angulos BAC, ADB *a* rectos, *b* ideoque æquales, & B communem, trigona BAC, ADB *c* ſimilia ſunt. Simili diſcurſu, ſimilia ſunt triangula BAC, ADC. *d* proinde ADB, ADC ſimilia erunt. Q.E.D.

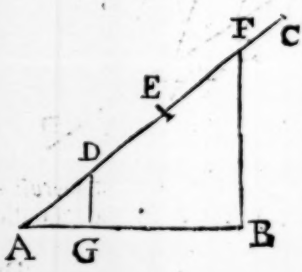
a *hyp.*
b 12. *ax.*
c 32. & 4. 6
d *Vid.* 21. 6.

Coroll.

Hinc 1. BD. DA *e* :: DA. DC.
2. BC. AC :: AC. DC, & CB. BA :: BA. BD.

e 1. *def.* 6.

PROP. IX.



A data recta linea AB *imperatam partem* $\frac{1}{3}$ (AG) *auferre.*

Ex A duc infinitam AC utcunq;, in qua *a* ſume tres, AD, DE, EF æquales utcunque,

a 3. 1.

b 31. 1. cunque, junge F B, cui ex D *b* duc parallelam DG. Dico factum.

c 2. 6. Nam GB. AG *c* :: FD. AD. ergo *d* com-

d 18. 5. ponendo AB. AG :: AF. AD. ergo cum AD = $\frac{1}{3}$ AF, erit AG = $\frac{1}{3}$ AB. Q.E.F.

PROP. IX.

Datam rectam lineam AB *infectam similiter secare (in* F, G,) *ut data altera* AC, *secta fuerit (in* D, E.) Extremitates sectæ & insectæ jungat recta BC.

a 31. 1. Huic ex punctis E, D *a* duc parallelas EG, DF rectæ secandæ occurrentes in G, & F. Dico factum.

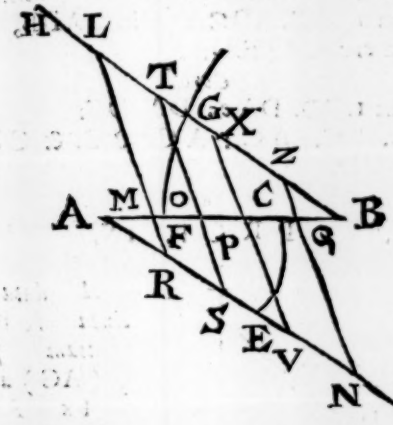
a Ducatur enim DH parall. AB. Estque AD.

b 2. 6. DE *b* :: AF. FG, & DE. EC *b* :: DI. IH *c* :: FG.

c 34. 1. & GB. Q.E.F.

7. 5.

Scholium.



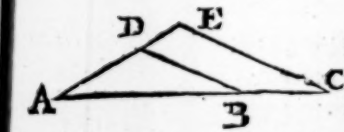
Hinc *discimus rectam datam* AB *in quotvis æquales partes* (puta 5.) *secare*, id quod facilius præstabitur sic;

Duc

Duc infinitam AD, eique parallelam BH etiam infinitam. Ex his cape partes æquales AR, RS, SU, UN; & BZ, ZX, XT, TL; in singulis una pauciores, quam desiderentur in AB; tum rectæ ducantur LR, TS, XV, ZN. hæ quinquisecabunt datam AB. a 33. 1. b *constr.* c 2. 6.

Nam RL, ST, UX, NZ *a* parallelæ sunt. ergo quum AR, RS, SU, UN *b* æquales sint, *c* erunt AM, MO, OP, PQ æquales. Similiter quia BZ = ZX, erit BQ = QP. ergo AB quinquisecta est. Q.E.F.

PROP. XI.



Datis duabus rectis lineis AB, AD, *tertiam proportionalem* DE *invenire.*

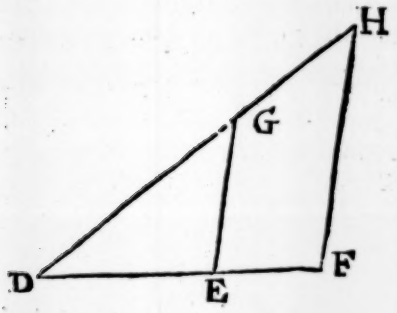
Junge BD, & ex AB protracta sume BC = AD. per C duc CE parall. BD. cui occurrat AD producta in E. Erit DE expetita. a 2. 6.

Nam AB. *a* BC (AD) :: AD. DE. Q.E.F. c 1 *cor.* 8. 6.

Vel sic, fac ang. ABC rectum, & ang. ACD etiam rectum. *b* erit AB. BC :: BC. BD.

PROP.

PROP. XII.

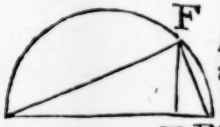


Tribus datis rectis lineis DE, EF, DG, *quartam proportionalem* GH *invenire.*

Connectatur EG. per F, duc FH parall. EG, cui occurrat DG producta ad H. liquet esse DE. EF *a* :: DG. GH. Q.E.F.

a 2. 6.

PROP. XIII.



Duabus datis rectis lineis AE, EB, *mediam proportionalem* EF *adinvenire.*

Super tota AB diametro describe semicirculum AFB. Ex E erige perpendicularem EF occurrentem peripheriæ in F. Dico AE. EF :: EF. EB. Ducantur enim AF, & FB. Ex

a 31. 3. trianguli *a* rectanguli AFB recto angulo deducta

b *cor.* 8. 6. est FE basi perpendicularis; *b* ergo AE. FE :: FE. EB. Q.E.F.

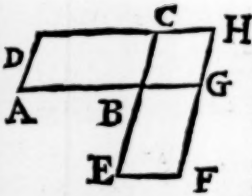
Vel (in eadem figura) sint AB, BF duæ datæ, *b* liquet esse AB. BF :: BF. BE.

Coroll.

Coroll.

Hinc, linea recta, quæ in circulo à quovis puncto diametri, ipsi diametro perpendicularis ducitur ad circumferentiam usque, media est proportionalis inter duo diametri segmenta.

PROP. XIV.



Æqualium, & unum ABC *uni* EBG *æqualem habentium angulum, parallelogrammorum* BD, BF, *reciproca sunt latera quæ circum æquales angulos.* (AB. BG :: EB. BC:) *Et quorum parallelogrammorum* BD, BF, *unum angulum* ABC *uni angulo* EBG *æqualem habentium, reciproca sunt latera quæ circum æquales angulos, illa sunt æqualia.*

Nam latera AB, BG circa æquales angulos faciant unam rectam: *a* quare EB, BC etiam in directum jacebunt. Producantur FG, DC; donec occurrant.

1. *Hyp.* AB, BG *b* :: BD. BH *c* :: BF. BH *d* :: BE. BC. *e* ergo, &c.

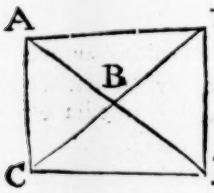
2. *Hyp.* BD. BH *f* :: AB. BG *g* :: BE. BC *h* :: BF. BH, *k* ergo Pgr. BD = BF. Q.E.D.

a *sch.* 15. 1.

b 1. 6.
c 7. 5.
d 1. 6.
e 11. 5.
f 1. 6.
g *hyp.*
h 1. 6.
k 11. & 9. 5

PROP.

PROP. XV.



Æqualium, & unum ABC, *uni* DBE *æqualem habentium angulum triangulorum* ABC, DBE, *reciproca ſunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC.) *Et quorum triangulorum* ABC, DBE, *unum angulum* ABC *uni* DBE *æqualem habentium reciproca ſunt latera, quæ circum æquales angulos* (AB. BE :: DB. BC.) *illa ſunt æqualia.*

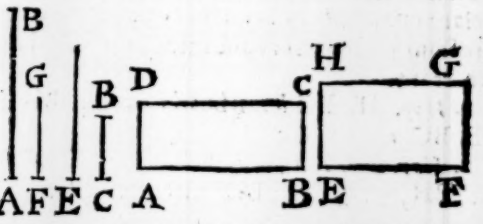
Latera CB, BD circa æquales angulos, ſtatuantur ſibi in directum; *a* ergo ABE eſt recta linea. ducatur CE.

a *ſch.* 15. 1.
b 1. 6.
c 7. 5.
d 1. 6.
e 11. 5.
f 1. 6.
g *hyp.*
h 1. 6.
k 11. & 9. 5.

1. *Hyp.* AB. BE *b* :: triang. ABC. CBE *c* :: triang. DBE. CBE. *d* :: DB. BC. *e* ergo, &c.

2. *Hyp.* Triang. ABC. CBE *f* :: AB. BE *g* :: DB. BC *h* :: triang DBE. CBE. *k* ergo triang. ABC = DBE. Q.E.D.

PROP. XVI.



Si quatuor rectæ lineæ proportionales fuerint (AB. FG :: EF. CB,) *quod ſub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale eſt ei, quod ſub mediis* EF, FG *comprehenditur, rectangulo* EG. *Et ſi ſub extremis comprehenſum rectangulum* AC *æquale fuerit ei, quod ſub mediis comprehenditur, rectangulo* EG, *illæ quatuor rectæ lineæ proportionales erunt* (AB. FG :: EF. CB.)

1. *Hyp.*

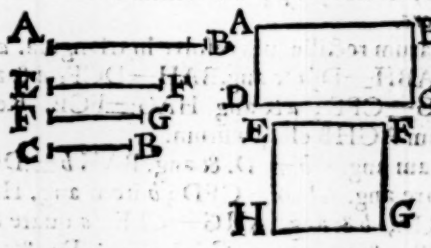
1. *Hyp.* Anguli B & F rect, ac *a* proinde pares sunt; atque ex hyp. AB. FG :: EF. CB. *b* ergo rectang. AC=EG. Q.E.D. a 12. ax. b 14. 6.

2. *Hyp. c* Rectang. AC=EG; atque ang. B=F; *d* ergo AB. FG :: EF. CB. Q.E.D. c hyp. d 14 6.

Coroll.

Hinc ad datam rectam lineam AB facile est datum rectangulum EG applicare, *e* faciendo AB. EF :: FG. BC. e 12. 6.

PROP. XVII.



Si tres rectæ lineæ sint proportionales (AB. EF :: EF. CB,) *quod sub extremis* AB, CB *comprehenditur rectangulum* AC, *æquale est ei, quod à media* EF *describitur, quadrato* EG. *Et si sub extremis* AB, CB *comprehensum rectangulum* AC, *æquale sit ei, quod à media* EF *describitur, quadrato* EG, *illæ tres rectæ lineæ proportionales erunt* (AB. EF :: EF. CB.)

Accipe FC=EF.

1. *Hyp.* AB. EF *a* :: EF (FG) CB. ergo Rectang. AC *b* = EG *c* = EFq. Q. E. D. a hyp. b 16 6.

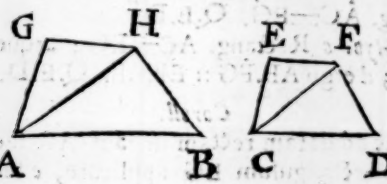
2. *Hyp.* Rectang. AC *d* = quadr. EG = EFq, *e* ergo AB. EF :: FG (EF.) BC. c 29 def. 1. d hyp. e 16. 6.

Coroll.

Sit A in B=Cq. ergo A. C :: C. B.

PROP.

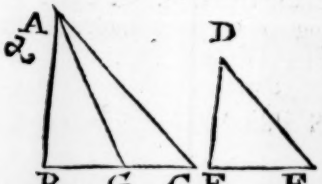
PROP. XVIII.



A data recta linea AB *dato rectilineo* CEFD *simile similiterque positum rectilineum* AGHB *describere.*

Datum rectilineum resolve in triangula. *a* fac
a 23. 1. ang. ABH = D; *a* & ang. BAH = DCF; *a* & ang. AHG = CFE; *a* & ang. HAG = FCE. Rectilineum AGHB est quæsitum.
b *constr.* Nam ang. B *b* = D. & ang. BAH *b* = DCF.
c 32. 1. *c* quare ang. AHB = CFD; *b* item ang. HAG = FCE, *b* & ang. AHG = CFE. *c* quare ang.
d 2. *ax.* G = E; & totus ang. GAB *d* = ECD; & totus GHB *d* = EFD. Polygona igitur sibi mutuo æquiangula sunt. Porro ob trigona æquiangula, AB. BH *e* :: CD. DF. & AG. GH. *e* :: CE.
e 4. 6. EF. item AG. AH. *e* :: CE. CF. & AH. AB *e* ::
f 22. 5. CF. CD. *f* unde ex æquo AG. AB :: CE. CD.
g 6. *def.* 6. eodem modo GH. HB :: EF. FD. *g* ergo polygona ABHG, CDFE similia similiterque posita existunt. Q E. F.

PROP. XIX.



Similia triangula ABC, DEF *sunt in duplicata ratione laterum homologorum* BC, EF.

a 11. 6. *a* Fiat BC. EF :: EF. BG. & ducatur AG.

Qui

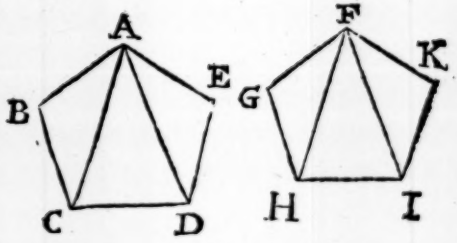
Quia AB.DE(*b* :: BC.EF)*c* :: EF.BG. & ang. B = E; *d* erit triang. ABG = DEF. verum triang. ABC. ABG *e* :: BC. BG; & *f* $\frac{BC}{BG}$ = $\frac{BC}{EF}$ bis; ergo triang. $\frac{ABC}{AEG}$ hoc eſt $\frac{ABC}{DEF}$ *g* = $\frac{BC}{EF}$ bis. Q. E. D.

b cor. 4. 6.
c conſtr.
d 15. 6.
e 1. 6.
f 10. *def.* 5.
g 11. 5.

Coroll.

Hinc, ſi tres lineæ BC, EF, BG proportionales fuerint; ut eſt prima ad tertiam, ita eſt triangulum ſuper primam BC deſcriptum ad triangulum ſuper ſecundam EF ſimile ſimiliterque deſcriptum. vel ita eſt triangulum ſuper ſecundam EF deſcriptum ad triangulum ſuper tertiam ſimile ſimiliterque deſcriptum.

PROP. XX.



Similia polygona ABCDE, FGHIK *in ſimilia triangula* ABC, FGH; *&* ACD, FHI, *&* ADE, FIK *dividuntur, & numero æqualia, & homologa totis.* (ABC. FGH :: ABCDE. FGHIK :: ACD. FHI :: ADE. FIK.) *Et polygona* ABCDE, FGHIK *duplicatam habent eam inter ſe rationem, quam latus homologum* BC *ad homologum latus* GH.

a *hyp.* 1. Nam ang. B *a* = G; & AB. BC *a* :: FG.
b *6. 6.* GH. *b* ergo triangula ABC, FGH æquiangula sunt. eodem modo, triangula AED, FKI assimilantur. cum igitur ang. BCA *b* = GHF; & ang. ADE *b* = FIK; totique anguli BCD, GHI; atque toti CDE, HIK *c* pares sint, *d* remanent ang. ACD = FHI; & ang. ADC = FIH; *e* unde etiam ang. CAD = HFI. ergo triangula ACD, FHI similia sunt. ergo, &c.
c *hyp.*
d *3. ax.*
e *32. 1.*

2. Quoniam igitur triangula BCA, GHF similia sunt, *f* erit $\frac{BCA}{GHF} = \frac{BC}{GH}$ bis. ob eandem causam $\frac{CAD}{HFI} = \frac{CD}{HI}$ bis. denique triang. $\frac{DEA}{IKF} = \frac{DE}{IK}$ bis. quare cum BC. GH *g* :: CD. HI *g* :: DE. IK, *b* erit triang. BCA. GHF :: CAD. HFI :: DEA. IKF :: *k* polyg. ABCDE. FGHIK :: $\frac{BC}{GH}$ bis.
f *19. 6.*
g *hyp. & 16. 5.*
h *cor. 23. 5.*
k *12. 5.*

Coroll.

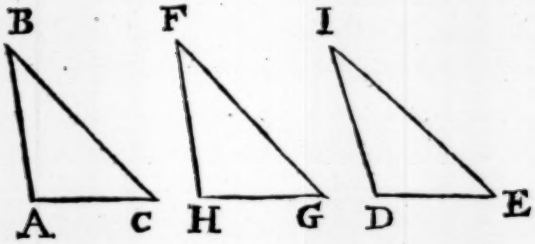
I. Hinc, si fuerint tres lineæ rectæ proportionales; ut est prima ad tertiam, ita erit polygonum super primam descriptum ad polygonum super secundam simile similiterque descriptum. vel ita erit polygonum super secundam descriptum ad polygonum super tertiam simile similiterque descriptum.

Unde elicitur methodus, *figuram quamvis recti lineam augendi vel minuendi in ratione data. Ut si velis pentagoni, cujus latus* CD, *aliud facere quintuplum. inter* AB, *&* 5 AB *inveni mediam proportionalem. Super hac* * *construe pentagonum simile dato. hoc erit quintuplum dati.*
* *18. 6.*

II. Hinc etiam, si figurarum similium homologa latera nota fuerint, etiam proportio figurarum innotescet; nempe inveniendo tertiam proportionalem.

PROP.

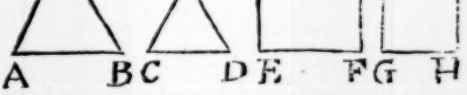
PROP. XXI.



Quæ (ABC, DIE) *eidem rectilineo* HFG *sunt similia, & inter se sunt similia.*

Nam ang. A *a* = H *a* = D. & ang. C *a* = G *a* 1. *def.* 6. *a* = E; & ang. B *a* = F *a* = I. *a* item AB. AC :: HF. HG :: DI. DE. *a* & AC. CB :: HG. GF :: DE. EI. & AB. BC :: HF. FG :: DI. IE. *a* ergo ABC, DIE similia sunt. Q.E.D.

PROP. XXII.



Si quatuor rectæ lineæ proportionales fuerint (AB. CD :: EF. GH) *& ab eis rectilinea similia similiterque descripta proportionalia erunt.* (ABI. CDK :: EM. GO.) *Et si à rectis lineis similia similiterque descripta rectilinea proportionalia fuerint* (ABI. CDK :: EM. GO.) *ipsæ etiam rectæ lineæ proportionales erunt.* (AB. CD :: EF. GH)

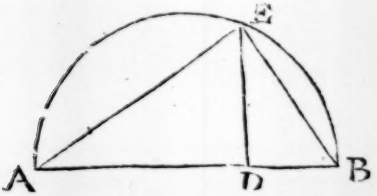
1. *Hyp.* $\frac{ABI}{CDK}\ a = \frac{AB}{CD}$ bis $= \frac{EF}{GH}$ bis $a = \frac{EM}{GO}$. *a* 19. 6. *d* ergo ABI. CDK :: EM. GO. Q.E.D.

2. *Hyp.* $\frac{AB}{CD}$ bis $= a\ \frac{ABI}{CDK}\ b = \frac{EM}{GO}\ c = \frac{EF}{GH}$. *b hyp.* *c* 20. 6. bis. *d* ergo AB. CD :: EF. GH. Q.E.D. *d cor.* 23. 6.

Schol.

Hinc deducitur, & demonſtratur ratio multiplicandi quantitates ſurdas. ex gr. Sit $\sqrt{5}$ multiplicandus in $\sqrt{3}$. dico provenire $\sqrt{15}$. Nam ex multiplicationis definitione debet eſſe, 1. $\sqrt{3}$ :: $\sqrt{5}$. product. ergo per hanc, q. 1. q. $\sqrt{3}$ :: q. $\sqrt{5}$. q. product. hoc eſt. 1. 3 :: 5. q. product. ergo q. product. eſt 15. quare $\sqrt{15}$. eſt productus ex $\sqrt{3}$ in $\sqrt{5}$. Q. E. D.

THEOR.



Petr. Herrig.

Si recta linea A B *ſecta ſit utcunque in* D, *rectangulum ſub partibus* A D, DB *contentum, eſt medium proportionale inter earum quadrata. Item rectangulum contentum ſub tota* A B, *& una parte* AD, vel DB, *eſt medium proportionale inter quadratum totius* A B, *& quadratum dictæ partis* A D, vel D B.

Super diametrum A B deſcribe ſemicirculum. ex D erige normalem DE occurrentem peripheriæ in E. junge AE, BE.

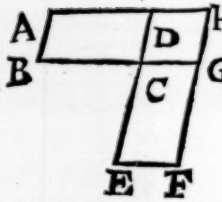
a *cor.* 8.6. Liquet eſſe A D. D E *a* :: D E. D B. *b* ergo
b 2. 6. A Dq. D Eq :: D Eq. D Bq. *c* hoc eſt, ADq.
c 17. 6. ADB :: ADB. DBq. Q. E. D.

d *cor.* 8.6. Porro, B A. A E *d* :: A E. A D. *e* ergo BAq.
e 22. 6. AEq :: AEq. ADq. *f* hoc eſt B Aq. B A D ::
f 17. 6. B A D. A Dq. Eodem modo A Bq. A B D :: ABD. BDq. Q. E. D.

a 1. 6. Vel ſic; ſit Z = A + E. liquet eſſe Aq. AE :: *a* A. E :: *a* AE. Eq. item Zq. Z A :: *a* Z. A. :: *a* ZA. Aq. & Zq. ZE :: *a* Z. E :: ZE. Eq.

PROP.

PROP. XXIII.



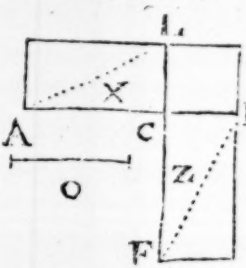
Æquiangula parallelogramma AC, CF *inter se rationem habent eam quæ ex lateribus componitur.* $\left(\frac{AC}{CF} = \frac{BC}{CG} + \frac{DC}{CE}\right)$.

Latera circa æquales angulos C *a* sibi in directum statuantur; & compleatur parallelogrammum CH. a *sch.* 15. 1.

Ratio $\frac{AC}{CF}$ *b* $= \frac{AC}{CH} + \frac{CH}{CF}$ *c* $= \frac{BC}{CG} + \frac{DC}{CE}$. b 20. *def.* 1. c 1. 6. Q. E. D.

Coroll.

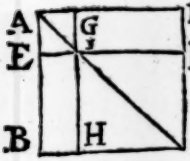
Hinc & ex 34. 1. patet primo, *Triangula, quæ unum angulum* (ad C) *æqualem habent, rationem habere ex rationibus rectarum,* AC ad CB, & LC ad CF, *æqualem angulum continentium.* *And Tarq.* 15. 5.



Patet secundo, *Rectangula ac* * *proinde & parallelogramma quæcunque rationem inter se habere compositam ex rationibus basis ad basim, & altitudinis ad altitudinem.* Neque aliter de triangulis ratiocinaberis. * 35. 1.

Patet tertio, *Quomodo triangulorum ac parallelogrammorum proportio exhiberi possit.* Sunto parallelogramma X & Z; quorum bases AC, CB; altitudines vero CL, CF. Fiat CL. CF :: CB. O. * erit X, Z :: AC. O. * 14. 6. & 1. 6.

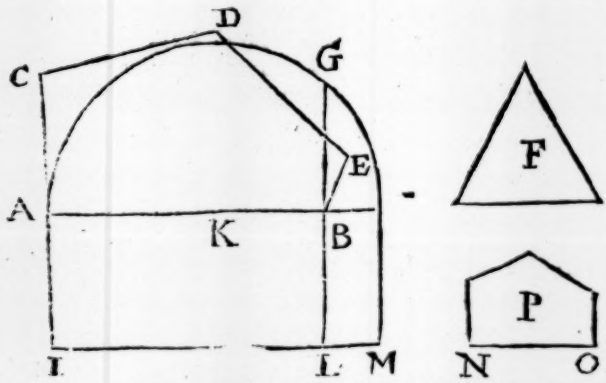
PROP. XXIV.



In omni parallelogrammo ABCD, *quæ circa diametrum* AC *sunt parallelogramma* EG, HF, *& toti & inter se sunt similia.*

Nam parallelogramma EG, HF habent singula unum angulum cum toto communem. *a* ergo toti & sibi mutuo æqui-
a 29. 1. angula sunt, *a* item tam triangula ABC, AEI, IHC, quam triangula ADC, AGI, IFC sunt
b 4. 6. inter se æquiangula. *b* ergo AE. EI :: AB. BC, *b* atque AE. AI :: AB. AC ; *b* & AI. AG :: AC.
c 22. 5. AD. *c* ex æquali igitur, AE. AG :: AB. AD.
d 1. *def.* 6. *d* ergo Pgr. EG. BD similia sunt. eodem modo HF, BD similia sunt. ergo, &c.

PROP. XXV.



Dato rectilineo ABEDC *simile similiterque positum* P, *idemque alteri dato* F *æquale, constituere.*

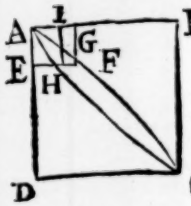
a 45. 1. *a* Fac rectang. AL = ABEDC. *b* item super
b 44. 1. BL fac triang. BM = F. Inter AB, BH *c* in-
c 13. 6. veni mediam proportionalem NO, super NO

d fac

d fac polygonum P ſimile dato ABEDC. Erit hoc æquale dato F. d 18. 6. e cor. 20. 6.

Nam ABEDC (AL.) P :: *e* AB. BH *f* :: AL. BM. ergo P *g* = BM *h* = F. Q.E.F. f 1. 6. g 14. 5. h conſtr.

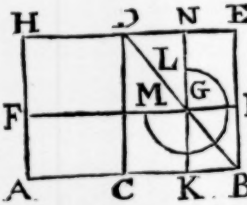
PROP. XXVI.



Si à parallelogrammo ABCD *parallelogrammum* AGFE *ablatum ſit, & ſimile toti, & ſimiliter poſitum, communem cum eo habens angulum* EAG, *hoc circa eandem cum toto diametrum* AC *conſiſtet.*

Si negas AC eſſe communem diametrum, eſto diameter AHC ſecans EF in H. & ducatur HI parall. AE. Parallelogramma EI, DB *a* ſimilia ſunt. *b* ergo AE. EH :: AD. DC *c* :: AE. EF. *d* proinde EH = EF. *f* Q.E.A. a 24. 6. b 1. def. 6. c hyp. d 9. 5. f 9. ax.

PROP. XXVII.



Omnium parallelogrammorum AD, AG *ſecundum eandem rectam lineam* AB *applicatorum, deficientiumque figuris parallelogrammis* CE, KI *ſimilibus, ſimiliterque poſitis, ei* AD, *quod à dimidia deſcribitur, maximum eſt* AD, *quod ad dimidium eſt applicatum, ſimile exſiſtens defectui* KI.

Nam quia GE *a* = GC, addito communi KI, *b* erit KE = CI *c* = AM. adde commune CG, *d* erit AG = Gnom. MBL. ſed Gnom. MBL *e* ⊐ CE (AD.) ergo AG ⊐ AD. Q.E.D. a 43. 1. b 2. ax. c 36. 1. d 2. ax. e 9. ax.

PROP. XXVIII.

Ad datam rectam lineam A B, *dato rectilineo* C *æquale parallelogrammum* AP *applicare deficiens figura parallelogramma* ZR, *quæ similis sit alteri parallelogrammo dato* D. * *Oportet autem datum rectilineum* C, *cui æquale* AP *applicandum est, non majus esse eo* AF, *quod ad dimidiam applicatur, similibus existentibus defectibus, & ejus* AF *quod ad dimidiam applicatur, & ejus* D, *cui simile deesse debet.*

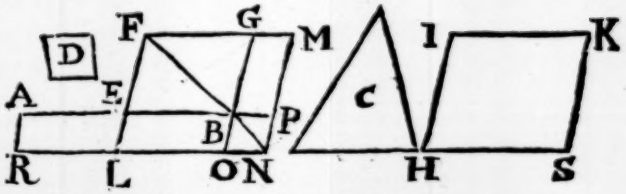
* 27. 6.

a 18. 6. Biseca A B in E. Super EB *a* fac Pgr. E G
b *scb.* 45. 1. simile dato D. *b* sitque EG = C+I. *c* fac Pgr.
c 25. 6. NT = I, & simile dato D, vel EG. duc diametrum FB. fac FO = KN; & FQ = KT. Per O, & Q duc parallelas SR, QZ. parallelogrammum A P est id quod quæritur.

Nam parallelogramma D, EG, OQ, NT,
d *const.* & 24. 6. ZR *d* sunt similia inter se. Et Pgr. EG *e* = NT + C *e* = O Q + C; *f* quare C = Gnom.
e *constr.* OBQ *g* = AO + PG *b* = AO + EP = AP.
f 3. *ax.* Q. E. F.
g 2. *ax.*
h 43. 1.

PROP.

PROP. XXIX.



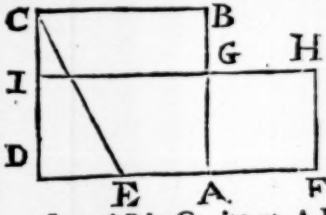
Ad datam rectam lineam AB, *dato rectilineo* C *æquale parallelogrammum* AN *applicare, excedens figura parallelogramma* OP, *quæ simili sit parallelogrammo alteri dato* D.

Biseca AB in E. super EB *a* fac Pgr. EG simile dato D. *b* sitque Pgr. HK = EG + C, & simile dato D vel EG. fac FELc = IH; *c* & FGM = IK. per L, M duc parallelas RN, MN; & AR parall. NM. Produc ABP, GBO. Duc diametrum FBN. Pgr. AN est quæsitum.

Nam parallelogramma D, HK, LM, EG *d* similia sunt. *e* ergo Pgr. OP simile est Pgro LM, vel D. item LM *f* = HK *f* = EG + C. *g* ergo C = Gnom. ENG. atqui AL *b* = LB *k* = BM. *l* ergo C = AN. Q.E.F.

a 18. 6.
b 25. 6.
c 3. 1.
d *constr.*
e 24. 6.
f *constr.*
g 3. *ax.*
h 36. 1.
k 43. 1.
l 2. & 1. *ax.*

PROP. XXX.



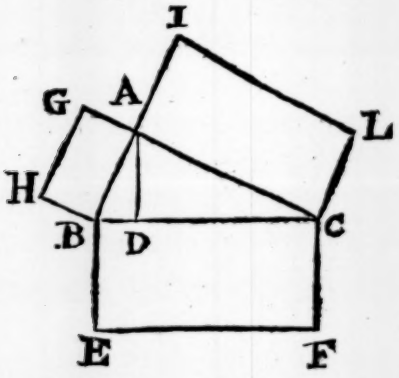
Propositam rectam lineam terminatam AB, *extrema ac media ratione secare.* (AB. AG :: AG. GB.)

a Seca AB in G, ita ut AB × BG = AGq. *b* ergo BA. AG :: AG. GB. Q.E.F.

a 11. 2.
b 17. 6.

PROP.

PROP. XXXI.



In rectangulis trianguli BAG, *figura quævis* BF *à latere* BC *rectum angulum* BAC *subtendente, descripta, æqualis est figuris* BG, AL, *quæ priori illi* BF *similes, & similiter positæ à lateribus* BA, AC *rectum angulum continentibus describuntur.*

Ab angulo recto BAC demitte perpendicularem AD. Quoniam DC. CA :: *a* CA. CB, *b* erit AL. BF :: DC. CB. Item ob DB. BA :: *a* BA. BC, *b* erit BG. BF :: DB. BC. *c* ergo AL + BG. BF :: DC + DB (BC.) BC. ergo AL+BG=BF. Q.E.D.

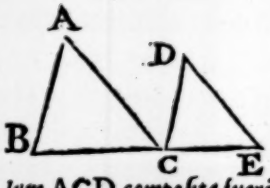
a *cor.* 8.6.
b *cor.* 20.6.
c 24. 5.
d *sch.* 14.5.

Coroll.

Ex hac propositione, addi possunt, & subtrahi figuræ quævis similes, eadem methodo, qua quadrata adduntur & subtrahuntur, in schol. 47. 1.

PROP.

PROP. XXXII.

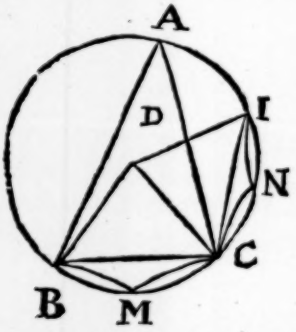


Si duo triangula ABC, DCE, *quæ duo latera duobus lateribus proportionalia habeant* (AB.AC :: DC.DE,) *secundum unum angulum* ACD *composita fuerint, ita ut homologa eorum latera sint etiam parallela* (AB ad DC, & AC ad DE) *tum reliqua illorum triangulorum latera* BC, CE *in rectam lineam collocata reperientur.*

Nam ang. A *a* = ACD *a* = D; & AB. AC *b* :: DC. DE. *c* ergo ang. B = DCE. ergo ang. B + A *d* = ACE. sed ang. B + A + ACB *e* = 2 Rect. *f* ergo ang. ACE + ACB = 2 Rect. *g* ergo BCE est recta linea. Q.E.D.

a 29. 1.
b *hyp.*
c 6. 6.
d 2. *ax.*
e 32. 1.
f 1. *ax.*
g 14. 1.

PROP. XXXIII.



In æqualibus circulis DBCA, HFGP, *anguli* BDC, FHG *eandem habent rationem cum peripheriis* BC, FG, *quibus insistunt; sive ad centra* (ut BDC, FHG,) *sive ad peripherias* A, E *constituti insistant: insuper vero & sectores* BDC, FHG, *quippe qui ad centra consistant.*

Duc

Duc rectas BC, FG. Accommoda CI=CB; & GL=FG=LP; & junge DI, HL, HP.

a 28. 3. Arcus BC *a*=CI, *a* item arcus FG, GL, LP
b 27. 3. æquantur. *b* ergo ang. BDC = CDI *b* & ang. FHG=GHL=LHP. Ergo arcus BI tam multiplex eſt arcûs BC, quam ang. BDI anguli BDC. pariterque æquemultiplex eſt arcus FP arcûs FG, atque ang. FHP anguli FHG. Ve-
c 27. 3. rum ſi arcus BI ⊏, =, ⊐ FP, *c* erit ſimiliter
d 6. *def.* 5. ang. BDI ⊏, =, ⊐ FHP. ergo arc. BC. FG *d* :: ang. BDC. FHG *e* :: $\frac{BDC}{2}$. $\frac{FHG}{2}$ *f* :: A. E.
e 15. 5.
f 20. 3.

Q. E. D.

Rurſus ang. BMC *g* = CNI; *b* atque idcirco
g 27. 3. ſegm. BCM = CIN. *k* item triang. BDC =
h 24. 3. CDI. *l* ergo ſector BDCM = CDIN. Simili
k 4. 1. ratione ſectores FHG, GHL, LHP æquantur.
l 2. *ax.* Quam igitur prout arcus BI ⊏, =, ⊐ FGP, ita
m 6. *def.* 5. ſimiliter ſector BDI ⊏, =, ⊐ FHP. *m* erit ſect. BDC. FHG :: arc. BC. FG. Q.E.D.

Coroll.

11. 5. Hinc 1. *Ut ſector ad ſectorem, ſic angulus ad angulum.*

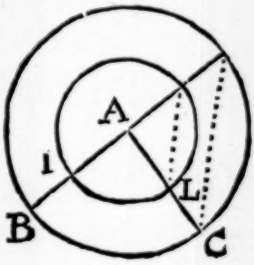
2. *Ang.* BDC *in centro eſt ad 4 rectos, ut arcus* BC *cui inſiſtit ad totam circumferentiam.*

Nam ut ang. BDC ad rectum, ſic arcus BC ad quadrantem. ergo BDC eſt ad 4 rectos, ut arcus BC ad 4 quadrantes, id eſt ad totam circumferentiam. item ang. A. 2 Rect :: arc. BC. periph.

Hinc 3. *Inæqualium circulorum arcus* IL, BC, *qui æquales ſubtendunt angulos, ſive ad centra,* ut IAL & BAC, *ſive ad peripheriam, ſunt ſimiles.*

Nam IL. periph. :: ang. IAL, (BAC.) 4 Rect. item arc. BC. periph :: ang. BAC. 4 Rect.

4 Rect. ergo IL. periph :: BC. periph. proinde arcus IL, & BC ſunt ſimiles. Unde



4. *Duæ ſemidiametri* AB, AC *à concentricis peripheriis arcus auferunt ſimiles* IL, BC.

LIB. VII.

Definitiones.

I. UNitas eſt, ſecundum quam unumquodque eorum quæ ſunt, unum dicitur.

II. Numerus autem eſt, ex unitatibus compoſita multitudo.

III. Pars eſt numerus numeri, minor majoris, quum minor metitur majorem.

Omnis pars ab eo numero nomen ſibi ſumit, per quem ipſa numerum, cujus eſt pars, metitur; ut 4 dicitur tertia pars numeri 12, quia metitur 12 per 3.

IV. Partes autem, cum non metitur.

Partes quæcunque nomen accipiunt à duobus illis numeris, per quos maxima communis duorum numerorum menſura utrumque eorum metitur. ut 10 dicitur $\frac{2}{3}$ numeri 15, eo quod maxima communis menſura, nempe 5, metitur 10 per 2, & 15 per 3.

V. Multiplex vero major minoris, cum majorem metitur minor.

VI. Par numerus eſt, qui bifariam dividitur.

VII. Impar vero numerus, qui bifariam non dividitur; vel, qui unitate differt à pari.

VIII. Pariter par numerus eſt, quem par numerus metitur per numerum parem.

IX. Pariter autem impar eſt, quem par numerus metitur per numerum imparem.

X. Impariter vero impar numerus eſt, quem impar numerus metitur per numerum imparem.

XI. Primus numerus eſt, quem ſola unitas metitur.

XII. Primi inter ſe numeri ſunt, quos ſola unitas, communis menſura, metitur.

XIII.

XIII. Compoſitus numerus eſt, quem numerus quiſpiam metitur.

XIV. Compoſiti autem inter ſe numeri ſunt, quos numerus aliquis, communis menſura, metitur.

In hac definitione & præcedenti unitas non eſt numerus.

XV. Numerus numerum multiplicare dicitur, cum toties compoſitus fuerit is qui multiplicatur, quot ſunt in ipſo multiplicante unitates, & procreatus fuerit aliquis.

Hinc, in omni multiplicatione unitas eſt ad multiplicatorem ut multiplicatus ad productum.

Nota, quod ſæpe cum multiplicandi ſunt quivis numeri, puta A *in* B, *literarum conjunctio productum denotat. Sic* AB = A *in* B. *item* CDE = C *in* D *in* E.

XVI. Cum autem duo numeri ſeſe multiplicantes aliquem fecerint, qui factus erit, planus appellabitur; Qui vero numeri ſeſe mutuo multiplicarint, latera illius dicentur. *Sic* 2 (C) in 3 (D) = 6 = CD *eſt numerus planus.*

XVII. Cum vero tres numeri mutuo ſeſe multiplicantes fecerint aliquem, qui procreatus erit, ſolidus appellabitur; Qui autem numeri mutuo ſeſe multiplicarint, latera illius dicentur. *Sic*, 2 (C) *in* 3 (D) *in* 5 (E) = 30 = CDE *eſt numerus ſolidus.*

XVIII. Quadratus numerus eſt, qui æqualiter æqualis, vel qui ſub duobus æqualibus numeris continetur. *Sit* A *latus quadrati*; *quadratus ſic notatur*, AA, *vel* Aq.

XIX. Cubus vero, qui æqualiter æqualis æqualiter, vel qui ſub tribus æqualibus numeris continetur. *Sit* A *latus cubi*; *cubus notatur ſic*, AAA, *vel* Ac.

In hac definitione, & tribus præcedentibus, unitas eſt numerus.

XX. Nu-

XX. Numeri proportionales ſunt, cum primus ſecundi, & tertius quarti æque multiplex eſt, vel eadem pars; vel denique cum pars primi ſecundum, & eadem pars tertii æque metitur quartum, vel vice verſa. A. B :: C. D. hoc eſt, 3. 9 :: 5. 15.

XXI. Similes plani, & ſolidi numeri ſunt, qui proportionalia habent latera.

Latera nempe non quælibet, ſed quædam.

XXII. Perfectus numerus eſt, qui ſuis ipſius partibus eſt æqualis.

Ut 6. & 28. *Numerus vero qui ſuis ipſius partibus minor eſt, abundans appellatur; qui vero major, diminutus. ut 12 eſt abundans, 15 eſt diminutus.*

XXIII. Numerus numerum metiri dicitur per illum numerum, quem multiplicans, vel à quo multiplicatus, illum producit.

In diviſione, unitas eſt ad quoticntem, ut dividens ad diviſum. Nota, quod numerus alteri lineola interjecta ſubſcriptus diviſionem denotat. Sic $\frac{A}{B}$ = A diviſ. per B. *item* $\frac{CA}{B}$ = C *in* A diviſ. per B.

Termini ſive radices proportionis dicuntur duo numeri, quibus in eadem proportione minores ſumi nequeunt.

Poſtulata.

1. POſtuletur, cuilibet numero quotlibet ſumi poſſe æquales, vel multiplices.

2. Quolibet numero ſumi poſſe majorem.

3. Additio, ſubtractio, multiplicatio, diviſio, extractioneſque radicum, ſeu laterum, numerorum quadratorum, & cuborum concedantur etiam, tanquam poſſibilia.

Axio-

Axiomata.

1. QUicquid convenit uni æqualium numerorum, convenit & reliquis æqualibus numeris.

2. Partes eidem parti, vel iiſdem partibus, eædem, ſunt quoque inter ſe eædem.

3. Qui numeri æqualium numerorum, vel ejuſdem, eædem partes fuerint, æquales inter ſe ſunt.

4. Quorum idem numerus, vel æquales, eædem partes fuerint, æquales inter ſe ſunt.

5. Unitas omnem numerum per unitates, quæ in ipſo ſunt, hoc eſt, per ipſummet numerum metitur.

6. Omnis numerus ſeipſum metitur per unitatem.

7. Si numerus numerum multiplicans, aliquem produxerit, metietur multiplicans productum per multiplicatum, multiplicatus autem eundem per multiplicantem.

Hinc nullus numerus primus planus eſt aut ſolidus, quadratus, vel cubus.

8. Si numerus numerum metiatur, & ille per quem metitur, eundem metietur per eas, quæ in metiente ſunt, unitates, hoc eſt, per ipſum numerum metientem.

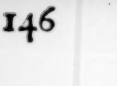
9. Si numerus numerum metiens, multiplicet eum, per quem metitur, vel ab eo multiplicetur, illum quem metitur, producit.

10. Numerus quotcunque numeros metiens, compoſitum quoque ex ipſis metitur.

11. Numerus quemcunque numerum metiens, metitur quoque omnem numerum quem ille metitur.

12. Numerus metiens totum & ablatum, metitur & reliquum.

PROP. I.



Si duobus numeris inæqualibus propositis (AB, CD) *detrahatur semper minor* CD *de majore* AB (& reliquus EB de CD &c.) *alterna quadam detractione, neque reliquus unquam præcedentem metiatur, quoad assumpta sit unitas* GB; *qui principio propositi sunt numeri* AB, CD *primi inter se erunt.*

Si negas, habeant AB, CD communem mensuram, numerum H. Ergo H metiens C D,
a 11. ax. 7. *a* etiam AE metitur; proinde & reliquum EB;
b 12. ax. 7. *a* ergo & C F, atque *b* idcirco reliquum F D;
a quare & ipsum EG. sed totum EB metiebatur;
b ergo & reliquum GB metitur, numerus uni-
c 9. ax. 1. tatem. *c* Q. E. A.

PROP. II.

Duobus numeris datis AB, CD *non primis inter se, maximam eorum communem mensuram* FD *reperire.*

Detrahe minorem numerum C D ex majori
a 6. ax 7. AB, quoties potes. Si nihil relinquitur, *a* patet ipsum C D esse maximam communem mensuram. Si relinquitur aliquid EB, deme hunc ex CD; & reliquum F D ex EB, & sic deinceps, donec aliquis F D præcedentem EB metiatur.
b 1. 7. (nam *b* hoc fiet antequam ad unitatem perveniatur.) Erit FD maxima communis mensura.
c constr. Nam F D *c* metitur E B, *d* ideoque & CF;
d 11. ax. 7 *e* proinde & totum CD; *d* ergo ipsum AE; atque
e 12. ax. 7. idcirco totum AB metitur. Liquet igitur F D communem esse mensuram. Si maximam esse

negas,

negas, ſit major quæpiam G. ergo G metiens CD, *d* metitur AE, *e* & reliquum EB, *d* ipſumque CF. *e* proinde & reliquum FD, *g* major minorem. *b* Q. E. A.

g *ſuppoſ.*
h *9. ax. 1.*

Coroll.

Hinc, numerus metiens duos numeros, metitur quoq; maximam eorum communem menſuram.

PROP. III.

A..... 12
B..... 8
D.... 4
C..... 6
E.. 2
F--.-

Tribus numeris datis A, B, C *non primis inter ſe, maximam eorum communem menſuram* E *reperire.*

Inveni D maximam communem menſuram duorum A, B. Si D metitur tertium **C**, liquet D maximam eſſe trium communem menſuram. Si D non metitur C, erunt ſaltem D, & C compoſiti inter ſe, ex coroll. præcedentis. Sit igitur ipſorum D, & C maxima communis menſura E. erit E is quem quæris.

Nam E *a* metitur C, & D; *a* ac D ipſos A, & B metitur; *b* ergo E metitur ſingulos A, B, C; nec major aliquis (F) eos metietur; nam ſi hoc affirmas, *c* ergo F metiens A, & B, eorum maximam communem menſuram D metitur. Eodem modo, F metiens D, & C, *c* eorum maximam communem menſuram E, *d* major minorem, metitur. *e* Q. E. A.

a *conſtr.*
b *11. ax. 7.*
c *cor. 27.*
d *ſuppoſ.*
e *9. ax. 1.*

Coroll.

Hinc, numerus metiens tres numeros, maximam quoq; eorum communem menſuram metitur.

PROP. IV.

A 6
B 7
B 18
B ..... 9.

Omnis numerus A, *omnis numeri* B, *minor majoris, aut pars est, aut partes.*

Si A & B primi sint inter se, *a* erit A tot partes numeri B, quot sunt in A unitates. (ut $6 = \frac{6}{7}$ 7.) Sin A metiatur B, *b* liquet A esse partem ipsius B. (ut $6 = \frac{1}{3}$ 18.) denique si A & B aliter compositi inter se fuerint, *c* maxima communis mensura determinabit, quot partes A conficiat ipsius B, ut $6 = \frac{2}{3}$ 9.

a 4. *def* 7.
b 3. *def.* 7.
c 4. *def.* 7.

PROP. V.

A ..... 6 D 4
6 6 4 4
B ..... G ..... C 12. E .... H .... F 8

Si numerus A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars; & simul uterque* (A+D) *utriusque simul* (BC+EF) *eadem pars erit, quæ unus* A *unius* BC

Nam si BC in suas partes BG, GC ipsi A æquales; atque EF in suas partes FH, HF ipsi D æquales resolvantur; *a* erit numerus partium in BC æqualis numero partium in EF. Quum igitur A + D *b* = BG + EH = GC + HF, erit A + D toties in BC + EF, quoties A in BC. Q. E. D.

a *hyp.*
b *const. & 2. ax* 1.

Vel sic brevius. Sit $a = \frac{x}{2}$ & $b = \frac{y}{2}$. quare 2a = x & 2b = y. *c* Ergo 2a + 2b = x + y. Ergo $a + b = \frac{x+y}{2}$.

PROP. VI.

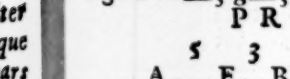
Si numerus AB *numeri* C *partes fuerit; & alter* DE *alterius* F *eædem partes; & simul uterq;* (AB+DE) *utriusq; simul* (C+F) *eædem partes erit, quæ unus* AB *unius* C.

Divide AB in ſuas partes AG, GB; & DE in ſuas DH, HE. Partium in utroque AB, DE æqualis eſt multitudo, ex hypoth. Quum igitur AG *a* ſit eadem pars numeri C, quæ DH numeri F, *b* erit AG+DH eadem pars compoſiti C+F, quæ unus AG unius C. *b* Eodem modo GB+HE eadem pars eſt ejuſdem C+F, quæ unus GB unius C; *c* ergo AB+DE eædem partes eſt ipſius C+F, quæ AB ipſius C. Q. E. D.

a hyp. b 5. 7. c 2. ax. 7.

Vel ſic. Sit $a=\frac{2}{3}x$, & $b=\frac{2}{3}y$, & $x+y=g$. ob 3 $a=2x$, & 3 $b=2y$, eſt 3 $a+3b=2x+2y=2g$. ergo $a+b=\frac{2}{3}g=\frac{2}{3}:x+y$.

PROP. VII.



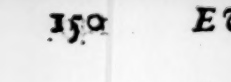
Si numerus AB *numeri* CD *pars fuerit, qualis ablatus* AE *ablati* CF; *& reliquus* EB *reliqui* FD *eadem pars erit, qualis totus* AB *totius* CD.

a Sit EB eadem pars numeri GC, quæ AB ipſius CD, vel AE ipſius CF. *b* ergo AE+EB eadem eſt pars ipſius CF+GC, quæ AE ipſius CF, vel AB ipſius CD. *c* ergo GF=CD. aufer communem CF, *d* manet GC=FD. *e* ergo EB eadem eſt pars reliqui FD (GC) quæ totus AB totius CB. Q. E. D.

a 1. *poſt.* 7. b 5. 7. c 6. *ax.* 1. d 3. *ax.* 1. e 2. *ax.* 7.

Vel ſic. Sit a+b=x, & c+d=y; atque tam x=3 y, quam a=3 c; dico b=3 d. Nam 3 c+3 d=3 y=x g=a+b. aufer utrinque 3 c g=a, & remanet 3 d=b. Q. E. D. f 1. 2. g *hyp.*

PROP. VIII.

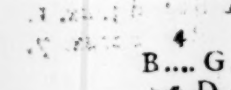


Si numerus AB *numeri* CD *partes fueris, quales ablatus* AE *ablati* CF; *& reliquus* EB *reliqui* ED *eædem partes erit, quales totus* AB *totius* CD.

Seca AB in AG, GB partes numeri CD; item AE in AH, HE partes numeri CF; & sume
a 3. *ax.* 1. GL = AH = HE; *a* quare HG = EL. & quia
b *constr.* *b* AG = GB, *c* etiam HG = LB. Cum igitur
c 3. *ax.* 1. totus AG eadem sit pars totius CD, quæ ablatus AH ablati CF; *d* erit reliquus HG, vel
d 7. 7. EL, eadem etiam pars reliqui FD, quæ AG ipsius CD. Eodem pacto, quia GB eadem pars est totius CD, quæ HE, vel GL, ipsius CF, *d* erit reliquus LB eadem pars reliqui FD, quæ GB totius CD; ergo EL + LB (EB) eædem est partes reliqui FD, quæ totus AB totius CD. Q. E. D.

Vel sic facilius. Sit a + b = x. & c + d = y.
e 9. *ax.* 7. Item tam $y = \frac{2}{3}x$, quam $c = \frac{2}{3}a$; vel *e* quod idem est, 3 y = 2 x; & 3 c = 2 a. Dico $d = \frac{2}{3}b$.
f 1. 2. Nam 3 c + 3 d *f* = 3 y = 2 x *f* = 2 a + 2 b.
g 1. *ax.* 1. *g* ergo 3 c + 3 d = 2 a + 2 b. aufer utrinque
h *hyp.* 3 c *h* = 2 a; & *k* manet 3 d = 2 b. *l* ergo $d = \frac{2}{3}b$.
k 3. *ax.* 1. Q. E. D.
l 8. *ax.* 7.

PROP. IX.



Si numerus A *numeri* BC *pars fuerit, & alter* D *alterius* EF *eadem pars; & vicissim quæ pars est, aut partes primus* A *tertii* D, *eadem pars erit, vel eædem partes, & secundus* BC *quarti* EF.

Poni-

Ponitur A⌐D. Sint igitur BG, GC, & EH, HF partes numerorum BC, EF, hæ ipſi A, illæ ipſi D pares utrinque multitudo partium æqualis ponitur. Liquet vero BG *a* eandem eſſe partem, aut eaſdem partes ipſius EH, quæ GC ipſius HF; *b* quare BC (BG+GC) ipſius EF (EH+HF) eadem pars eſt aut partes, quæ unus BG (A) unius EH (D.) Q. E. D. a 1. ax. 7. & 4. 7. b 5. vel 6. 7

Vel ſic; ſit $a=\frac{b}{3}$, & $c=\frac{d}{3}$, vel $3a=b$, & $3c=d$; *c* eſtque $\frac{c}{a}=(\frac{3c}{3a}=)\frac{b}{d}$. c 5. 15.

PROP. X.

A .. G .. B 4
C 6
5 5
D ..... H ..... E 10
F 15

Si numerus AB *numeri* C *partes fuerit, & alter* DE *alterius* F *eædem partes; & viciſſim quæ partes eſt primus* AB *tertii* DE, *aut pars, eædem partes erit & ſecundus* C *quarti* F, *aut pars.*

Ponitur AB⌐DE, & C⌐F. Sint AG, GB, & DH, HE partes numerorum C, & F, tot nempe in AB, quot in DE. Conſtat AG ipſius C eandem eſſe partem, quæ DH ipſius F. *a* quare viciſſim AG ipſius DH, pariterque GB ipſius HE, & *b* proinde conjunctim AB ipſius DE eadem pars erit, aut partes, quæ C ipſius F. Q. E. D. a 9. 7. b 5. & 9. 7.

Vel ſic; ſit $a=\frac{2}{3}b$, & $c=\frac{2}{3}d$. vel $3a=2b$, & $3c=2d$. Eſt $\frac{c}{a}=\frac{3c}{3a}=\frac{2d}{2b}=\frac{d}{b}$.

PROP. XI.

4 3
A .... E ... B 7.
8 6
C ..... F ..... D 14

Si fuerit, ut totus AB *ad totum* CD, *ita ablatus* AE *ad ablatum* CF; *& reliquus* EB *ad reliquum*

FD *erit, ut totus* AB *ad totum* CD.

a 4. 7. b 20. *def.* 7. c 7. vel 8. 7. Sit primo A B ⊐ C D ; *a* ergo AB vel pars eſt, vel partes numeri CD; *b* eademque pars eſt, vel partes ipſe AE ipſius CF; *c* ergo reliquus EB reliqui FD eadem pars eſt, aut partes, quæ totus A B totius C D. *b* ergo A B. C D :: E B. F D. Sin fuerit A B ⊏ C D ; eodem modo erit juxta modo oſtenſa, CD. AB :: FD. EB. ergo invertendo, AB. CD :: EB. FD.

PROP. XII.

A, 4. C, 2. E, 3.
B, 8. D, 4. F, 6.

Si ſint quotcunque numeri proportionales (A. B :: C. D :: E F) *erit quemadmodum unus antecedentium* A *ad unum conſequentium* B, *ita omnes antecedentes* (A + C + E) *ad omnes conſequentes* (B + D + F.)

Sint primo, A, C, E minores quam B, D, F.

a 20. *def.* 7. b 5. & 6. 7. c 20. *def.* 7. ergo (propter eaſdem rationes) *a* erit A eadem pars aut partes ipſius B, quæ C ipſius D. *b* ergo conjunctim A + C eadem erit pars aut partes ipſius B + D, quæ unus A unius B. Similiter A + C + E eadem pars eſt, aut partes ipſius B + D + F, quæ A ipſius B. *c* ergo A + C + E. B + D + F :: A. B. Q. E. D. Sin A, C, E, ipſis B, D, F majores ponantur, idem oſtendetur invertendo.

PROP. XIII.

A, 3. C, 4.
B, 9. D, 12.

Si quatvor numeri proportionales ſint (A. B :: C. D.) *& viciſſim proportionales erunt* (A. C :: B. D.)

Sint primo A & C ipſis B & D minores, atque A ⊐ C. Ob eandem proportionem, *a* erit A eadem pars, aut partes ipſius B, quæ C ipſius D. *b* ergo viciſſim A ipſius C eadem pars eſt, aut partes, quæ B ipſius D, ergo A. C :: B. D. Sin

a 20. *def.* 7. b 9. & 10. 7

A ⊏

A⌐C; atque A & C majores ſtatuantur, quam B & D, eadem res erit, proportiones invertendo.

PROP. XIV.

A, 9. D, 6. B, 6. E, 4. C, 3. F, 2.

Si ſint quotcunque numeri A, B, C, *& alii totidem* D, E, F *illis æquales multitudine, qui bini ſumantur, & in eadem ratione* (A.B :: D.E. & B.C :: E.F) *etiam ex æqualitate in eadem ratione erunt.* (A.C :: D.F.)

Nam quia A.B :: D.E, *a* erit viciſſim, A.D :: B.E :: *a* C.F. *a* ergo iterum permutando, A,C :: D.F. Q.E.D. a 13. 7.

PROP. XV.

1. D. B ... 3. E ..... 6.

Si unitas numerum quempiam B *metiatur; æque autem alter numerus* D *alterum quendam numerum* E *metiatur; & viciſſim æque unitas tertium numerum* D *metietur, & ſecundus* B *quartum* E.

Nam quia 1 eſt eadem pars ipſius B, quæ D ipſius E, *a* erit viciſſim 1 eadem pars ipſius D, quæ B ipſius E. Q.E.D. a 9. 7.

PROP. XVI.

B, 4. A, 3. A, 3. B, 4. AB, 12. BA, 12.

Si duo numeri A, B *ſeſe mutuo multiplicantes fecerint aliquos* AB, BA, *geniti ex ipſis* AB, BA *æquales inter ſe erunt.*

Nam quia AB=A in B, *a* erit 1 in A toties, quoties B in AB. *b* ergo viciſſim 1 in B toties erit, quoties A in AB. atqui quoniam BA=B in A, *a* erit 1 in B toties, quoties A in BA. ergo quoties 1 in AB, toties 1 in BA; & *c* proinde AB=BA. Q.E.D. a 15. *def.* 7. b 15. 7. c 4. *ax.* 7.

PROP.

PROP. XVII.

A, 3. B, 2. C, 4. AB, 6. AC, 12.

Si numerus A *duos numeros* B, C *multiplicans fecerit aliquos* AB, AC; *geniti ex ipsis eandem rationem habebunt, quam multiplicati.* (AB. AC :: B. C.)

a 15. *def.* 7. Nam quia AB = A in B, *a* erit 1 toties in A, quoties B in AB. *a* item quia AC = A in C, erit 1 toties in A, quoties C in AC. ergo quo-
b 20. *def.* 7 ties B in AB, toties C in AC, quare B. AB ::
c 13. 7. C. AC. *c* ergo vicissim, B. C :: AB. AC. Q. E. D.

PROP. XVIII.

C, 5. A, 3. $\overline{AC}$, 15. C, 5. B, 9. $\overline{BC}$, 45.

Si duo numeri A, B, *numerum quempiam* C *multiplicantes fecerint aliquos* AC, BC; *geniti ex ipsis eandem rationem habebunt, quam multiplicantes.* (A. B :: AC. BC.)

a 16. 7. Nam AC *a* = CA; & BC *a* = CB; sic idem C multiplicans A & B producit AC, & BC.
b 17. 7. *b* ergo A. B :: AC. BC. Q. E. D.

Schol.

Ex his pendet modus vulgaris reducendi fractiones ($\frac{3}{5}, \frac{7}{9}$) ad eandem denominationem. Nam duc 9 tam in 3, quam in 5, proveniunt $\frac{27}{45} = \frac{3}{5}$. quoniam ex his, 3. 5 :: 27. 45. item duc 5 in 7, & 9, prodeunt $\frac{35}{45} = \frac{7}{9}$. quia 7. 9 :: 35. 45.

PROP. XIX.

A, 4. B, 6. AD, 48. C, 8. D, 12. BC, 48.

Si quatuor numeri proportionales fuerint, (A. B :: C. D;) *qui ex primo & quarto fit numerus* AD, *aequalis est ei, qui ex secundo & tertio fit, numero* BC.

BC. *Et ſi qui ex primo & quarto ſit numerus* AD, *æqualis ſit ei, qui ex ſecundo & tertio ſit, numero* BC, *ipſi quatuor numeri proportionales erunt.* (A. B :: C. D.)

1. *Hyp.* Nam AC. AD *a* :: C. D *b* :: A. B *c* :: AC. BC. *d* ergo AD = BC. Q. E. D.
a 17. 7. b *hyp.* c 18. 7. d 9. 5.

2. *Hyp.* Quoniam *e* AD = BC, erit AC. AD *f* :: AC. BC. Sed AC. AD *g* :: C. D. & AC. BC *b* :: A. B. *k* ergo C. D :: A. B. Q. E. D.
e *hyp.* f 7. 5. g 17. 7. h 18. 7. k 11. 5.

PROP XX.

A. 4. B. 6. C. 9. AC, 36. BB, 36. D, 6.

Si tres numeri proportionales ſuerint (A. B :: B. C.) *qui ſub extremis continetur* (AC) *æqualis eſt ei, qui à medio efficitur* (BB.) *Et ſi qui ſub extremis continetur* (AC) *æqualis fuerit ei* (Bq) *qui ſub medio, ipſi tres numeri proportionales erunt* ($\frac{A}{B} :: \frac{B}{C}$.)

1. *Hyp.* Nam ſume D = B. *a* ergo AB :: D (B.) C. *b* quare AC = BD, *a* vel BB. Q. E. D.
a 1. *ax* 7. b 19. 7.

2. *Hyp.* Quia AC *c* = BD, *d* erit A. B :: D (B.) C. Q. E. D.
c *hyp.* d 19. 7.

PROP. XXI.

A .. G .. B 5. E ..... 10. C .. H . D 3. F ..... 6.

Numeri AB, CD *minimi omnium eandem cum eis rationem habentium* (E, F) *metiuntur æque numeros* E, F *eandem cum eis rationem habentes, major quidem* AB *majorem* E, *minor vero* CD *minorem* F.

Nam AB. CD *a* :: E. F. *b* ergo viciſſim AB. E :: CD. F. *c* ergo AB eadem pars eſt, vel partes ipſius E, quæ CD ipſius F. Non partes; nam ſi ita, ſint AG, GB partes numeri E; & CH, HD partes numeri F. *c* ergo AG. E ::
a *hyp.* b 13. 7. c 20. *def.* 7.

CH.

d 13. 7. CH. F; & permutando, AG. CH *d* :: E. F *e* ::
e *hyp.* AB. CD. ergo AB, CD non ſunt minimi in ſua ratione, contra hypoth. ergo, &c.

PROP. XXII.

A, 4. D, 12. B, 3. E, 8. C, 2. F, 6.

Si fuerint tres numeri A, B, C, *& alii ipſis multitudine æquales* D, E, F, *qui bini ſumantur, & in eadem ratione; fuerit autem perturbata eorum proportio* (A. B :: E. F & B. C :: D. E;) *etiam ex æqualitate in eadem ratione erunt* (A. C :: D. F.)

a *hyp.* Nam quia A. B *a* :: E. F, *b* erit AF = BE; &
b 19. 7. quia B. C :: *a* D. E, *b* erit BE = CD. *c* ergo
c 1. *ax.* 1. AF = CD. *d* quare A. C :: D. F. Q. E. D.
d 19. 7.

PROP. XXIII.

A, 9. B, 4. C --- D --- E --

Primi inter ſe numeri A, B, *minimi ſunt omnium eandem cum eis rationem habentium.*

Si fieri poteſt, ſint C & D minores quam A & B, atque in eadem ratione.
a 21. 7. *a* ergo C metitur A æque, ac D metitur B, puta per eundem numerum E: quoties igitur
b 23. *def* 7. 1 in E, *b* toties erit C in A. *c* quare viciſſim quo-
c 15. 7. ties 1 in C, toties E in A. ſimili diſcurſu quoties 1 in D, toties E in B. ergo E utrumque A & B metitur; qui proinde inter ſe primi non ſunt, contra Hypoth.

PROP. XXIV.

A, 9. B, 4. C --- D --- E --

Numeri A, B, *minimi omnium eandem cum eis rationem habentium, primi inter ſe ſunt.*

Si fieri poteſt, habeant A & B communem menſuram C; is metiatur A
a 9. *ax.* 7. per D, & B per E; *a* ergo CD = A, & CE = B.

b quare

b quare A. B :: D. E. Sed D & E minores ſunt quam A & B, utpote eorum partes. Ergo A & B non ſunt minimi in ſua ratione, contra hypoth. b 17. 7.

PROP. XXV.

A, 9. B, 4. C, 3. D --

Si duo numeri A, B *primi inter ſe fuerint, qui unum eorum* A *metitur numerus* C, *ad reliquum* B *primus erit.*

Nam ſi affirmes aliquem D numeros B & C metiri, *a* ergo D metiens C, metitur A. ergo A & B non ſunt primi inter ſe, contra Hypoth. a 11. *ax.* 7.

PROP. XXVI.

A, 5. B, 3. AB, 15. C, 8. E.... F....

Si duo numeri A, B *ad quempiam* C *primi fuerint, etiam ex illis genitus* AB *ad eundem* C *primus erit.*

Si fieri poteſt, ſit ipſorum AB, & C communis menſura numerus E. ſitque $\frac{AB}{E}$ = F; *a* ergo AB = EF; *b* quare E.A :: B. F. Quia vero A primus eſt ad C quem E metitur, *c* erunt E & A primi inter ſe; *d* adeoque in ſua proportione minimi, & *e* proinde æque metiuntur B, & F; nempe E ipſum B, & A ipſum F. Quum igitur E utrumque B, C metiatur, non erunt illi primi inter ſe, contra Hypoth. a 9. *ax.* 7. b 19. 7. c 25. 7. d 23. 7. e 21. 7.

PROP. XXVII.

A, 4. B, 5. Aq, 16. D, 4.

Si duo numeri, A, B, *primi inter ſe fuerint, etiam ex uno eorum genitus* (Aq) *ad reliquum* B *primus erit.*

Sume D = A; ergo *a* ſinguli D, & A primi ſunt ad B. *b* quare A D, vel Aq, ad B primus eſt. Q. E. D. a 1. *ax.* 7. b 26. 7.

PROP.

PROP. XXVIII.

A, 5. C, 4. B, 3. D, 2. AB, 15. CD, 8.

Si duo numeri A, B *ad duos numeros* C, D, *uterque ad utrumque, primi fuerint, & qui ex eis gignentur* AB, CD, *primi inter se erunt.*

a 26. 7. Nam quia A & B ad C primi sunt, *a* erit AB ad C primus. Eadem ratione erit AB ad D
b 26. 7. primus. *b* ergo AB ad CD primus est. Q. E. D.

PROP. XXIX.

A, 3. B, 2. Aq, 9. Bq, 4. Ac, 27. Bc, 8.

Si duo numeri A, B *primi inter se fuerint, & multiplicans uterque seipsum fecerit aliquem* (Aq, & Bq,) *& geniti ex ipsis* (Aq, Bq) *primi inter se erunt; & si qui in principio* A, B *genitos ipsos* Aq, Bq *multiplicantes fecerint aliquos* (Ac, Bc;) *& hi primi inter se erunt: & semper circa extremos hoc evenict.*

a 27. 7. Nam quia A primus est ad B, *a* erit Aq ad B primus, & quia Aq primus ad B, *a* erit Aq ad Bq primus. Rursus quia tam A ad B & Bq,
b 28. 7. quam Aq ad eosdem B, & Bq primi sunt, *b* erit A × Aq, id est Ac, ad B × Bq, id est Bc, primus. Et sic porro de reliquis.

PROP. XXX.

8 5
A ..... B ..... C 13. D ---- AB, BC

Si duo numeri primi inter se fuerint, etiam uterque simul (AC) *ad quemlibet illorum* AB, BC *primus erit. Et si uterque simul* AC *ad unum aliquem illorum* AB *primus fuerit, etiam qui in principio numeri* AB, BC *primi inter se erunt.*

1. *Hyp.* Nam si AC, AB compositos velis,
a 12. ax. 7. sit D communis mensura. *a* Is metietur reliquum BC. ergo AB, BC non sunt primi inter se, contra Hypoth.

2. *Hyp.*

2. *Hyp.* Positis AC, AB inter se primis, vis D ipsorum AB, BC communem esse mensuram. *b* Is igitur totum AC metitur. quare AC, AB non sunt primi inter se, contra Hypoth. b 10. *ax.* 7.

Coroll.

Hinc numerus, qui ex duobus compositus, ad unum illorum primus est, ad reliquum quoque primus est.

PROP. XXXI.

A 5, B, 8. *Omnis primus numerus* A *ad omnem numerum* B, *quem non metitur, primus est.*

Nam si communis aliqua mensura metiatur utrumque A, B; *a* non erit A primus numerus, contra Hypoth. a 11. *def.* 7.

PROP. XXXII.

A, 4. B, 6. AB, 24. D, 3. E, 8. *Si duo numeri* A, B, *se mutuo multiplicantes fecerint aliquem* AB; *genitum autem ex ipsis* AB *metiatur aliquis primus numerus* D; *is etiam unum eorum, qui à principio,* A, vel B *metietur.*

Pone numerum D non metiri A; sit vero $\frac{AB}{D} = E$. *a* ergo AB = DE. *b* quare D. A :: B. E. *c* est vero D ad A primus. *d* ergo D & A minimi sunt in sua ratione; *e* proinde D metitur B, æque ac A metitur E. liquet igitur propositum. a 9. *ax.* 7. b 19. 7. c *byp. &* 31. 7. d 23. 7. e 21. 7.

PROP. XXXIII.

A, 12. B, 2. *Omnem compositum numerum* A, *aliquis primus numerus* B *metitur.*

Unus vel plures numeri *a* metiantur A, quorum minimus sit B, is primus erit, nam a 13. *def.* 7

a 13. *def.* 7. nam si dicetur compositus, *a* eum minor aliquis
b 11. *ax* 7. metietur, *b* qui proinde ipsum A metietur; quare B non est minimus eorum, qui A metiuntur; contra Hypoth.

PROP. XXXIV.

A, 9. *Omnis numerus* A, *aut primus est, aut eum aliquis primus metitur.*

Nam A necessario vel primus est, vel compositus. Si primus, hoc est quod asserimus. Si compositus,
a 33. 7. *a* ergo eum aliquis primus metitur. Q. E. D.

PROP. XXXV.

A, 6. B, 4. C, 8.
D, 2. H -- I -- K ---
E, 3. F, 2. G, 4. L ---

Numeris datis quotcunque A, B, C *reperire minimos omnium* E, F, G *eandem rationem cum eis habentium.*

Si A, B, C primi sint inter se, ipsi in sua ratione minimi *a* erunt. Si compositi sint, *b* esto
a 23. 7.
b 3. 7. eorum maxima communis mensura D, qui ipsos metiatur per E, F, G. Hi minimi erunt in ratione A, B, C.

c 9. *ax.* 7. Nam D ductus in E, F, G *c* producit ABC.
d 17 7. *d* ergo hi & illi in eadem sunt ratione. Jam puta
e 21. 7. alios H, I, K minimos esse in eadem; *e* qui propterea æque metientur A, B, C nempe per numerum L. *f* ergo L in H, I, K ipsos A, B, C procreabit. *g* ergo ED = A = HL. *h* unde E. H :: L. D. Sed E *k* ⊏ H; *l* ergo L ⊏ D. ergo D non est maxima communis mensura ipsorum A, B, C; contra Hypoth.
f 9. *ax.* 7.
g 1. *ax.* 1.
h 19. 7.
k *suppos.*
l 20. *def.* 7.

Coroll.

Hinc, maxima communis mensura quotlibet numc-

numerorum metitur ipſos per numeros, qui minimi ſunt omnium eandem rationem cum ipſis habentium. Ex quo patet methodus vulgaris reducendi fractiones ad minimos terminos.

PROP. XXXVI.

Duobus numeris datis A, B, *reperire, quem illi minimum metiuntur, numerum.*

A, 5. B, 4. AB, 20. D------ E--- F---

1. *Caſ.* Si A, & B primi ſint inter ſe, eſt AB quæſitus. Nam liquet A & B metiri AB. Si fieri poteſt, metiantur A & B aliquem D⌝AB; puta per E, & F. *a* ergo AE=D=BF. *b* quare A. B:: F. E. Quia vero A, & B *c* primi ſunt inter ſe, *d* adeoque in ſua ratione minimi, *e* æque metientur A ipſum F, ac B ipſum E. Atqui B. E *f*:: AB. AE (D.) *g* ergo AB etiam metietur D, ſeipſo minorem. Q. E. A.

a 9. *ax.* 7. & 1. *ax.* 1.
b 19. 7.
c *hyp.*
d 23. 7.
e 21. 7.
f 17. 7.
g 20. *def.* 7.

A, 6. B, 4. C, 3. D, 2. AD, 12. F---- G---- H----

2. *Caſ.* Sin A, & B inter ſe compoſiti fuerint, *h* reperiantur C, & D minimi in eadem ratione. *k* ergo A D=BC. Erit A D, vel B C quæſitus.

h 35. 7.
k 19. 7.

Nam *l* liquet B, & A ipſum A D, vel B C metiri Puta A, & B meriti F⌝A D, nempe A per G, & B per H. *m* ergo A G = F = B H. *n* unde A. B:: H. G *o*:: C. D. *p* proinde æque metitur C ipſum H, ac D ipſum G. atqui D. G *q*:: A D. A G (F.) ergo A D *r* metitur F, major minorem Q. E. A.

l 7. *ax.* 7.
m 9. *ax.* 7.
n 19. 7.
o *conſtr.*
p 21. 7.
q. 17. 7.
r 20. *def.* 7.

Coroll.

Hinc, ſi duo numeri multiplicent minimos eandem rationem habentes, major minorem, & minor majorem, producetur numerus minimus, quem illi metiuntur.

PROP. XXXVII.

A, 2. B, 3.
E, ... 6.
C---F---D

Si duo numeri A, B *numerum quempiam* CD *metiantur; etiam minimus* E, *quem illi metiuntur, eundem* CD *metietur.*

Si negas, aufer E ex CD, quoties fieri poteſt, & relinquatur FD⌐E. quum igitur A & B *a* metiantur E, *b* & E ipſum CF, *c* etiam A, & B metiuntur CF; *a* metiuntur autem totum CD; *d* ergo etiam reliquum FD metiuntur. ergo E non eſt minimus, quem A, & B metiuntur, contra hyp.

a *hyp.*
b *conſtr.*
c 11. *ax.* 7.
d 12. *ax.* 7.

PROP. XXXVIII.

A, 3, B, 4, C, 6.
D, 12.

Tribus numeris datis A, B, C, *reperire minimum, quem illi metiuntur.*

a Reperi D minimum, quem duo A, & B metiuntur; quem ſi tertius C metiatur, patet D eſſe quæſitum. Quod ſi C non metiatur D, ſit E minimus, quem C, & D metiuntur. Erit E requiſitus.

a 36. 7.

A, 2. B, 3. C, 4.
D, 6. E, 12.
F---

Nam ſingulos A, B, C metiri E conſtat ex 11. ax. 7. Quod vero nullum alium F minorem metiantur, facile oſtenditur. Nam ſi affirmas, *b* ergo D metitur F; *b* proinde E eundem F metitur, major minorem. Quod eſt abſurdum.

b 37. 7.

Coroll.

Hinc, ſi tres numeri numerum quempiam metiantur; etiam minimus, quem illi metiuntur, eundem metietur.

PROP. XXXIX.

A, 12. B,4, C,3. *Si numerum* A *quispiam numerus* B *metiatur, ille* A *quem* B *metitur, partem habebit* C, *à metiente* B *denominatam.*

Nam quia $\frac{A}{B}$ *a*=C, *b* erit A=BC. *c* ergo $\frac{A}{C}$ =B. Q. E. D.

a *hyp.*
b 9. *ax.* 7.
c 7. *ax.* 7.

PROP. XL.

A, 15. B, 3. C, 5. *Si numerus* A *partem habuerit quamlibet* B, *metietur illum numerus* C, *à quo ipsa pars* B *denominatur.*

Nam quia BC *a*=A, *b* erit $\frac{A}{C}$ =B. Q. E. D.

a *hyp. &* 9. *ax.* 7.
b 7. *ax.* 7.

PROP. XLI.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ G, 12. H ---. *Numerum reperire* G, *qui minimus cum sit, habeat datas partes,* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

a Inveniatur G minimus, quem denominatores 2, 3, 4 metiuntur. *b* Liquet G habere partes, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$. Si fieri potest, H⊐G habeat easdem partes; *c* ergo 2, 3, 4 metiuntur H, & proinde G non est minimus, quem 2, 3, 4 metiuntur. contra constr.

a 38. 7.
b 39. 7.
c 40. 6.

LIB. VIII.

PROP. I.

A, 8. B, 12. C, 18. D, 27.
E - F -- G ... H ....

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D ; *extremi vero ipsorum* A, D *primi inter se fuerint* ; *ipsi* A, B, C, D *minimi sunt omnium eandem cum eis rationem habentium.*

Nam, si fieri potest, sint alii totidem E, F, G, H
a 14. 7. minores in illa ratione. *a* ergo ex æquali A. D ::
b 23. 7. E. H. ergo A, & D primi numeri, *b* adeoque in
c 21. 7. sua ratione minimi, *c* æque metiuntur E, & H, seipsis minores. Q. E. A.

PROP II.

I.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.
Ac, 8. AqB, 12. ABq, 18. Bc, 27.

Numeros reperire deinceps proportionales minimos, quotcunque jusserit quispiam, in data ratione A ad B.

Sint A, & B minimi in data ratione. Erunt Aq, AB, Bq tres minimi deinceps in ratione A ad B.

a 17. 7. Nam AA AB *a* :: A. B *a* :: AB. BB. item
b 24. 7. quia A, & B *b* primi sunt inter se, *c* erunt Aq, Bq
c 29. 7. inter se primi ; *d* proinde Aq, AB, Bq sunt
d 1. 8. :: minimi in ratione A ad B.

Dico porro, Ac, AqB, ABq, Bc in ratione A ad B quatuor esse minimos. Nam AqA,
e 17. 7. AqB *e* :: A. B *e* :: ABA (AqB.) ABB. *e* atque A, B :: ABq. BBq. (Bc) Quum igitur Ac, &

B

Bc *f* inter ſe primi ſint, *g* erunt Ac, AqB, f 29. 7.
ABq, Bc quatuor ∷ minimi in ratione A ad B. g 1. 8.
Eodem modo quotvis proportionales inveſtiga-
bis. Q. E. F.

Coroll.

1. Hinc, ſi tres numeri minimi ſunt proportionales, extremi quadrati erunt; ſi quatuor, cubi.

2. Extremi quotcunque proportionales per hanc propoſ. inventi in data ratione minimi, inter ſe primi ſunt.

3. Duo numeri, minimi in data ratione, metiuntur omnes medios quotcunque minimorum in eadem ratione; quia ſcilicet producuntur ex illorum multiplicatione in alios quoſdam numeros.

4. Hinc etiam liquet ex conſtructione, ſeries numerorum 1, A, Aq, Ac; 1, B, Bq, Bc; Ac, AqB, ABq, Bc, conſtare æquali multitudine numerorum; ac proinde extremos numeros quotcunque minimorum continue proportionalium, eſſe ultimos totidem continue proportionalium ab unitate. ut extremi Ac, Bc continue proportionalium Ac, AqB, ABq, Bc, ſunt ultimi totidem proportionalium ab unitate 1, A, Aq, Ac; & 1, B, Bq, Bc.

5. 1, A, Aq, Ac; & B, BA, BAq; ac Bq, ABq ſunt ∷ in ratione 1 ad A. item, B, Bq, Bc; & A, AB, ABq; ac Aq, AqB ſunt ∷ in ratione 1 ad B.

PROP. III.

A, 8. B, 12. C, 18. D, 28. *Si ſint quotcunque numeri*

A, B, C, D *deinceps proportionales, minimi omnium eandem cum eis rationem habentium; illorum extremi* A, D *ſunt inter ſe primi.*

[2] 2. 8. Nam ſi *a* inveniantur totidem numeri minimi in ratione A ad B, illi non alii erunt, quam A, B, C, D; ergo juxta 2 coroll. præcedentis extremi A & D primi ſunt inter ſe. Q. E. D.

PROP. IV.

A, 6. B, 5. C, 4. D, 3. *Rationibus datis quotcunque in minimis terminis*
H, 4. F, 24. E, 20. G, 15.
I -- K -- L -- -
(A ad B, & C ad D) *reperire numeros deinceps minimos in datis rationibus.*

[a] 36. 7. *a* Reperi E minimum, quem B, & C metiuntur; & B ipſum E *b* æque metiatur, ac A alterum
[b] 3. poſt. 7. F, puta per eundem H. *b* item C ipſum E, ac D alterum G æque metiantur: erunt F, E, G minimi in datis rationibus. Nam AH *c* = F; &
[c] 9. ax. 7. BH *c* = E. *d* ergo A. B :: AH. BH *e* :: F. E.
[d] 18. 7. Similiter C. D :: E. G. ſunt igitur F, E, G
[e] 7. 5. deinceps proportionales in datis rationibus. Imo minimi ſunt in iiſdem: nam puta alios I, K, L
[f] 21. 7. minimos eſſe. *f* ergo A & B ipſos I & K, *f* pariterque C & D ipſos K & L æque metiuntur. ergo
[g] 37. 7. B, & C eundem K metiuntur. *g* Quare etiam E eundem K metitur, ſeipſo minorem. Q.E.A.

A, 6. B, 5. C, 4. D, 3. E, 5. F, 7.
H, 24. G, 20. I, 15. K, 21.

Datis vero tribus rationibus A ad B, & C ad D, ac E ad F. reperi, ut prius, tres H, G, I minimos deinceps in rationibus A ad B, & C ad
[b] 3 poſt. 7. D. tunc ſi E numerum I metiatur, *b* ſume alterum K, quem F æque metiatur; erunt quatuor H, G, I, K, deinceps minimi, in datis rationibus, quod non aliter probabimus, quam in priori parte.

A, 6.

A, 6. B, 5. C, 4. D, 3. E, 2. F, 7.
H, 24. G, 29. I, 15.
M, 48. L, 40. K, 30. N, 105.

Sin E non metiatur I, ſit K minimus, quem E, & I metiuntur; & quoties I ipſum K, toties G ipſum L, & H ipſum M metiatur. quoties vero E ipſum K, toties F ipſum N metiatur. Erunt M, L, K, N minimi deinceps in datis rationibus; quod demonſtrabimus, ut prius.

PROP. V.

C, 4. E, 3.
D, 6. F, 16. ED, 18.
$\overline{CD}$, 24. $\overline{EF}$, 48.

Plani numeri C D, E F *rationem habent ex lateribus compoſitam.* ($\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}$.)

Nam quia CD. ED *a* :: C. E; *a* & ED. EF :: D. F. atque $\frac{CD}{EF}$ *b* $= \frac{CD}{ED} + \frac{FD}{EF}$, *c* erit ratio $\frac{CD}{EF} = \frac{C}{E} + \frac{D}{F}$. Q. E. D.

a 17. 7.
b 20 *def.* 5.
c 11. 5.

PROP. VI.

A, 16. B, 24. C, 36. D, 54. E, 81.
F, 4. G, 6. H, 9.

Si ſint quotcunq; numeri deinceps proportionales A, B, C, D, E; *primus autem* A *ſecundum* B *non metiatur, neque alius quiſpiam ullum metietur.*

Quoniam A non metitur B, *a* neque quilibet proxime ſequentem metietur, quia A. B :: B. C :: C. D, &c. *b* Accipe tres F, G, H minimos in ratione A ad B. quoniam igitur A non metitur B, *a* neque F metietur G. *c* ergo F non eſt unitas. *d* ſed F, & H inter ſe primi ſunt; ergo quum *e* ſit ex æquo A. C :: F. H, & F non metiatur H, neque A ipſum C metietur; proinde nec B ipſum D, nec C ipſum E, &c. quia A. C *e* :: B. D *e* :: C. E, &c. Eodem modo ſumptis

a 20. *def.* 6.
b 35. 7.
c 5. *ax.* 7.
d 3. 8.
e 14. 7.

sumptis quatuor vel quinque minimis in ratione A ad B, ostendetur A ipsos D, & E; ac B ipsos E, & F non metiri, &c. Quare nullus alium metietur. Q. E. D.

PROP. VII.

A, 3. B, 6. C, 12. D, 24. E, 48.

Si sint quotcunque numeri deinceps proportionales A, B, C, D, E; *primus autem* A *extremum* E *metiatur; is etiam metitur secundum* B.

a 6. 7. Si negas A metiri B, *a* ergo nec ipsum E metietur, contra Hypoth.

PROP. VIII.

A, 24. C, 36. D, 54. B, 81. G, 8. H, 12. I, 18. K, 27. E, 32. L, 48. M, 72. F, 108.

Si inter duos numeros A, B *medii continua proportione cecideriut numeri* C, D; *quot inter eos medii continua proportione cadunt numeri, tot & inter alios* E, F *eandem cum illis habentes rationem, medii continua proportione cadent.* (L, M.)

a 35. 7. *a* Sume G, H, I, K minimos ∴ in ratione
b 14. 7. A ad C; *b* erit ex æquali, G. K :: A. B *c* :: E. F.
c *hyp.* Atqui G, & K *d* primi sunt inter se; *e* quare G
d 3. 8. æque metitur E, ac K ipsum F. per eundem nu-
e 21. 7. merum metiatur H ipsum L, & I ipsum M.
f *constr.* *f* itaque E, L, M, F ita se habent ut G, H, I, K; hoc est ut A, B, C, D. Q. E. D.

PROP. IX.

1. E, 2. F, 3. G, 4. H, 6. I, 9. A, 8. C, 12. D, 18. B, 27.

Si duo numeri A, B, *sint inter se primi, & inter eos medii continua proportione cecideriut numeri* C, D; *quot inter eos medii continua*

tinua proportione ceciderint numeri, totidem (E,G, & F,I) *& inter utrumque eorum ac unitatem medii continua proportione cadent.*

Conſtat 1, E, G, A; & 1, F, I, B eſſe ∷; & totidem quot A, C, D, B, nimirum ex 4 coroll. 2. 8. Q. E. D.

PROP. X.

A, 8. I, 12. K, 18. B, 27.
E, 4. DF, 6. G, 9.
D, 2. F, 3.
1.

Si inter duos numeros A, B, *& unitatem continue proportionales ceciderint numeri* (E, D, & F, G,) *quot inter utrumque ipſorum, & unitatem deinceps medii continua proportione cadunt numeri, totidem & inter ipſos medii continua proportione cadent,* I, K.

Nam E, DF, G; & A, DqF (I,) DG (K,) B ſunt ∷, per 2. 8. ergo, &c.

PROP. XI.

A, 2. B, 3.
Aq, 4. AB, 6. Bq, 9.

Duorum quadratorum numerorum Aq, Bq *unus medius proportionalis eſt numerus* AB. *& quadratum* Aq *ad quadratum* Bq, *duplicatam habet lateris* A *ad latus* B *rationem.*

a Liquet Aq, AB, Bq, eſſe ∷. *b* proinde etiam $\frac{Aq}{Bq} = \frac{A}{B}$ bis. Q. E. D.

a 17 7.
b 10. *def* 5.

PROP.

PROP. XII.

Ac, 27. AqB, 36. ABq, 48. Bc, 64. A, 3. B, 4. Aq, 9. AB, 12. Bq, 16.

Duorum cuborum numerorum Ac, Bc *duo media proportionales sunt numeri* AqB, ABq. *Et cubus* Ac *ad cubum* Bc *triplicatam habet lateris* A *ad latus* B *rationem.*

a 2. 1. *a* Nam Ac, AqB, ABq, Bc sunt ∴ in ratione
b 10. *def.* 5. A ad B. *b* proinde $\frac{AC}{BC} = \frac{A}{B}$ ter. Q. E. D.

PROP. XIII.

A, 2. B, 4. C, 8. Aq, 4. AB, 8. Bq, 16. BC, 32. Cq, 64. Ac, 8. AqB, 16. ABq, 32. Bc, 64. BqC, 128. BCq, 256. Cc, 512.

Si sint quotlibet numeri deinceps proportionales, A, B, C; *& multiplicans quisque seipsum faciat aliquos; qui ab illis producti fuerint* Aq, Bq, Cq *proportionales erunt: & si numeri primum positi* A, B, C *multiplicantes jam factos* Aq, Bq, Cq, *fecerint aliquos* Ac, Bc, Cc; *ipsi quoque proportionales erunt, & semper circa extremos hoc eveniet.*

a 2. 8. Nam Aq, AB, Bq, BC, Cq *a* sunt ∴ *b* ergo
b 14. 7. ex æquo Aq. Bq :: Bq, Cq. Q. E. D.

a Item Ac, AqB, ABq, Bc, BqC, BCq, Cc sunt ∴, *b* ergo iterum ex æquo, Ac. BC :: Bc. Cc. Q. E. D.

PROP. XIV.

Aq, 4. AB, 12. Bq, 36. A, 2. B, 6.

Si quadratus numerus Aq *quadratum numerum* Bq *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *& si unius quadrati latus* A *metietur latus alterius* B, *& quadratus* Aq *quadratum* Bq *metietur.*

a 2. & 11. 8 1. *Hyp.* Nam Aq AB *a* :: AB. Bq; cum igitur ex hyp. Aq metiatur Bq; idem Aq secundum

cundum AB *b* metietur. atqui A q. AB :: A. B. b 7. 8.
c ergo etiam A metitur B. Q. E. D. c 20 *def.* 7.

2. *Hyp.* A metitur B. *c* ergo tam Aq ipſum AB, *c* quam AB ipſum Bq metitur; *d* & proinde d 11. *ax.* 7.
Aq metitur Bq. Q. E. D.

PROP. XV.

A, 2. B, 6. *Si cubus numerus* Ac, 8. AqB, 24. ABq, 72. Bc, 216. *cubum numerum* Bc *metiatur, & latus unius* (A) *metietur latus alterius* (B:) *Et ſi latus* A *unius cubi* Ac *latus* B *alterius* Bc *metiatur, & cubus* Ac *cubum* Bc *metietur.*

1. *Hyp.* Nam Ac, AqB, ABq, Bc *a* ſunt ÷. a 2. & 12. 8.
ergo Ac, *b* metiens extremum Bc, *c* etiam ſecundum AqB metietur. atqui Ac. AqB :: A. B. b *hyp.* c 7. 8.
d ergo etiam A metietur B. Q. E. D.

2. *Hyp.* A metitur B; *d* ergo Ac metitur AqB, d 20 *def.* 7.
iſque ABq, & hic Bc; *e* ergo Ac metietur Bc. e 11. *ax.* 7.
Q. E. D.

PROP. XVI.

A, 4. B, 9. *Si quadratus numerus* Aq Aq, 16. Bq, 81. *quadratum numerum* Bq *non metiatur, neque* A *latus unius alterius latus* B *metietur: & ſi* A *latus unius quadrati* Aq *non metiatur* B *latus alterius* Bq, *neque quadratus* Aq *quadratum* Bq *metietur.*

1. *Hyp.* Nam ſi affirmes A metiri B, *a* etiam a 14. 8.
Aq ipſum Bq metietur, contra hyp.

2. *Hyp.* Vis Aq metiri Bq; *a* ergo A ipſum B metietur, contra hyp.

PROP.

PROP. XVII.

A, 2. B, 3. Ac, 8. Bc, 27. *Si cubus numerus* Ac *cubum numerum* Bc *non metiatur, neque* A *latus unius latus* B *alterius metietur. Et ſi latus* A *unius cubi* Ac *latus* B *alterius* Bc *non metiatur, neque cubus* Ac *cubum* Bc *metietur.*

a 15. 8. 1. *Hyp.* Dic A metiri B; *a* ergo Ac metietur Bc. contra *Hypoth.*

2. *Hyp.* Dic Ac metiri Bc; *a* ergo A ipſum B metietur. contra Hyp.

PROP. XVIII.

C, 6. D, 2. CD, 12. E, 9. F, 3. DE, 18. EF, 27. *Duorum ſimilium planorum numerorum* CD, EF, *unus medius proportionalis eſt numerus* DE; *& planus* CD *ad planum* EF *duplicatam habet lateris* C *ad latus homologum* E *rationem.*

* 21. *def.* 7. Quoniam * ex hyp. C. D :: E. F; permu-
a 17. 7. tando erit C. E :: D F. atqui C. E *a* :: CD.
b 11. 5. DE; *a* & D. F :: DE. EF. *b* ergo CD. DE :: DE. EF. Q. E. D.

c 20. *def.* 5. *c* Ergo ratio CD ad EF duplicata eſt rationis CD ad DE; hoc eſt rationis C ad E, vel D ad F.

Coroll.

Hinc perſpicuum eſt, inter duos ſimiles planos cadere unum medium proportionalem, in ratione laterum homologorum.

PROP.

PROP. XIX.

CDE, 30. DEF, 60 FGE, 120. FGH, 240.
CD, 6. DF, 12. FG, 24.
C, 2. D, 3. E, 5. F, 4. G, 6. H, 10.

Duorum ſimilium ſolidorum CDE, FGH, *duo medii proportionales ſunt numeri* DFE, FGE. *Et ſolidus* CDE *ad ſolidum* FGH *triplicatam rationem habet lateris homologi* C *ad latus homologum* F.

Quoniam ex * hyp. C. D :: F. G; & D. E :: G. H; erit *a* permutando C. F :: D. G *a* :: E. H. atqui CD. DF *b* :: C. F; & DF. FG *b* :: D. G. *c* quare CD. DF :: DF. FG :: E. H. *d* ergo CDE. DFE :: DFE. FGE :: E. H. :: FGE. FGH. ergo inter CDE, FGH cadunt duo medii proportionales, DFE, FGE. Q. E. D. *e* Liquet igitur rationem CDE ad FGH triplicatam eſſe rationis CDE ad DFE, vel C ad F. Q. E. D.

* 21. *def.* 7. a 13. 7. b 17. 7. c 11. 5. d 17. 7. e 10. *def.* 1.

Coroll.

Hinc, inter duos ſimiles ſolidos cadunt duo medii proportionales, in ratione laterum homologorum.

PROP. XX.

A, 12. C, 18. B, 27.
D, 2. E, 3. F, 6. G, 9.

Si inter duos numeros A, B, *unus medius proportionalis cadat numerus* C, *ſimiles plani erunt illi numeri*, A, B.

a Accipe D, & E minimos in ratione A ad C, vel C ad B. *b* ergo D æque meritur A, ac E ipſum C, puta per eundem F. *b* item D æque metitur C ac E ipſum B, puta per eundem G. *c* ergo DF = A, & EG = B. *d* quare A, & B plani ſunt numeri. Quia vero EF *c* = C *c* = DG; *e* erit D. E :: F. G, & viciſſim D. F :: E. G. *f* ergo plani numeri A, & B etiam ſimiles ſunt. Q. E. D.

a 35. 7. b 21. 7. c 9. *ax.* 7. d 16. *def.* 7. e 19 7. f 21. *def.* 7.

PROP.

PROP. XXI.

A 16. C, 24. D, 36. B, 54. E, 4. F, 6. G, 9. H, 2. P, 2. M, 4. K, 3. L, 3. N, 6. *Si inter duos numeros* A, B *duo medii proportionales cadant numeri* C, D; *similes solidi erunt illi numeri*, A, B.

a 2. 8. *a* Sume E, F, G minimos ∴ in ratione A ad
b 20. 8. C. *b* ergo E, & G sunt numeri plani similes.
c 21. *def.* 7 hujus latera sint H & P; illius K & L: *c* ergo H.
d *cor* 18. 8. K :: P. L :: *d* E. F. Atqui E, F, G ipsos A, C,
e 21. 7. D *e* æque metiuntur, puta per eundem M; iidemque ipsos, C, D, B æque metiuntur, puta
f 9. *ax.* 7. per eundem N *f* ergo A = EM = HPM, *f* &
g 17. *def.* 7. B = GN = KLN; *g* quare A & B solidi sunt numeri. Quoniam vero C *f* = FM; & D *f* =
h 17. 7. FN, erit M. N *h* :: F M. F N *k* :: C D *l* :: E.
k 7. 5. F :: H. K :: P. L. *m* ergo A, & B sunt numeri
l *constr.* solidi similes. Q. E. D.
m 21. *def.* 7

LEMMA.

AE,	EF,	CG,	DH,	*Si proportionales*
A,	B,	C,	D,	*numeri* A, B, C, D
E,	F,	G,	H.	*proportionales numeros* AE, BF, CG, DH *metiantur per numeros* E, F, G, H, *erunt ei* [E, F, G, H] *proportionales.*

a 19. 7. Nam ob AEDH *a* = BFCG, *a* & AD = BC,
b 1. *ax.* 7. *b* erit $\frac{AEDH}{AD} = \frac{BFCG}{BC}$, *c* hoc est EH = FG.
c 9. *ax.* 7. *a* ergo E. F :: G. H. Q. E. D.

Corell.

d 15. *def.* 7. Hinc $\frac{Bq}{Aq} = \frac{B}{A}$ in $\frac{B}{A}$. *d* Nam 1. B :: B. Bq. *d* &
e *lem. præc.* 1. A :: A. Aq. *e* ergo 1. $\frac{B}{A}$ :: $\frac{B}{A}$. $\frac{Bq}{Aq}$. *d* ergo $\frac{Bq}{Aq}$ $\frac{B}{A} \times \frac{B}{A}$. Similiter $\frac{B}{Ac}$ in $\frac{Bq}{Ac} = \frac{BC}{Acc}$. & sic de reliquis.

PROP.

PROP. XXII.

Aq, B, C. 4, 8, 16. *Si tres numeri,* Aq, B, C *deinceps sint proportionales, primus autem* Aq *sit quadratus, & tertius* C *quadratus erit.*

Nam ob Aq C a = Bq, b erit C = $\frac{Bq}{Aq}$ c = Q. $\frac{B}{A}$

Liquet vero $\frac{B}{A}$ esse numerum, d ob $\frac{Bq}{Aq}$, vel C numerum. ergo si tres, &c.

a 20. 7.
b 7. ax. 7.
c cor. lem. præc.
d *hyp.* & 14. 8.

PROP. XXIII.

Ac, B, C, D. 8, 12, 18. 27. *Si quatuor numeri* Ac, B, C, D *deinceps sint proportionales, primus autem* Ac *sit cubus; & quartus* D *cubus erit.*

Nam quia Ac D a = BC, b erit D = $\frac{BC}{Ac}$ c = $\frac{B}{Ac}$ x C; hoc est (ob Ac C = d Bq, & b proinde C = $\frac{Bq}{Ac}$) D = $\frac{B}{Ac}$ x $\frac{Bq}{Ac}$ e = $\frac{Bc}{Acc}$ e = C : $\frac{B}{Aq}$

e liquet vero ipsum $\frac{B}{Aq}$ esse numerum, quia $\frac{Bc}{Acc}$ vel D numerus ponitur; ergo si quatuor numeri, &c.

a 19. 7.
b 7. ax. 7.
c cor. lem. præc.
d 20. 7.
e 15. 8.

PROP. XXIV.

A, 16. 24. B, 36. C, 4. 6. D, 9. *Si duo numeri* A, B *rationem habeant inter se, quam quadratus numerus* C *ad quadratum numerum* D, *primus autem* A *sit quadratus: & secundus* B *quadratus erit.*

Inter C, & D numeros quadratos, * adeoque inter A, & B eandem rationem habentes, a cadit unus

* 8. 8.
a 11. 8.

b *hyp.* unus medius proportionalis. Ergo *b* cum A quadratus ſit, *c* etiam B quadratus erit. Q.E.D.

c 22. 8.

Coroll.

1. Hinc ſi fuerint duo numeri ſimiles AB, CD (A. B :: C.D) primus autem AB ſit quadratus, etiam ſecundus CD quadratus erit.

* 11.& 18. 8. * Nam AB. CD :: Aq. Cq.

2. Liquet ex his, proportionem cujuſvis numeri quadrati ad quemlibet non quadratum, exhiberi nullo modo poſſe in duobus numeris quadratis. unde non erit, Q. Q :: 1. 2. nec 1. 5. :: Q. Q. &c.

PROP. XXV.

C, 64. 96, 144. D, 216. A, 8. 12. 18. B, 27.

Si duo numeri A, B *rationem inter ſe habeant, quam cubus numerus* C *ad cubum numerum* D, *primus autem* A *ſit cubus, & ſecundus* B *cubus erit.*

a 12. 8. b 8. 8. c *hyp.* d 23. 8. *a* Inter C, & D cubos, *b* adeoque inter A & B eandem rationem habentes, cadunt duo medii proportionales. ergo propter A *c* cubum, *d* etiam B cubus erit. Q. E. D.

Coroll.

1. Hinc etiam ſi fuerint duo numeri A B C, DEF (A.B :: D.E. & B.C :: E.F;) primus autem ABC cubus fuerit, etiam ſecundus DEF cubus erit.

* 12.& 19. 8. * Nam ABC. DEF :: Ac=Dc.

2. Patet etiam ex his, proportionem cujuſvis numeri cubi ad quemlibet numerum non cubum non poſſe reperiri in duobus numeris cubis.

PROP. XXVI.

A, 20. C, 30. B, 45. D, 4. E, 6. F, 9.

Similes plani numeri A, B *rationem inter ſe habent, quam quadratus numerus ad quadratum numerum.*

a 18. 8. Inter A, & B *a* cadit unus medius proportio-

nalis

nalis C. *b* ſume tres D, E, F minimos ∺ in ratione A ad C: *c* Extremi D, F quadrati erunt atqui ex æquali A. B *d* :: D. F. ergo A. B :: Q. Q. C. E. D.

b 2. 8.
c *cor.* 2. 8.
d 11. 7.

PROP. XXVII.

A, 16. C, 24. D, 36. B, 54. E, 8. F, 12. G, 18. H, 27. *Similes ſolidi numeri* A, B, *rationem habent inter ſe, quam cubus numerus ad cubum numerum.*

a Inter A, & B cadunt duo medii proportionales, puta C & D: *b* ſume quatuor E, F, G, H minimos ∺ in ratione A ad C. *b* Extremi E, H cubi ſunt. At A. B *c*: E. H :: C. C. Q. E. D.

a 19. 8.
b 2. 8.
c 14. 7.

Schol.

1. Ex his infertur, nullos numeros habentes proportionem ſuperparticularem, vel ſuperbi partientem, vel duplam, aut aliam quamcunque multiplam non denominatam à numero quadrato, eſſe ſimiles planos. *Vide Clavium.*

2. Nec duo quivis primi numeri, neque duo quicunque inter ſe primi, qui quadrati non ſint, ſimiles eſſe poſſunt.

LIB. IX.

PROP. I.

A, 6. B, 54.
Aq, 36. 108. AB, 324.

SI duo similes plani numeri A, B *multiplicantes se mutuo faciant quendam* AB, *productus* AB *quadratus erit.*

a 17. 7. Nam A. B *a* :: Aq. AB; cum igitur
b 18. 8. inter A, & B *b* cadat unus medius proportionalis, *c* etiam inter Aq, & AB cadet unus med. pro-
c 8. 8. port. ergo cum primus Aq sit quadratus, *d* etiam
d 22. 8. tertius AB quadratus erit. Q. E. D.

Vel sic. Sint ab, cd similes plani, nempe a. b ::
x 19. 7. c. d. *x* ergo ad = bc. quare abcd, vel adbc *y* =
y 1. *ax* 7. adad = Q : ad.

PROP. II.

Si duo numeri A, B *se*
A, 6. B, 54. *mutuo multiplicantes faci-*
Aq, 36. AB, 324. *ant* AB *quadratum, similes plani erunt,* A, B.

a 17. 7. Nam A. B *a* :: Aq. AB; quare cum inter Aq,
b 11. 8. AB *b* cadat unus medius proportionalis, *c* etiam
c 8. 8. unus inter A, & B medius cadet. *d* ergo A, & B
d 20. 8. sunt similes plani. Q. E. D.

PROP. III.

A, 2. Ac, 8. Acc, 64. *Si cubus numerus* Ac *seipsum multiplicans procreet aliquem* Acc, *productus* Acc *cubus erit.*

a 15. *def* 7. Nam 1. A *a* :: A. Aq *b* :: Aq. Ac. ergo inter 1, &
b 17. 7. Ac cadunt duo medii proportionales. Sed 1. Ac *a*
c 8. 8. :: Ac. Acc. *c* ergo inter Ac, & Acc cadunt etiam duo

duo medii proportionales. Proinde cum Ac ſit cubus, *d* erit Acc cubus. Q. E. D. d 23. 8.

Vel ſic; aaa (Ac) in ſe ductus facit aaaaaa. (Acc;) hic cubus eſt, cujus latus aa.

PROP. IV.

Ac, 8. Bc, 27. Acc, 64. AcBc, 216. *Si cubus numerus* Ac *cubum numerum* Bc *multiplicans, faciat aliquem* AcBc, *factus* AcBc *cubus erit.*

Nam Ac. Bc *a*:: Acc. AcBc. ſed inter Ac & Bc *b* cadunt duo medii proportionales; *c* ergo inter Acc, & Ac Bc totidem cadunt. itaque cum Acc ſit cubus, *d* erit AcBc etiam cubus. Q.E.D.

a 17. 7.
b 12. 8.
c 8. 8.
d 23. 8.

Vel ſic. AcBc = aaabbb (ababab) = C: ab.

PROP. V.

Ac, 8. B, 27. Acc, 64. AcB, 216. *Si cubus numerus* Ac *numerum quendam* B *multiplicans, faciat cubum* AcB; *& multiplicatus* B *cubus erit.*

Nam Acc. AcB *a*:: Ac. B. Sed inter Acc, & AcB *b* cadunt duo medii proportionales. *c* ergo totidem cadent inter Ac, & B. quare cum Ac cubus ſit, *d* etiam B cubus erit. Q. E. D.

a 17. 7.
b 12. 8.
c 8. 8.
d 23. 8.

PROP. VI.

A, 8. Aq, 64. Ac, 512. *Si numerus* A *ſe ipſum multiplicans faciat* Aq *cubum; & ipſe* A *cubus erit.*

Nam quia Aq *a* cubus, & AqA (Ac) *b* cubus, *c* erit A cubus. Q E. D.

a *hyp.*
b 19. *def.* 7.
c 5. 9.

PROP. VII.

A, 6. B, 11. AB, 66. D, 2. E, 3. *Si compoſitus numerus* A *numerum quempiam* B *multiplicans, quempiam faciat* AB, *factus* AB *ſolidus erit.*

a 13 *def* 7. b 9 *ax.* 7. c 17. *def* 7. Quoniam A compositus est, *a* metitur eum a-liquis D, puta per E. *b* ergo A = DE; *c* quare DEB = AB solidus est. Q. E. D.

PROP. VIII.

1. a, 3. a^2, 9. a^3, 27. a^4, 81. a^5, 243. a^6, 729.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, a^4, &c.) *tertius quidem ab unitate* a^2 *quadratus est, & unum intermittentes omnes* (a^1, a6, a8, &c.) *quartus autem* a^3 *est cubus, & duos intermittentes omnes* (a6, a9, &c.) *septimus vero* a6, *cubus simul & quadratus; & quinque intermittentes omnes* (a^{12}, a18, &c.)

Nam 1. a^2 = Q. a. & a^4 = aaaa = Q. aa. & a6 = aaaaaa = Q. aaa, &c.

2. a^3 = aaa = C. a. & a6 = aaaaaa = C. aa. & aaaaaaaaa = C. aaa, &c.

3. a6 = aaaaaa = C. aa = Q. aaa. ergo, &c.

a *hyp* b 20 7. c 22. 8 d 23. 8 Vel juxta Euclidem; quia 1. a *a* :: a. a^2, *b* erit a^2 = Q: a. ergo cum a^2, a^3, a^4 sint ∷ *c* erit tertius a^4 etiam quadratus, pariterq; a6, a8, &c. Item quia 1. a *a* :: a^2, a^3. erit a^3 *b* = a^2 in a = C: a *d* ergo quartus ab a^3, nempe a6, etiam cubus erit, &c. ergo a6 cubus simul & quadratus exiitit, &c.

PROP. IX.

1. a, 4. a^2, 16. a^3, 64. a^4, 256, &c.

1. a, 8. a^2, 64. a^3, 512. a^4, 4096.

Si ab unitate quotcunque numeri deinceps proportionales fuerint (1, a, a^2, a^3, &c.) *qui vero* (a) *post unitatem sit quadratus; & reliqui omnes.* a^2, a^3, a^4, &c. *quadrati erunt. At si* a, *qui post unitatem sit cubus & reliqui omnes* a^2, a^3, a^4, &c. *cubi erunt.*

1. *hyp.* Nam a^2, a^4, a6 &c. quadrati sunt ex præc. item quia a ponitur quadratus, *a* erit a 22. 8 tertius a^3 quadratus, pariterque a^5, a7, &c. ergo omnes.

2. *Hyp.*

2. *Hyp.* a cubus ponitur, *b* ergo a^4, a7. a 10 b 13. 8.
cubi ſunt: atqui ex præced. a^3, a6, a9, &c cubi
ſunt. denique quia 1. a :: a. aa, *c* erit $a^2 = Q$. c 20. 7.
a. cubus autem in ſe *d* facit cubum; ergo a^2 cu- d 3. 9.
bus eſt, & *e* proinde ab eo quartus a^5, pariterque e 23. 8.
a8, a^{11}, &c. cubi ſunt. ergo omnes. Q. E. D.

Clarius forſitan ſic; Sit quadrati a latus b. ergo ſeries a, a^2, a^3, a^4, &c. aliter exprimetur ſic, bb, b^4, b6, b8, &c. liquet vero hos omnes quadratos eſſe; & ſic etiam exprimi poſſ.; Q. b, Q: bb, Q: bbb, Q: bbbb, &c.

Eodem modo, ſi b latus fuerit cubi a, ſeries ita nominari poteſt; b^3, b6, b9, b^{12}, &c. vel C: b, C: b^2, C: b^3, C: b^4, &c.

PROP. X.

1, a, a^2, a^3, a^4, a^5, a6. *Si ab unitate quot-*
1, 2, 4, 8, 16, 32, 64. *cunque numeri deinceps proportionales fuerint* (1, a, a^2, a^3, &c) *qui vero poſt unitatem* (a) *non ſit quadratus, neque alius ullus quadratus erit, præter* a^2 *tertium ab unitate, & unum intermittentes omnes* (a4, a6, a8.) *At ſi* a, *qui poſt unitatem, non ſit cubus, neque ullus alius cubus erit præter* a^3 *quartum ab unitate, & duos intermittentes omnes*, a6, a9, a12, *&c.*

1. *Hyp* Nam ſi fieri poteſt, ſit a^5 quadratus numerus. quoniam igitur a. a^2. *a* :: a^4. a^5, atq; a *hyp.*
inverſe a^5. a^4 :: a^2. a; ſintque a^5, & a^4 *b* qua- b *ſuppoſ. & 8. 9.*
drati, primuſque a^2 quadratus, *c* erit a etiam c 24. 8.
quadratus, contra Hyp.

2. *Hyp.* Si fieri poteſt, ſit a^4 cubus. quoniam igitur *d* ex æquo a^4. a6 :: a. a^3, atque in- d 14. 7.
verſe a6. a^4 :: a^3. a; *b* ſintque a6, & a^4 cubi, e 25. 8.
& primus a^3 cubus, *c* etiam a cubus erit, contra Hypoth.

PROP. XI.

1, a, a^2, a^3, a^4, a^5, a6. *Si ab u-*
1, 3, 9, 27, 81, 243, 719. *nitate quot-*
cunq; numeri
deinceps proportionales fuerint (1, a, a^2, a^3, &c.)
minor majorem metitur per aliquem eorum qui in
proportionalibus sunt numeris.

a 5 ax. 7. & 20. def. 7. Quoniam 1. a :: a. aa, *a* erit $\frac{aa}{a} = a = \frac{aaa}{aa}$.

b 14. 7. item quia 1. aa *b* :: a. aaa. *a* erit $\frac{aaa}{a} = aa = \frac{a^4}{aa} = \frac{a^5}{a^3}$ &c. denique quia 1. a^3 *b* :: a. a^4, *a* erit $\frac{a^4}{a} = a^3 = \frac{a^6}{a^3}$ &c.

Coroll.

Hinc, si numerus qui metitur aliquem ex proportionalibus, non sit unus proportionalium, neque numerus per quem metitur, erit aliquis ex proportionalibus.

PROP. XII.

1, a, a^2, a^3, a^4, *Si ab unitate quotcunq;*
1, 6, 36, 216, 1296. *numeri deinceps proportio-*
B, 3. *nales fuerint* (1, a, a^2, a^3, a^4,) *quicunque pri-*
morum numerorum B *ultimum* a^4 *metiuntur, iidem*
(B) *& eum* (a) *qui unitati proximus est, metientur.*

a 31. 7. Dic B non metiri a, *a* ergo B ad a primus est;
b 27. 7. *b* ergo B ad a^2 primus est; & *c* proinde ad a^4
c 26. 7. quem metiri ponitur. Q. E. A.

Coroll.

1. Itaque omnis numerus primus ultimum metiens, metitur quoque omnes alios ultimum præcedentes.

2. Si

2. Si aliquis numerus non metiens proximum unitati, metiatur ultimum, erit numerus compoſitus.

3. Si proximus unitati ſit primus numerus, nullus alius primus numerus ultimum metietur.

PROP. XIII.

1, *a*, a^2, a^3, a^4.
1, 5, 25, 125, 625.
H--G--F--E--

Si ab unitate quotcunque numeri deinceps proportionales fuerint (a, a^2, a^3, *&c.*) *qui vero poſt unitatem* (a) *primus ſit; maximum nullus alius metietur, præter eos qui ſunt in numerâ proportionalibus.*

Si fieri poteſt, alius quiſpiam E metiatur a^4, nempe per F, *a* erit F alius extra *a*, a^2, a^3. a *cor* 12. 9.
Quia vero E metiens a^4 non metitur *a*, *b* erit b 2 *cor.* 12.
E numerus compoſitus; *c* ergo eum aliquis pri- 9.
mus metitur, *d* qui proinde ipſum a^4 metitur; c 33. 7.
e ideoque alius non eſt, quam *a*. ergo *a* meti- d 11. *ax.* 7.
tur E. Eodem modo oſtendetur F compoſitus e 3 *cor.* 12.
numerus, metiens a^4, adeoque *a* ipſum F metiri. 9.
itaque quum EF $f = a^4 = a$ in a^3, *g* erit *a* E :: F. f 9. *ax.* 7.
a^3. ergo cum *a* metiatur E, *b* æque F metietur g 19. 7.
a^3, puta per eundem G. *k* Nec G erit *a*, vel a^2. h 20. *def.* 7.
ergo, ut prius, G eſt numerus compoſitus, & *a* k *cor.* 11. 9.
eum metitur. quum igitur FG $f = a^3 = a^2$ in *a*,
g erit *a*. F :: G. a^2; & proinde, quia A metitur
F, *b* æque G metietur a^2, ſcilicet per eundem H;
k qui non eſt *a*. ergo quum GH $= a^2 = aa$. l 20. 7.
l erit H. *a* :: *a*. G. ergo quia *a* metitur G (ut m 20. *def.* 7
prius) *m* etiam H metietur *a*, numerum primum. Q. F. N.

PROP. XIV.

A. 30. B, 2. C, 3. D, 5. E -- F - -

Si minimum numerum A primi numeri B, C, D metiantur; nullus alius numerus primus E illum metietur, præter eos, qui à principio metiebantur.

a 9. ax. 7. Si fieri potest, sit $\frac{A}{E} = F$. *a* Ergo A = EF.

b 32. 7. *b* Ergo singuli primi numeri B, C, D ipsorum E, F unum metiuntur; non E, qui primus ponitur; ergo F, minorem scilicet ipso A; contra Hypoth.

PROP. XV.

A, 9. B, 12 C, 16. D, 3. E, 4.

Si tres numeri A, B, C deinceps proportionales, fuerint minimi omnium eandem cum ipsis rationem habentium; duo quilibet compositi, ad reliquum primi erunt.

a 35. 7. *a* Sume D, & E minimos in ratione A ad B.
b 2. 8. *b* ergo A = Dq; *b* & C = Eq, *b* & B = DE. Quia
c 24. 7. vero D ad E *c* primus est, *d* erit D + E primus ad
d 30. 7. singulos D, & E. * ergo D in D + E *e* = Dq +
* 26. 7. DE (ſ A + B) ad E primus est, *g* ideoque ad C
e 3. 2. vel Eq. Q. E. D. Pari pacto DE + Eq (B + C)
f *prius*. ad D primus est, & proinde ad A = Dq. Q. E. D.
g 27. 7. Denique quia B ad D + E *b* primus est; is ad
h 26. 7. hujus quadratum *k* Dq + 2 DE + Eq (A + 2
k 4. 2. B + C) primus erit. *l* quare idem B ad A + B + C,
l 30. 7. *l* adeoque ad A + C primus erit. Q. E. D.

PROP.

PROP. XVI.

A, 3. B, 5. C --- *Si duo numeri* A, B *primi inter se fuerint, non erit ut primus* A *ad secundum* B, *ita secundus* B *ad alium quempiam* C.

Dic A. B :: B. C. ergo quum A & B in sua ratione *a* minimi sint, A *b* metietur B æque ac B ipsum C; sed A *c* seipsum etiam metitur; ergo A & B non sunt primi inter se, contra Hypoth.

a 23. 7.
b 21. 7.
c 6. *ax.* 7.

PROP. XVII.

A, 8. B, 12. C, 18. D, 27. E ---

Si fuerint quotcunque numeri deinceps proportionales A, B, C, D, *extremi autem ipsorum* A, D *primi inter se sint; non erit ut primus* A *ad secundum* B, *ita ultimus* D *ad alium quempiam* E.

Dic A. B :: D. E. ergo vicissim A. D :: B. E. ergo quum A & D in sua ratione *a* minimi sint, *b* metietur A ipsum B; *c* quare B ipsum C, & C sequentem D, *d* adeoq; A eundem D metietur. Ergo A & D non sunt primi inter se, contra Hypoth.

a 23. 7.
b 21. 7.
c 20. *def.* 7.
d 11. *ax* 7.

PROP. XVIII.

A, 4. B, 6. C, 9. Bq, 36. *Duobus numeris datis* A, B, *considerare an possit ipsis tertius proportionalis* C *inveniri.*

Si A metiatur Bq per aliquem C, *a* erit AC = Bq. unde *b* liquet esse A. B :: B. C. Q.E.F.

a 9. *ax* 7.
b *per* 20. 7.

A, 6. B, 4. Bq, 16. Sin A non metiatur Bq, non erit aliquis tertius proportionalis.

Nam dic A. B :: B. C. *a* ergo AC = Bq. *c* proinde $\frac{Bq}{A}$ = C. Scilicet A metitur Bq, contra Hypoth.

c 7. *ax.* 7.

PROP.

PROP. XIX.

A, 8. B, 12. C, 18. D, 27. BC, 216.

Tribus numeris datis A, B, C, *considerare an possit ipsis quartus proportionalis* D *inveniri.*

a 9. *ax* 7. b *ax.* 19. 7. Si A metiatur BC per aliquem D, *a* ergo AD = BC; *b* constat igitur esse A. B :: C. D. Q. E. F.

Sin A non metiatur BC, non datur quartus proportionalis; quod ostendetur, prout in præcedenti.

PROP. XX.

A, 2. B, 3. C, 5. D, 30. G----

Primi numeri plures sunt omni propositâ multitudine primorum numerorum A, B, C.

a 38. 7. b 33. 7. c *suppos.* d *constr.* e 12. *ax* 7. *a* Sit D minimus, quem A, B, C metiuntur. si D+1 primus sit, res patet; si compositus, *b* ergo aliquis primus, puta G, metitur D+1, qui non est aliquis trium A, B, C; nam si ita, quum is *c* totum D+1, & *d* ablatum D metiatur, *e* idem reliquam unitatem metietur. Q. E. A. Ergo propositorum primorum numerorum multitudo aucta est per D+1; vel saltem per G.

PROP. XXI.

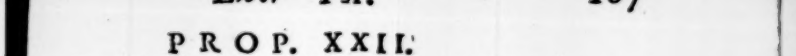
5 5 3 3 2 2
A ..... E ..... B ... F ... C .. G ,. D 20.

Si pares numeri quotcunque AB, BC, CD *componantur, totus* AD *par erit.*

a 6. *def.* 7. b 12. 7. c 6. *def.* 7. *a* Sume EB = ½ AB & FC = ½ BC, & GD = ½ CD. *b* liquet EB + FC + GD = ½ AD. *c* ergo AD est par numerus. Q. E. D.

PROP.

PROP. XXII.

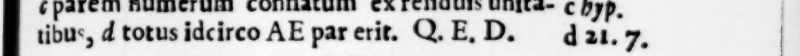


Si impares numeri quotcunque AB,BC,CD,DE *componantur, multitudo autem ipſorum ſit par, totus* AE *par erit.*

Detracta unitate ex ſingulis imparibus, *a* manebunt AF, BG, CH, DL numeri pares, & *b* proinde compoſitus ex ipſis par erit; adde his *c* parem numerum conflatum ex reſiduis unitatibus, *d* totus idcirco AE par erit. Q. E. D.

a 7. *def.* 7.
b 21. 9.
c *hyp.*
d 21. 7.

PROP. XXIII.

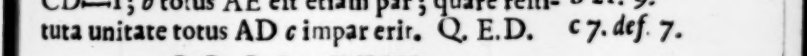


Si impares numeri quotcunque AB, BC, CD *componantur, multitudo autem ipſorum ſit impar; & totus* AD *impar erit.*

Nam dempto CD uno imparium, reliquorum aggregatus AC *a* eſt par numerus. huic adde CD—1; *b* totus AE eſt etiam par; quare reſtituta unitate totus AD *c* impar erit. Q. E. D.

a 22. 9.
b 21. 9.
c 7. *def.* 7.

PROP. XXIV.



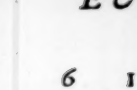
Si à pari numero AC *par* AB *detrahatur, & reliquus* BC *par erit.*

Nam ſi BD(BC—1) impar fuerit, *a* erit BC (BD+1) par. Q. E. D. Sin BD parem dicas, propter AB *b* parem, *c* erit AD par; *a* ideoque AC (AD+1) impar, contra Hypoth. ergo BC eſt par. Q. E. D.

a 7. *def.* 7.
b *hyp.*
c 21. 9.

PROP.

PROP. XXV.

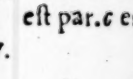


Si à pari numero AB *impar* AC *detrahatur, & reliquus* CB *impar erit.*

a 7. *def.* 7. b 24. 9. c 7. *def.* 7.

Nam AC—1 (AD) *a* est par. *b* ergo DB est par. *c* ergo CB (DB—1) est impar. Q.E.D.

PROP. XXVI.

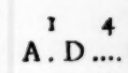


Si ab impari numero AB *impar* CB *detrahatur, reliquus* AC *par erit.*

a 7. *def.* 7. b 24 9.

Nam AB—1 (AD) & CB—1 (CD) *a* sunt pares. *b* ergo AD—CD (AC) est par. Q.E.D.

PROP. XXVII.

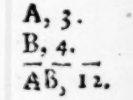


Si ab impari numero AB *par detrahatur* CB, *reliquus* AC *impar erit.*

a 7. *def* 7. b 24. 9. c 7. *def* 7.

Nam AB—1 (DB) *a* est par; & CB ponitur par. *b* ergo reliquus CD par est. *c* ergo CD+1 (CA) est impar. Q. E. D.

PROP. XXVIII.



Si impar numerus A *parem numerum* B *multiplicans fecerit aliquem* AB, *factus* AB *par erit.*

a *hyp* & 15. *def* 7 b 21. 9.

Nam AB *a* componitur ex impari A toties accepto, quoties unitas continetur in B pari. *b* ergo AB est par numerus.

Schol.

Eodem modo, si A sit numerus par, erit AB par.

PROP.

PROP. XXIX.

A, 3. B, 5. AB, 15. *Si impar numerus* A, *imparem numerum* B *multiplicans fecerit aliquem* AB, *factus* AB *impar erit.*

Nam AB *a* componitur ex B impari numero toties accepto, quoties unitas includitur in A etiam impari. *b* ergo AB est impar. Q. E. D.

a 15 *def* 7.
b 23. 9.

Schollum.

B, 12 / A, 3 (C, 4. 1. *Numerus* A *impar numerum* B *parem metiens, per numerum parem* C *eum metitur.*

Nam si C impar dicatur, quoniam *a* B = AC, *b* erit B impar, contra Hypoth.

a 9 *ax* 7.
b 29. 9.

B, 15 / A, 3 (C, 5. 2. *Numerus* A *impar numerum* B *imparem metiens, per numerum* C *imparem eum metitur.*

Nam si C dicatur par; *a* erit AC, vel B par, contra Hypoth.

a 28. 9.

B, 15 / A, 3 (C, 5. 3 *Omnis numerus* (A & C) *metiens imparem numerum* B, *est impar.*

Nam si utervis A, vel C dicatur par, *a* erit B numerus par, contra Hypoth.

a 28 9.

PROP. XXX.

B, 24 / A, 3 (C, 8. D, 12 / A, 3 E, 4.

Si impar numerus A *parem numerum* B *metiatur, & illius dimidium* D *metietur.*

a Sit $\frac{B}{A} = C$. *b* ergo C est numerus par. Sit igitur $E = \frac{1}{2}C$, erit $Ec = CA$ *d* $= 2Ee = 2D$. *f* ergo $EA = D$; & *g* proinde $\frac{D}{A} = E$. Q. E. D.

a *hyp.*
b 1. *schol.*
c 9. *ax* 7.
d 1. 2.
e *hyp.*
f 7 *ax* 1.
g 7 *ax* 7.

PROP.

PROP. XXXI.

A, 5. B, 8. C, 16. D --- *Si impar numerus* A *ad aliquem numerum* B *primus sit; & ad illius duplum* C *primus erit.*

Si fieri poteſt, aliquis D metiatur **A**, & C.
a 3. *ſchol.* 29. 9. *a* ergo D metiens imparem A impar erit, *b* ideoque ipſum B paris C ſemiſſem metietur. ergo
b 30. 9. **A**, & B non ſunt primi inter ſe, contra Hypoth.

Coroll.

Sequitur hinc, numerum imparem, qui ad aliquem numerum progreſſionis duplæ primus eſt, primum quoque eſſe ad omnes numeros illius progreſſionis.

PROP. XXXII.

1. A, 2. B, 4. **C**, 8. D, 16. *Numerorum* A, B, C, D, &c. *à binario duplorum unuſquiſque pariter par eſt tantum.*

a 6. *def.* 7. Conſtat omnes A, B, C, D *a* pares eſſe; atque
b 20. *def.* 7. *b* ∴ nimirum in ratione dupla, & *c* proinde
c 11. 9. quemque minorem metiri majorem per aliquem
d 8. *def.* 7. ex illis. *d* Omnes igitur B, C, D ſunt pariter pa-
e 13. 9. res. Sed quoniam A primus eſt, *e* nullus extra eos eorum aliquem metietur. Ergo pariter pares ſunt tantum. Q. E. D.

PROP. XXXIII.

A, 30. B, 15. D --- E -- *Si numerus* A *dimidium* B *habeat imparem*, A *pariter impar eſt tantum.*

a *hyp.* b 9. *def.* 7. Quoniam impar numerus B *a* metitur A per 2
c 8. *def.* 7. parem, *b* eſt B pariter impar. Dic etiam pariter
d 9. *ax.* 7. parem. *c* ergo eum par aliquis D per parem E
e 19. 7. metitur. unde 2 B *d* = A *d* = DE. *e* quare 2.

E ::

E :: D. B. ergo ut 2 *f* metitur parem E, *g* sic D par imparem B metitur. Q. F. N.

f 6. *def.* 7.
g 20. *def.* 7.

PROP. XXXIV.

A, 24. *Si par numerus* A, *neque à binario duplus sit, neque dimidium habeat impa-rem; pariter par est, & pariter impar.*

Liquet A esse pariter parem, quia dimidium imparem non habet. Quia vero si A bifarietur, & rursus ejus dimidium, & hoc semper fiat, tandem incidemus in aliquem *a* imparem (quia non in binarium, quoniam A à binario duplus non ponitur) is metietur A per parem numerum (nam *b* alias ipse A impar esset, contra Hypoth.) ergo A est etiam pariter impar. Q. E. D.

a 7. *def.* 7.
b 2 *sch.* 29.

PROP. XXXV.

A ..... 8.
4 8
B .... F ..... G 12.
C ..... 18.
9 6 4 8
D ..... H ..... L .. K ..... N 27.

Si sint quotcunque numeri deinceps proportionales A, BG, C, DN, *detrahantur autem* FG *à secundo, &* KN *ab ultimo, æquales ipsi primo* A; *erit ut secundi excessus* BF *ad primum* A, *ita ultimi excessus* DK *ad omnes* A, BG, C *ipsum antecedentes.*

Ex DN deme NL = BG, & NH = C.

Quoniam DN. C. (HN) *a* :: HN. BG. (LN) *a* :: LN. A. (KN.) *b* erit dividendo ubique, DH. HN :: HL. LN :: LK. KN. quare DK. C + BG + A :: LK (*d* BF.) KN. (A.) Q. E. D.

a *hyp.*
b 17. 5.
c 12. 5.
d 3 *ax.* 1.

Coroll.

Hinc *e* componendo, DN + BG + C. A + BG + C :: BG. A.

e 18. 5.

PROP.

PROP. XXXVI.

1. A, 2. B 4. C, 8. D, 16.
E, 31. G, 62. H, 124. L, 248 F, 456.
M, 31. N, 465.
P---- Q---

Si ab unitate quotcunque numeri 1, A, B, C, D, *deinceps exponantur in dupla proportione, quoad totus compositus* E *fiat primus & totus hic* E *in ultimum* D *multiplicatus faciat aliquem* F; *factus* F *erit perfectus*

Sume totidem, E, G, H, L etiam in proportione dupla continae; ergo *a* ex æquo A. D :: E. L. *b* ergo AL = DE *c* = F. *d* ergo L = $\frac{F}{2}$ quare E, G, H, L, F sunt :: in ratione dupla. Sit G — E = M, & F — E = N. *e* ideo M. E :: N E + G + H + L. *f* at M = E. *g* ergo N = E + G + H + L *h* ergo F = 1 + A + B + C + D + E + G + H + L = E + N. Quinetiam quia D *k* metitur DE (F,) *l* etiam singuli 1, A, B, C *m* metientes D, *m* nec non E, G, H, L metiuntur F. Porro nullus alius eundem F metitur Nam si aliquis, sit P, qui metiatur F per Q, *n* ergo P Q = F = D E. *o* ergo E. Q :: P. D. ergo cum A primus numerus metiatur D, & *p* proinde nullus alius P. eundem metiatur, *q* consequenter E non metitur Q. quare cum E primus ponatur, *r* idem ad Q primus erit. *s* ergo E & Q in sua ratione minimi sunt, & *t* propterea E ipsum P ac Q ipsum D æque metiuntur. *u* ergo Q est aliquis ipsorum A, B, C. Sit igitur B; ergo cum ex æquo sit B. D :: E. H; *x* ideoque BH = DE = F = PQ. *x* adeoque Q. B :: H. P. *y* erit H = P. ergo P est etiam aliquis ipsorum A, B, C, &c. contra Hypoth. ergo nullus alius præter numeros prædictos eundem F metietur: & proinde F est numerus perfectus. Q. E. D.

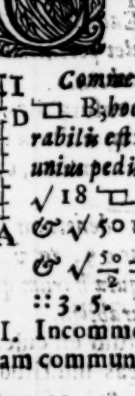
a 14. 7. b 19. 7. c hyp. d 7. ax. 7. e 35. 9. f 3 ax. 1. g 14. 5. h 2. ax. 1. k 7. ax. 7. l 11. ax. 7. m 11. 9. n 9 ax. 7. o 19. 7. p 13. 9. q 20. def 7 r 31. 7. s 23. 7. t 21. 7. u 13. 9. x 19. 7. y 14. 5. z 22. def 7.

LIB.

LIB. X.

Definitiones.

I. COmmenſurabiles magnitudines dicuntur, quas eadem menſura metitur.

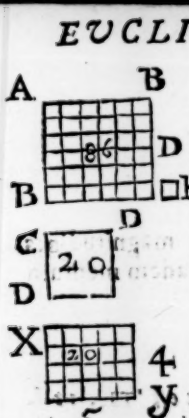


Commenſurabilitatis nota eſt □, *ut* A □ B, *hoc eſt, linea* A 8 *pedum commenſurabilis eſt lineæ* B 13 *pedum; quia* D *linea unius pedis ſingulas* A *&* B *metitur. Item* √18 □ √50; *quia* √2 *ſingulas* √18, *&* √50 *metitur. Nam* $\sqrt{\frac{18}{2}} = \sqrt{9} = 3$. *&* $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$. *quare* √18. √50 :: 3. 5.

II. Incommenſurabiles autem ſunt, quorum nullam communem menſuram contingit reperiri.

Incommenſurabilitas ſignificatur nota □. *ut* √6 □ √25 (5;) *hoc eſt* √6 *incommenſurabilis eſt numero* 5, *vel magnitudini hoc numero deſignatæ; quia harum nulla eſt communis menſura, ut poſtea patebit.*

III. Rectæ lineæ potentia commenſurabiles ſunt, cum quadrata earum idem ſpatium metitur.



Hujuſce commenſurabilitatis nota eſt ⌐¬, *ut* AB ⌐¬ CD; *h.e. linea* AB *ſex pedum potentia commenſurabilis eſt lineæ* CD, *quæ exprimitur per* √ 20. *quia ſpatium* E *unius pedis quadrati metitur tam* AB□ (30) *quam rectangulum* XY (20.) *cui æquale eſt quadratum lineæ* CD (√ 20.) *Eadem nota* ⌐¬ *nonnunquam valet potentia tantum commenſurabilia.*

IV. Incommenſurabiles vero potentia, cum quadratis eorum nullum ſpatium, quod ſit communis eorum menſura, contingit reperiri.

Hujuſmodi incommenſurabilitas denotatur ſic; 5 ⌐¬ *v* √ 8; *hoc eſt, numeri vel lineæ* 5, *&* *v* √ 8 *ſunt incommenſurabiles potentia*; *quia harum quadrata* 25, *&* √ 8 *ſunt incommenſurabilia.*

V. Quæ cum ita ſint, manifeſtum eſt cuicunque rectæ propoſitæ, rectas lineas multitudine infinitas, & commenſurabiles eſſe, & incommenſurabiles; aliàs quidem longitudine & potentia, aliàs vero potentia ſolum. Vocetur autem propoſita recta linea Rationalis.

Hujus nota eſt ῥ.

VI. Et huic commenſurabiles, ſive longitudine & potentia, ſive potentia tantum, Rationales, ῥ.

VII. Huic vero incommenſurabiles Irrationales vocentur.

Hæ ſic denotantur ἀῥ.

VIII. Et quadratum, quod à propoſita recta fit, dicatur Rationale ῥν.

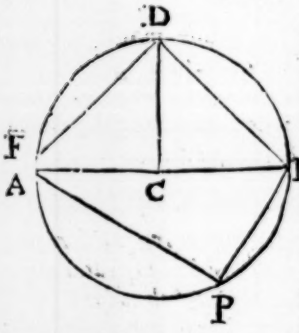
IX. Et huic commenſurabilia quidem Rationalia ῥα.

X. Huic

X. Huic vero incommensurabilia, Irrationalia dicantur, ῥα.

XI. Et rectæ, quæ ipsa possunt, Irrationales, ῥ.

Schol.



Ut postremæ 7 definitiones exemplo aliquo illustrentur, sit circulus ADBP, *cujus semidiameter* CB; *huic inscribantur latera figurarum ordinatarum, Hexagoni quidem* BP, *Trianguli* AP, *quadrati* BD, *pentagoni* FD. *Itaque si juxta 5 defin. semidiameter* CB *sit Rationalis expositæ, numero 2. expressa, cui reliquæ* BP, AP, BD, FD *comparandæ sunt, a erit* BP *a* = BC = 2. *quare* BP *est* ῥ ⌐¬ BC, *juxta 6. def. Item* AP *b* = √ 12 (*nam* ABq (16) — BPq (4) = 12) *quare* AP *est* ῥ ⌐¬ BC, *etiam juxta 6. def. atque* APq (12) *est* ῥν, *per def. 9. Porro* BD *b* = √ DCq + BCq = √8; *unde* BD *est* ῥ ⌐¬ BC; *&* BDq ῥν. *Denique,* FDq = 10 — √ 20 (*ut patebit ex praxi ad 10. 13. tradenda*) *erit* ῥν, *juxta 10 def. & proinde* FD = √ : 10 — √ 20 *est* ῥ, *juxta 11 defin.*

a cor. 15. 4.

b 67. 1.

Postulatum.

POstuletur, quamlibet magnitudinem toties posse multiplicari, donec quamlibet magnitudinem ejusdem generis excedat.

Axiomata.

1. MAgnitudo quotcunque magnitudines metiens, compositam quoque ex ipsis metitur.

2. Magnitudo quamcunque magnitudinem metiens, metitur quoque omnem magnitudinem quam illa metitur.

3. Magnitudo metiens totam magnitudinem & ablatam, metitur & reliquam.

PROP. I.

Duabus magnitudinibus inæqualibus AB, C *propositis, si à majore* AB *auferatur majus quam dimidium* (AH) *& ab eo* (HB) *quod reliquum est, rursus detrahatur majus quam dimidium* (HI,) *& hoc semper fiat; relinquetur tandem quædam magnitudo* IB, *quæ minor erit proposita minore magnitudine* C.

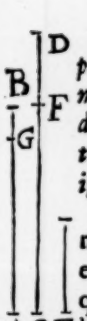
a *post.* 10. *a* Accipe C toties, donec ejus multiplex DE proxime excedat AB; sintque DF=FG=GE=C. Deme ex AB plusquam dimidium AH, & à reliquo HB plusquam dimidium HI; & sic deinceps, donec partes AH, HI, IB æque multæ sint partibus DF, FG, GE. Jam liquet FE, quæ non minor est quam $\frac{1}{2}$ DE, majorem esse quam HB, quæ minor est quam $\frac{1}{2}$ AB ⊐ DE. Pariterque GE quæ non minor est quam $\frac{1}{2}$ FE, major est quam IB ⊐ $\frac{1}{2}$ HB. ergo C, vel GE ⊏ IB. Q. E. D.

Idem demonstrabitur, si ex AB auferatur dimidium AH, & ex reliquo HB rursus dimidium HI, & ita deinceps.

PROP.

PROP. II.

Si duabus magnitudinibus inæqualibus propoſitis (AB, CD) *detrahatur ſemper minor* AB *de majore* CD, *alterna quadam detractione, & reliqua minime præcedentem metiatur; incommenſurabiles erunt ipſæ magnitudines.*

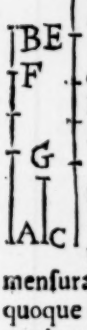


Si fieri poteſt, ſit aliqua E communis menſura. Quoniam igitur AB detracta ex CD, quoties fieri poteſt, relinquit aliquam FD ſe minorem, & FD ex AB relinquit GB, & ſic deinceps, *a* tandem relinquetur aliqua GB ⊐ E. ergo E *b* metiens AB, *c* ideoque CF, *b* & totam CD; *d* etiam reliquam FD metitur. *c* proinde & AG; *d* ergo & reliquam GB, ſeipſa minorem. Q. E. A.

a 1. 10.
b *hyp.*
c 2. *ax.* 10.
d 3. *ax.* 10.

PROP. III.

Duabus magnitudinibus commenſurabilibus datis, AB, CD, *maximam earum communem menſuram* FB *reperire.*



Deme AB ex CD, & reliquum ED ex AB, & FB ex ED, donec FB metiatur ED; (quod tandem fiet, *a* quia per Hyp. AB ⊏ CD) erit FB quæſita.

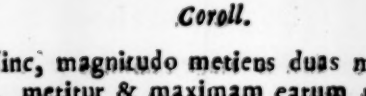
Nam FB *b* metitur ED, *c* ideoque ipſam AF; ſed & ſeipſam, *d* ergo etiam AB, & *c* propterea GE, *d* adeoque & totam CD. Proinde FB communis eſt menſura ipſarum AB, CD. Dic G communem quoque eſſe menſuram, hac majorem; ergo G metiens AB, & CD, *e* metitur CE, & *f* reliquam ED, *e* ideoque AF, & *f* proinde reliquam FB, major minorem. Q. E. A.

a 2. 10.
b *conſtr.*
c 2. *ax.* 10.
d 1. *ax.* 10.
e 2. *ax.* 10.
f 3. *ax.* 10.

Coroll.

Hinc, magnitudo metiens duas magnitudines, metitur & maximam earum menſuram communem.

PROP. IV.



Tribus magnitudinibus commenſurabilibus datis A, B, C; *maximam earum menſuram communem invenire.*

a 3. 10. *a* Inveni D maximam communem menſuram duarum quarumcunque A, B; *a* item E ipſarum D & C maximam communem menſuram; erit E quæſita.

b *conſtr.* & 2. *ax.* 10. c *cor.* 3. 10. *a* Nam perſpicuum eſt E metiens D & C *b* metiri tres A, B, C. Puta aliam F hac majorem eaſdem metiri. *c* ergo F metitur D; *c* proinde & E, ipſorum D, C maximam communem menſuram, major minorem. Q. E. A.

Coroll.

Hinc quoque, magnitudo metiens tres magnitudines, metitur quoque maximam earum communem menſuram.

PROP. V.

A D. 4. *Commenſurabiles magnitudines* A, B *inter ſe rationem habent, quam numerus ad numerum.*
C F. 1.
B E. 3.

a 3. 10. *a* Inventa C ipſarum A, B maxima communi menſura; quoties C in A & B, toties 1 continetur in numeris D & E. *b* ergo C. A :: 1. D; quare inverſe A. C :: D. 1. *b* atqui etiam C.

b 20. *def.* 7.

B ::

B :: 1. E. *c* ergo ex æquali A.B :: D. E :: N.N. Q. E. D. c 22. 5.

PROP. VI.

E ——— F. 1. *Si duæ mag-*
A ——————— C. 4. *nitudines* A, B
B ——————— D. 3. *inter ſe proportionem habeant, quam numerus* C *ad numerum* D; *commenſurabiles erunt magnitudines* A, B.

Qualis pars eſt 1 numeri C, *a* talis fiat E ipſius A. Quoniam igitur E. A *b* :: 1. C. atq; A. B *c* :: C. D; *d* ex æquo erit E. B :: 1. D. ergo quum 1 *e* metiatur numerum D, *f* etiam E metitur B; ſed & ipſum A *g* metitur. *h* ergo A ⊓ B. Q. E. D.

a *ſch.* 10. 6.
b *conſtr.*
c *hyp.*
d 22. 5.
e 5. *ax* 7.
f 20. *def.* 7.
g *conſtr.*
h 1. *def.* 10.

PROP. VII.

A ——————— *Incommenſurabiles*
B ——————— *magnitudines* A, B *inter ſe proportionem non habent, quam numerus ad numerum.*

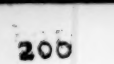
Dic A. B :: N. N. *a* ergo A ⊓ B, contra Hypoth. a 6. 10.

PROP. VIII.

A ——————— *Si duæ magnitudines*
B ——————— A, B *inter ſe proportionem non habeant, quam numerus ad numerum, incommenſurabiles erunt magnitudines.*

Puta A ⊓ B *a* ergo A. B :: N. N, contra Hypoth. a 5. 10.

PROP. IX.



Quæ à rectis lineis longitudine commensurabilibus fiunt quadrata, inter se proportionem habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem habentia, quam quadratus numerus ad quadratum numerum, & latera habebunt longitudine commensurabilia. Quæ vero à rectis lineis longitudine incommensurabilibus fiunt quadrata, inter se proportionem non habent, quam quadratus numerus ad quadratum numerum: & quadrata inter se proportionem non habentia, quam quadratus numerus ad quadratum numerum, neque latera habebunt longitudine commensurabilia.

1. *Hyp.* A. □ B. Dico Aq. Bq :: Q. Q. Nam *a* sit A. B :: num. E. num. F. ergo $\frac{Aq}{Eq}$ (*b* $\frac{A}{B}$ bis) *c* = $\frac{E}{F}$ bis. *d* = $\frac{Eq}{Fq}$ *e* ergo Aq. Bq :: Eq. Fq :: Q. Q. Q. E. D.

a *per* 5. 10.
b 20. 6.
c *sch.* 23. 5.
d 11. 8.
e 11. 5.

2. *Hyp.* Aq. Bq :: Eq. Fq :: Q. Q. Dico A □ B. Nam $\frac{A}{B}$ bis (*f* $\frac{Aq}{Bq}$) *g* = $\frac{Eq}{Fq}$ *h* = $\frac{E}{F}$ bis. *i* ergo A. B :: E. F :: N. N. *k* quare A □ B. Q. E. D.

f 20. 6.
g *hyp.*
h 11. 8.
i *sch.* 23. 5.
k 6. 10.

3. *Hyp.* A □ B. Nego esse Aq. Bq :: Q. Q. Nam dic Aq. Bq :: Q. Q. Ergo A □ B, ut modo ostensum est, contra Hypoth.

4. *Hyp.* Non Aq. Bq :: Q. Q. Dico A □ B. Nam pura A □ B; ergo Aq. Bq :: *Q. Q.* ut modo diximus, contra Hypoth.

Coroll.

Lineæ □ sunt etiam ⌐¬; at non contra. Sed lineæ □ non sunt idcirco ⌐¬. Lineæ vero ⌐¬ sunt etiam □.

PROP.

PROP. X.

Si quatuor magnitudines proportionales fuerint (C. A :: B. D;) *prima vero* C *ſecundæ* A *fuerit commenſurabilis; & tertia* B *quartæ* D *commenſurabilis erit. Et ſi prima* C *ſecundæ* A *fuerit incommenſurabilis, & tertia* B *quartæ* D *incommenſurabilis erit.*

C A B D

Si C ▭ A, *a* ideo erit C. A :: N. N :: B. D. *b* ergo B ▭ D. Sin C ▭ A, ergo *c* non erit C. A :: N. N :: B. D. *d* quare B ▭ D. Q. E. D.

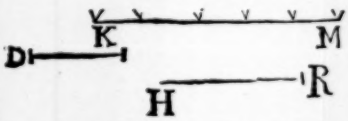
a 5. 10.
b 6. 10.
c 7. 10.
d 8. 10.

LEMMA 1.

Duos numeros planos invenire, qui proportionem non habeant, quam quadratus numerus ad quadratum numerum.

Huic Lemmati ſatisfacient duo quilibet numeri plani non ſimiles, quales ſunt numeri habentes proportionem ſuperparticularem, vel ſuperbipartientem, vel duplam; vel etiam duo quivis numeri primi. vid. Schol. 27. 8.

LEMMA 2.



Invenire lineam HR, *ad quam data recta linea* KM *ſit in ratione datorum numerorum* B, C.

a Divide KM in partes æquales æque multas unitatibus numeri B. harum tot, quot unitates ſunt in numero C, *b* componant rectam HR. liquet eſſe KM. HR :: B. C.

a ſch. 10. 6.
b 3, 1.

LEMMA 3.

Invenire lineam D, *ad cujus quadratum datæ rectæ* KM *quadratum ſit in ratione datorum numerorum* B, C.

Fac

a 2. *lem.* 10. Fac B. C *a* :: KM. HR. ac inter KM, & HR
10. *b* inveni mediam proportionalem D. Erit KMq.
b 13. 6. Dq *c* :: KM. HR *d* :: B. C.
c 20. 6.
d *constr.*

PROP. XI.

A ——— B. 20.
E ——— C. 16.
D ———

Propositæ rectæ lineæ A *invenire duas rectas lineas incommensurabiles; alteram quidem* D *longitudine tantum, alteram vero* E *etiam potentia.*

a 2 *lem.* 10 1. Sume numeros B. C, *a* ita ut non sit B. C ::
10. Q. Q. *b* fiatque B. C :: Aq. Dq. *c* liquet A ⊥ D. Sed Aq *d* ⊥ Dq. Q. E. F.
b 3 *lem.* 10 2. *d* Fac A. E :: E. D. Dico Aq ⊥ Eq.
10. Nam A. D *e* :: Aq Eq. ergo cum A ⊥ D,
c 9. 10. ut prius, *f* erit Aq ⊥ Eq. Q. E. F.
d 6. 10.
d 13. 6.
e 20. 6.

PROP. XII.

f 10. 10. *Quæ* (A, B) *eidem magnitudini* C *sunt commensurabiles, & inter se sunt commensurabiles.*

a 5. 10. Quia A ⊥ C, & C ⊥ B, *a* sit A. C :: N N :: D. E. atque C. B :: N. N :: F. G. *b* sumantur tres numeri H, I, K minimi in rationibus D ad E, & F ad G. Jam quia A. C *c* :: D. E *c* :: H. I. ac C. B *c* :: F. G. *c* :: I. K. *d* erit ex æquali A. B :: H. K :: N. N. *e* ergo A ⊥ B. Q. E. D.

D, 18. E, 8.
F, 2 G, 3.
b 4. 8. H, 5. I, 4 K, 6.
A B C

c *constr.*
d 22. 5.
e 6. 10.

Schol.

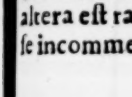
Hinc, omnis recta linea rationali lineæ commensurabilis, est quoque ῥ rationalis. Et omnes rectæ rationales inter se commensurabiles sunt, saltem potentia. Item, omne spatium rationali spatio commensurabile, est quoque rationale; & omnia spatia rationalia inter se commen-

12. 10. & *def* 6.

def. 9.

menſurabilia ſunt. Magnitudines vero, quarum altera eſt rationalis, altera irrationalis, ſunt inter ſe incommenſurabiles. *def.* 7. & 10

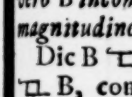
PROP. XIII.



Si ſint duæ magnitudines A, B; *& altera quidem* A *eidem* C *ſit commenſurabilis, altera vero* B *incommenſurabilis, incommenſurabiles erunt magnitudines* A, B.

Dic B ⊓ A. ergo cum C *a* ⊓ A, *b* erit C ⊓ B, contra Hypoth. a *hyp.* b 12. 10.

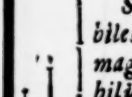
PROP. XIV.



Si ſint duæ magnitudines commenſurabiles A, B; *altera autem ipſarum* A *magnitudini cuipiam* C *incommenſurabilis fuerit; & reliqua* B *eidem* C *incommenſurabilis erit.*

Puta B ⊓ C. ergo cum A ⊓ *a* B, *b* erit A ⊓ C, contra Hyp. a *hyp.* b 12. 10.

PROP. XV.

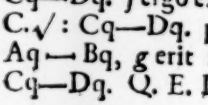


Si quatuor rectæ lineæ proportionales fuerint (A B :: C. D;) *prima vero* A *tanto plus poſſit quam ſecunda* B, *quantum eſt quadratum rectæ lineæ ſibi commenſurabilis longitudine; & tertia* C *tanto plus poterit, quam quarta* D, *quantum eſt quadratum rectæ lineæ ſibi longitudine commenſurabilis. Quod ſi prima* A *tanto plus poſſit quam ſecunda* B, *quantum eſt quadratum rectæ lineæ ſibi incommenſurabilis longitudine; & tertia* C *tanto plus poterit, quam quarta* D, *quantum eſt quadratum rectæ lineæ ſibi longitudine incommenſurabilis.*

Nam quia A. B *a* :: C. D. *b* erit Aq. Bq :: Cq. Dq. *c* ergo dividendo Aq—Bq. Bq :: Cq—Dq. a *hyp.* b 22. 6. c 17. 5.

d 22. 6. Dq. Dq. *d* quare √: Aq—Bq. B :: √: Cq—Dq.
e cor. 4. 5. D. *c* invertendo igitur B. √ : Aq—Bq :: D. √:
f 22. 5. Cq—Dq. *f* ergo ex æquali A. √ : Aq — Bq :: C. √ : Cq—Dq. proinde si A □, vel □ √
g 10. 10. Aq — Bq, *g* erit similiter C □, vel □ √: Cq—Dq. Q. E. D.

PROP. XVI.



Si duæ magnitudines commensurabiles AB, BC *componantur, & tota magnitudo* AC *utrique ipsarum* AB, BC *commensurabilis erit: quod si tota magnitudo* AC *uni ipsarum* AB, vel BC *commensurabilis fuerit; & quæ à principio magnitudines* AB, BC *commensurabiles erunt.*

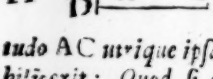
a 3. 10. 1. *Hyp. a* Sit D ipsarum AB, BC communis
b 1. ax 10. mensura. *b* ergo D metitur AC. *c* ergo AC □
c 1. def. 10. AB, & BC. Q. E. D.

2. *Hyp. a* Sit D communis mensura ipsarum
d 3. ax. 10. AC, AB; *d* ergo D metitur AC—AB (BC;) *c* proinde AB □ BC. Q. E. D.

Coroll.

Hinc etiam, si tota magnitudo ex duabus composita, commensurabilis sit alteri ipsarum, eadem & reliquæ commensurabilis erit.

PROP. XVII.



Si duæ magnitudines incommensurabiles AB, BC *componantur, & tota magnitudo* AC *utrique ipsarum* AB, BC *incommensurabilis erit: Quod si tota magnitudo* AC *uni ipsarum* AB *incommensurabilis fuerit, & quæ à principio magnitudines* AB, BC *incommensurabiles erunt.*

1. *Hyp.*

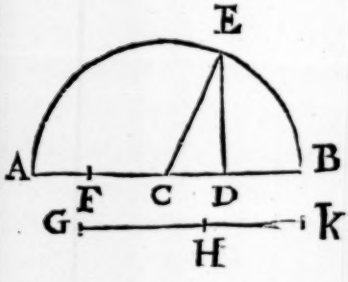
1. *Hyp.* Si fieri poteſt, ſit D ipſarum AC, AB communis menſura. *a* ergo D metitur AC—AB (BC) *b* ergo AB ⊥ BC, contra Hypoth. a 3. *ax.* 10. b 1. *def.* 10.

2. *Hyp.* Dic AB ⊥ BC. *c* ergo AC ⊥ AB, contra Hypoth. c 16. 10.

Coroll.

Hinc etiam, ſi tota magnitudo ex duabus compoſita, incommenſurabilis ſit alteri ipſarum, eadem & reliquæ incommenſurabilis erit.

PROP. XVIII.



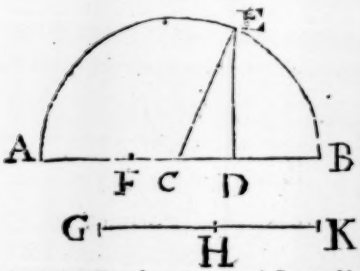
Si fuerint duæ rectæ lineæ inæquales AB, GK; *quartæ autem parti quadrati, quod ſit à minori* GK, *æquale parallelogrammũ* ADB *ad majorem* AB *applicetur, deficiens figura quadrata, & in partes* AD, DB *longitudine commenſurabiles ipſam dividat; major* AB *tanto plus poterit quam minor* GK, *quantum eſt quadratum rectæ lineæ* FD *ſibi longitudine commenſurabilis. Quod ſi major* AB *tanto plus poſſit, quam minor* GK, *quantum eſt quadratum rectæ lineæ* FD *ſibi longitudine commenſurabilis; quartæ autem parti quadrati, quod ſit à minori* GK, *æquale parallelogrammum* A D B *ad majorem* AB *applicetur, deficiens figura quadrata, in partes* A D, D B *longitudine commenſurabiles ipſam dividet.* a 10. 1. b 28. 6.

a Bileca GK in H, & *b* fac rectang. ADB = GHq: abſcinde AF = DB. Eſtque ABq *c* = 4 ADB *d* (4 GHq, vel GKq) + FDq. Jam c 8. 2. d *conſtr.* & 4. 2. primo

primo, Si AD ▭ DB, erit AB e ▭ DB e ▭ 2 DB f (AF + DB, vel AB — FD) g ergo AB ▭ FD. Q. E. D. Sin ſecundo, AB ▭ FD, h erit ideo AB ▭ AB — FD (2 DB) k ergo AB ▭ DB. l quare AD ▭ DB. Q. E. D.

e 16 10.
f *conſtr.*
g *cor.* 16 10.
h *cor.* 16 10.
k 12. 10.
l 16. 10.

PROP. XIX.



Si fuerint duæ rectæ lineæ inæquales, AB, GK; *quarta autem parti quadrati, quod fit à minore* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata; & in partes incommenſurabiles longitudine* AD, DB, *ipſam* AB *dividat; major* AB *tanto plus poterit, quam minor* GK, *quantum eſt quadratum rectæ lineæ* FD, *ſibi longitudine incommenſurabilis. Quod ſi major* AB *tanto plus poſſit, quam minor* GK, *quantum eſt quadratum rectæ lineæ* FD *ſibi longitudine incommenſurabilis; quartæ autem parti quadrati, quod fit à minore* GK, *æquale parallelogrammum* ADB *ad majorem* AB *applicetur, deficiens figura quadrata; in partes longitudine incommenſurabiles* AD, DB *ipſum* AB *dividet.*

Facta puta, & dicta eadem, quæ in præcedenti. Itaque primo. Si AD ▭ DB, a erit propterea AB ▭ DB; b quare AB ▭ 2 DB (AB — FD) c ergo AB ▭ FD. Q. E. D.

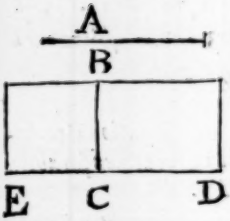
a 17. 10.
b 13. 10

Secundo, Si AB ▭ FD; c ergo AB ▭ AB — FD (2 BB;) d quare AB ▭ DB, & e proinde AD ▭ DB. Q. E. D.

c *cor.* 17. 10.
d 13. 10.
e 17. 10.

PROP.

PROP. XX.



Quod ſub rationalibus longitudine commenſurabilibus rectis lineis BC, CD, *ſecundum aliquem prædictorum modorum, continetur rectangulum* BD, *rationale eſt.*

Exponatur A, ῥ. & *a* deſcribatur BE quadratum ex BC. Quoniam DC. CE (BC) *b* :: BD. BE. & DC *c* ▭ BC; *d* erit rectang. BD ▭ quad. BE. ergo quum quad. BE *e* ▭ Aq; *f* erit BD ▭ Aq. proinde rectang. BD eſt ῥν. Q. E. D.

a 46. 1.
b 1. 6.
c *hyp.*
d 10. 10.
e *hyp. & 9.*

Not. *Tria ſunt genera linearum rationalium inter ſe commenſurabilium. Aut enim duarum linearum rationalium longitudine inter ſe commenſurabilium altera æqualis eſt expoſitæ rationali; aut neutra rationali expoſitæ æqualis eſt, longitudine tamen ei utraque eſt commenſurabilis; aut denique utraque expoſitæ rationali commenſurabilis eſt ſolum potentia. Hi ſunt modi illi, quos innuit præſens theorema.*

def. 10.
f 12. 10.

In numeris, ſit BC, $\sqrt{8}$ ($2\sqrt{2}$) & CD, $\sqrt{18}$ ($3\sqrt{2}$) erit rectang. BD $= \sqrt{144} = 12$.

PROP. XXI.

Si rationale DB *ad rationalem* DC *applicetur, latitudinem* CB *efficit rationalem, & ei* DC *ad quam applicatum eſt* DB, *longitudine commenſurabilem.*

Exponatur G, ῥ. & deſcribatur DA quadratum ex BC. quoniam BD. DA *a* :: BC. CA; atque BD. DA *b* ſunt ῥα, *c* ideoque ▭; *d* erit

a 1. 6.
b *hyp.*
c *ſch.* 12. 10
d 10. 10.

BC

c *ſch.*12.10 BC ▭ CA. at CD (CA) *b* eſt ῥ. *c* ergo BC eſt ῥ. Q. E. D.

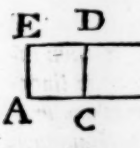
In numeris, ſit rectang. DB, 12; & DC, √8. erit CB, √18. atqui √18 = 3√2. & √8 = 2√2.

LEMMA.

A ——— *Duas rectas rationales potentia ſolum commenſurabiles invenire.*
B ———
C ———

a 11. 10. Sit A expoſita ῥ. *a* Sume B ⌐ A, *a* & C ⌐ B.
b *ſch.*12 10 *b* liquet B, & C eſſe quæſitas.

PROP. XXII.



Quod ſub rationalibus DC, CB *potentia ſolum commenſurabilibus rectis lineis continetur rectangulum* DB, *irrationale eſt; & recta linea* H *ipſum potens, irrationalis; vocetur autem Media.*

Sit G expoſita ῥ. & deſcribatur DA quadratum ex DC; ſitque Hq = DB. Quoniam AC: CB *a* :: DA. DB. *b* atque AC ▭ CB, *c* erit DA ▭ DB (Hq.) *d* atqui Gq ▭ DA. *e* ergo Hq ▭ Gq. *f* ergo H eſt ῥ. Q. E. D. vocetur autem Media, quia AC. H :: H. CB.

a 1. 6.
b *hyp.*
c 10. 10.
d *hyp & 9. def.* 10.
e 13. 10.
f *def.* 11. 10

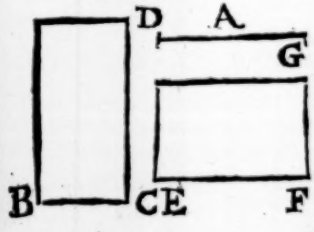
In numeris, ſit DC, 3; & CB, √6. erit rectangulum DB (Hq) √54. quare H eſt *v* √54. Mediæ nota eſt *μ*, Medii vero *μν*; pluraliter *μα*.

SCHOL.

Omne rectangulum, quod poteſt contineri ſub duabus rectis rationalibus potentia ſolum commenſurabilibus, eſt Medium; quamvis contineatur ſub duabus rectis irrationalibus: atque omne

omne Medium poteſt contineri ſub duabus rectis rationalibus potentia tantum commenſurabilibus, ut exemp. gr. $\sqrt{24}$ eſt $\mu\nu$. quia continetur ſub $\sqrt{3}$, & $\sqrt{8}$, qui ſunt ρ^{c} ▭. etſi poſſet contineri ſub $v\sqrt{6}$, & $v\sqrt{96}$ Irrationalibus; nam $\sqrt{24} = v\sqrt{576} = v\sqrt{6}$ in $v\sqrt{96}$.

PROP. XXIII.

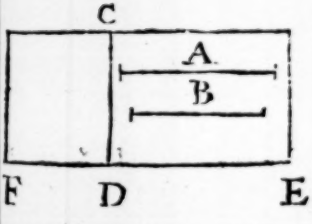


Quod (BD) *à mediâ* A *ſit, ad rationalem* BC *applicatum, latitudinem* CD *rationalem efficit, & ei* BC, *ad quam applicatum eſt* BD *longitudine incommenſurabilem.*

Quoniam A eſt μ, *a* erit Aq rectangulo alicui (EG) æquale contento ſub EF, & FG ρ^{c} ▭. *b* ergo BD = EG. *c* quare BC. EF :: FG. CD. *d* ergo BCq. EFq :: FGq. CDq. ſed BCq, & EFq *e* ſunt $\rho\alpha$, *f* ideoque ▭. *g* ergo FGq ▭ CDq. Ergo quum FG ſit ρ^{c}, *h* erit CD ρ^{c}. Por. 10, quia EF. FG *k* :: EFq. EG (BD;) ob EF ▭ FG, *l* erit EFq ▭ BD. verum EFq *m* ▭ CDq. *n* ergo rectang. BD ▭ CDq. quum igitur CDq. BD *o* :: CD. BC. *p* erit CD ▭ BC. ergo, &c.

a *ſch.* 12. 10
b 1. *ax.* 1.
c 14. 6.
d 22. 6.
e *hyp.*
f *ſch.* 12. 10
g 10. 10.
h *ſch.* 12. 10
k 1. 6.
l 10. 10.
m *ſch.* 12. 10.
n 13. 10.
o 1. 6.
p 10. 10.

PROP. XXIV.



Mediæ A *commenſurabilis* B, *media eſt.*

Ad CD ρ^{c} *a* [illegible] rectang. CE = Aq; *a* & rectang. CF = Bq. Quoniam Aq (CE) eſt $\mu\nu$, *b* & CD ρ^{c}, *c* erit latitudo

a 11. 6.
b *hyp.*
c 23. 10.

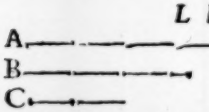
d 1. 6. DE ρ̊ ⊐ CD Quoniam vero CE. CF *d* ::
e *hyp.* ED. DF, & CE *e* ⊏⊐ CF, *f* erit ED ⊏⊐ DF.
f 10. 10. *g* ergo DF est ρ̊ ⊏⊐ CD. *h* ergo rectang. CF
g 2. & 13. 10. (Bq) est $\mu\nu$ & proinde B est μ. Q. E. D.

h 22. 10. Nota *quod signum* ⊐ *plerumque valet potentia tantum commensurabile, ut in hac demonstratione, & in præced. &c. quod intellige, ut ex usu erit, & juxta citationes.*

Coroll.

Hinc liquet spatium medio spatio commensurabile medium esse.

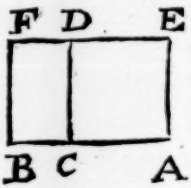
LEMMA.



Duas rectas medias A, B *longitudine commensurabiles; item duas* A, C *potentia tantum commensurabiles invenire.*

a *lem.* 22. 10. & 13. 6 *a* Sit A μ quævis; sume *b* B ⊏⊐ A; *c* & C ⊐ A.
b 2. *lem.* 10 10. *d* Factum esse liquet.

PROP. XXV.

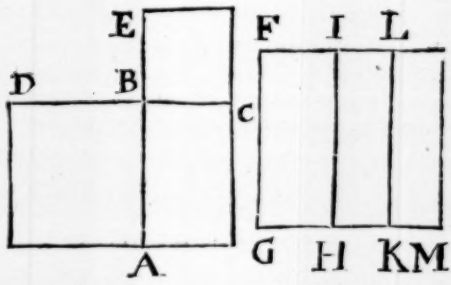


c 3. *lem* 10 10. *Quod sub* DC, CB *mediis longitudine commensurabilibus rectis lineis continetur rectangulum* DB, *medium est.*

d *constr.* & 24. 10. Super DC construatur quadratum DA. Quoniam AC.
a 1. 6. (DC) CB *a* :: DA. DB. & DC
b 10. 10. ⊏⊐ CB; *b* erit DA ⊏⊐ DB. *c* ergo DB est $\mu\nu$.
c 24. 10. Q. E. D.

PROP.

PROP. XXVI.



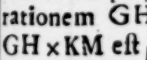
Quod ſub mediis potentia tantum commenſurabilibus rectis lineis AB, BC *continetur rectangulum* AC, *vel rationale eſt, vel medium.*

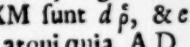
Super rectas AB, BC *a* deſcribe quadrata AD, CE. atque ad FG $\dot{\rho}$, *b* fac rectangula FH = AD, *b* & IK = AC, *a b* & LM = CE.

Quadrata AD, CE, hoc eſt, rectangula FH, LM *c* ſunt $\mu\alpha$, & $\sqsubset$; ergo eandem habentes rationem GH, KM ſunt *d* $\dot{\rho}$, & *e* $\sqsubset$. *f* ergo GH × KM eſt $\dot{\rho}\nu$. atqui quia AD, AC, CE, hoc eſt FH, IK, LM *g* ſunt $\div$; & *b* proinde GH, HK, KM etiam $\div$, *k* erit HKq = GH × KM; *l* ergo HK eſt $\dot{\rho}$; vel $\sqsubset$, vel $\sqsubset\!\!\!\lrcorner$ IH (GF;) ſi $\sqsubset$, *m* ergo rectang. IK vel AC eſt $\dot{\rho}\nu$. Sin $\sqsubset\!\!\!\lrcorner$. *n* ergo AC eſt $\mu\nu$. Q. E. D.

a 46. 1.
b *cor.* 16. 6
c *hyp.* & 24. 20.
d 23. 10.
e 10. 10.
f 20. 10.
g *ſch.* 22. 6.
h 1. 6.
k 17. 6.
l 12. 10.
m 20. 10.
n 22. 10.

LEMMA.





Si A, *&* E *ſint* $\sqsubset\!\!\!\lrcorner$ *tantum;*

Erunt primo Aq, Eq, Aq + Eq, Aq — Eq *a* $\sqsubset$. *Erunt ſecundo*, Aq, Eq, Aq + Eq, Aq — Eq $\sqsubset$ AE, & 2 AE. Nam A. E *b* :: Aq. AE *b* :: AE. Eq. ergo cum A *c* $\sqsubset$ E. *d* erit Aq $\sqsubset$ AE, *e* & 2 AE. item Eq *d* $\sqsubset$ AE, *e* & 2 AE. *c* quare cum Aq + Eq $\sqsubset$ Aq, & Eq; & Aq — Eq $\sqsubset$ Aq, &

hyp. & 10.
a 16. 6.
b 1.
c *hyp* 1
d 10. 10.
e 4. 10.

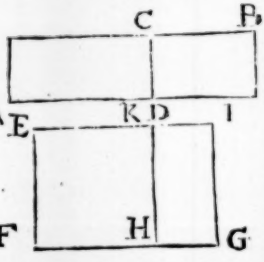
f 14. 10. Eq, f erunt Aq+Eq, f & Aq—Eq ▭ AE, & 2 AE.

Hinc erunt tertio, Aq, Eq, Aq+Eq, Aq—Eq, 2 AE g ▭ Aq+Eq+2 AE; & Aq+Eq—2 AE.

g 14. 10. & 17. 10. g & Aq+Eq+2 AE ▭ Aq+Eq—2 AE. b (Q. A—E.)

h *cor.* 7. 20.

PROP. XXVII.



Medium AB *non superat medium* AC *rationali* DB.

a *cor.* 16. 6. Ad EF ρ^e, a fac EG = AB, a & EH = AC Rectangula AB, AC, hoc est, EG, EH b sunt $\mu\alpha$, c ergo FG, & FH sunt ρ^e ⌐ EF.

b *hyp.*

c 23. 10.

d 3. *ax.* 1. itaque si KG, d id est DB sit $\rho^e v$. e erit HG ▭

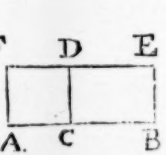
e 21. 10. HK; f quare HG ⌐ FH. g ergo FGq ▭ FHq.

f 13. 10. sed FH est ρ^e. h ergo FG est ρ. verum prius

g *lem.* 26. 10. erat FG $\acute{\rho}$. Quæ repugnant.

h *sch.* 12. 10

SCHOL.



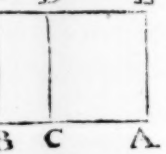
1. *Rationale* AE *superat rationale* AD *rationali* CE.

a *hyp.*

b *cor.* 16. 10.

Nam AE a ▭ AD; b ergo AE ▭ CE. c quare CE est $\rho^e v$. Q. E. D.

c *sch.* 12. 10



2. *Rationale* AD *cum rationali* CF *facit rationale* AF.

a *sch* 12. 10

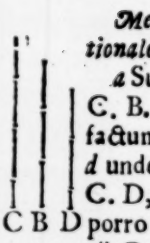
b 16. 10.

c *sch.* 12. 10

Nam AD a ▭ CF; b quare AF ▭ AD, & CF. c proinde AF est $\rho^e v$. Q. E. D.

PROP. XXVIII.

Medias invenire (C, *&* D) *quæ rationale* CD *contineant.*



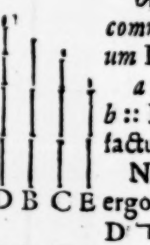
a Sume A, & B ρ^c ⌙. *b* fac A. C :: C. B. *c* atque A. B :: C. D. Dico factum. Nam AB (Cq) *d* est $\mu\nu$; *d* unde C est μ. quum vero A. B *e* :: C. D, *f* erit C ⌙ D. *g* ergo D est μ. porro permutando A. C :: B. D. *e* hoc est C. B :: B. D. *b* ergo Bq = CD. atqui Bq *e* est $\rho^c\nu$. *b* ergo CD est $\rho^c\nu$. Q. E. F.

In numeris, sit A, $\sqrt{2}$, & B, $\sqrt{6}$. ergo C est $v\sqrt{12}$. fac $\sqrt{2}.\sqrt{6} :: v\sqrt{12}$. D. vel $v\sqrt{4}.v\sqrt{36} :: v\sqrt{12}$. D. erit D, $v\sqrt{108}$. atqui $v\sqrt{12}$ in $v\sqrt{108} = v\sqrt{1296} = \sqrt{36} = 6$. ergo CD est 6. item C. D :: 1. $\sqrt{3}$. quare C ⌙ D.

a *lem.* 21.
b 13. 6.
c 12. 6.
d 22. 10.
e *constr.*
f 10. 10.
g 24. 10.
h 17. 6.
h *sch.* 12. 10

PROP. XXIX.

Medias invenire potentia tantum commensurabiles D, *&* E, *quæ medium* DE *contineant.*



a Sume A, B, C ρ^c ⌙. Fac A. D *b* :: D. B. *c* & B. C :: D. E. Dico factum.

Nam AB *d* = Dq & AB *c* est $\mu\nu$; ergo D est μ. & B *f* ⌙ C. *g* ergo D ⌙ E. *b* ergo E est μ. porro, B. C *f* :: D. E, & permutando B. D :: C. E. *k* hoc est D. A :: C. E. *l* ergo DE = AC. Sed AC *m* est $\mu\nu$. ergo DE est $\mu\nu$. Q. E. D.

In numeris sit A, 20; & B, $\sqrt{200}$; & C, $\sqrt{80}$. Ergo D est $\sqrt{\sqrt{80000}}$; & E $v\sqrt{12800}$. Ergo DE = $\sqrt{\sqrt{1024000000}} = \sqrt{32000}$. & D. E :: $\sqrt{10}$. 2. quare D ⌙ E.

a *lem.* 21.
b 13. 6.
c 12. 6.
d 17. 6.
e 22. 10.
f *constr.*
g 10. 10.
h 24. 10.
k *constr.* & *cor.* 4. 5.
l 16. 6.
m 22. 6.

SCHOL.

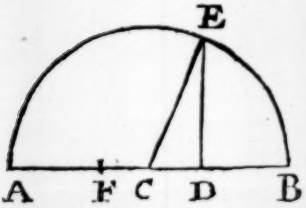
A, 6. C, 12.
B, 4. D, 8.
$\overline{AB}$, 24. $\overline{CD}$, 96.

Invenire duos numeros planos similes vel dissimiles.

Sume quoscunque quatuor numeros proportionales, A. B :: C. D. liquet AB, & CD esse similes planos. Planos autem dissimiles quotcunque reperies ope scholii 27. 8.

A, 6. C, 5.
B, 4. D, 8.
$\overline{AB}$, 24. $\overline{CD}$, 40.

LEMMA.



1. *Duos numeros quadratos* (DEq & CDq) *invenire, ita ut compositus ex ipsis* (CEq) *quadratus etiam sit.*

Sume AD, DB numeros planos similes (quorum ambo pares sint, vel ambo impares) ut in rum AD, 24. & DB, 6. Horum summa, (AB) est 30; differentia (FD) 18, cujus semissis
a 18. 8. (CD) est 9. *a* Habent vero plani similes AD, DB unum medium numerum proportionalem, nempe DE. patet igitur singulos numeros CE,
b 47. 1. CD, DE rationales esse; proinde CEq (*b* CDq + DEq) est numerus quadratus requisitus.

Facile itaque invenientur duo numeri quadrati, quorum excessus sit quadratus, vel non quadratus numerus, nempe ex eadem constructione,
c 3. ax. 1. *c* erit CEq − CDq = DEq.

Quod si AD, DB sint numeri plani dissimiles,

les, non erit media proportionalis (DE) numerus rationalis; proinde quadratorum CEq, CDq excessus (DEq) non erit numerus quadratus.

LEMMA 2.

2. *Duos numeros quadratos* B, C *invenire, ita ut compositus ex ipsis* D, *non sit quadratus. item, quadratum numerum* A *dividere in duos numeros* B, C *non quadratos.*

A, 3. B, 9. C, 36. D, 45.

1. Sume numerum quemlibet quadratum B, sitque C=4 B; & D=B+C. Dico factum.

Nam B est ◻. ex constr. item quia B. C :: 1. 4 :: ◻. ◻. *a* erit C etiam quadratus. Sed quoniam B+C. (D) C :: 5. 4 :: non ◻. ◻. *b* non erit D numerus quadratus. ◻. E. F.

a 24. 8.
b *cor.* 24. 8.

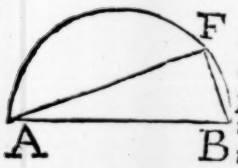
A, 36. B, 24. C, 12. D, 3. E, 2. F, 1.

2. Sit A numerus quivis quadratus. Accipe D, E, F numeros planos dissimiles, sitque D=E+F. fac D. E :: A. B. & D. F. :: A. C. Dico factum.

Nam quia D. E+F :: A. B+C. & D=E+F, *a* erit A=B+C. Jam dic B quadratum esse. *b* ergo A & B, & *c* proinde D & E, sunt numeri plani similes, contra Hypoth. idem absurdum sequetur, si C dicatur quadratus. ergo, &c.

a 14. 5.
b 21. *def.* 7.
c 26. 8.

PROP. XXX.

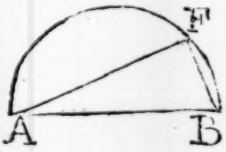


Invenire duas rationales A B, A F *potentia tantum commenſurabiles, ita ut major* A B *plus poſſit, quam minor* A F, *quadrato rectæ lineæ* B F *longitudine ſibi commenſurabilis.*

a 1. *lem.* Exponatur AB, ρ. *a* Su-
29. 10. me CD, CE numeros quadratos, ita ut CD − CE
b 3. *lem.* (ED) ſit non Q. *b* Fiatque CD. ED :: ABq. AFq.
10. 10. In circulo ſuper AB diametrum deſcripto *c* ap-
c 1. 4. tetur AF, ducaturq; BF. Sunt AB, AF, quas petis.
d *conſtr.* Nam ABq. AFq *d* :: CD. ED. *e* ergo ABq
e 6. 10. □ AFq. verum AB eſt ρ. *f* ergo AF eſt ρ. ſed
f *ſch.* 12. 10 quia CD eſt Q: at ED non Q: *g* erit AB □
g 9. 10. AF. porro, ob ang. *h* rectum AFB, eſt ABq
h 31. 3. *k* = AFq + BFq; cum igitur ABq, AFq ::
k 47. 1. CD. ED. per converſionem rationis erit ABq.
l 9. 10. BFq :: CD. CE :: Q. Q. *l* ergo AB □ BF. Q. E. F.

In numeris; ſit AB, *6*; CD, *9*. CE, *4*; quare ED, *5*. Fac 9. 5 :: 36. (Q. 6) AFq. erit AFq 20. proinde AF √ 20. ergo BFq = 36 − 20 = 16. quare BF eſt 4.

PROP. XXXI.



Invenire duas rationales A B, A F *potentia tantum commenſurabiles, ita ut major* A B *plus poſſit, quam minor* A F, *quadrato rectæ lineæ* B F *ſibi longitudine incommenſurabilis.*

a 2. *lem.* Exponatur AB, ρ. *a* acci-
29. 10. pe numeros CE, ED quadratos, ita ut CD = CE + ED ſit non Q. & in reliquis imitare conſtructionem præcedentis. Dico factum.

Nam,

Nam, ut ibi, AB, AF sunt ῥ ⌐¬. item ABq. BFq :: C D. E D. ergo cum C D sit non Q. *b* erunt AB, BF ⌐¬. Q. E. F. b 9. 10.

In numeris, sit AB, 5. C D, 45. CE = 36; ED = 9. Fac 45. 9 :: 25 (ABq.) 5 (AFq.) ergo AF = √ 5. proinde BFq = 45 − 25 = 20. quare BF = √ 20.

PROP. XXXII.

A ——·——·——·—— *Invenire duas medias*
B ——·——·——·—— **C**, **D** *potentia tantum*
C ——·——·—— *commensurabiles, quæ*
D ——·—— *rationale* CD *contineant, ita ut major* C *plus possit, quam minor* D, *quadrato rectæ lineæ sibi longitudine commensurabilis.*

a Accipe A, & B ῥ ⌐¬; ita ut √ Aq − Bq ⌐¬ A. *b* Fiatque A. C :: C. B, *c* atque A. B :: C. D. Dico factum. a 30. 10. b 13. 6. c 12. 6.

Nam quia A, & *d* B sunt ῥ ⌐¬, *e* erit C (*f* √ AB) *μ*. item *g* ideo C ⌐¬ D. *b* ergo D etiam *ν*. porro quia A. B *d* :: C. D; & permutatim A. C :: B. D :: C. B; & Bq *d* est ῥ *ν*, erit CD *k* (Bq) ῥ *ν*. Denique quia √ Aq − Bq *d* ⌐¬ A, *l* erit √ Cq − Dq ⌐¬ C. ergo, &c. Sin √ Aq − Bq ⌐¬ Aq, erit √ Cq − Dq ⌐¬ C. d constr. e 22. 10. f 17. 6. g 10. 10. h 24. 10. k 17. 6. l 15. 10.

In numeris, sit A, 8; B, √ 48 (√ : 64 − 16) ergo C = √ AB = *v* √ 3072. & D = *v* √ 1728. quare CD = *v* √ 5308416 = √ 2304.

PROP. XXXIII.

A ——·——·——·—— *Invenire duas medias*
D ——·——·——·—— **D**, E *potentia solum*
B ——·——·—— *commensurabiles, quæ*
C ——·——·—— *medium* D E *contineant, ita ut major* D *plus possit, quam minor* E, *quadrato rectæ lineæ sibi longitudine commensurabilis.*

Sume

a 30. 10. *a* Sume A, & C ρ̊, ⊐; ita ut √ Aq—Cq ⊏ A. *b* sume etiam B ⊐ A, & C; & fac A. D *c* :: D. B *d* :: C. E. Erunt D, & E quæsitæ.

b *lem.* 21. 10.

c 13. 6. Nam quoniam A, & C *e* sunt ρ̊, *e* & B ⊐ A & C, *f* erit B ρ̊, & D (√ AB) *g* erit μ. *e* Quia vero A. D :: C. E. erit permutando A. C :: D. E. ergo cum A ⊐ C, *h* erit D ⊐ E. *k* ergo E est μ. porro, *l* quia D. B :: C. E; *l* & BC est μν, etiam DE ei *m* æquale est μν. denique propter A. C :: D. E. *e* quia √ Aq — Cq ⊏ A, *n* erit √ Dq — Eq ⊏ D. ergo, &c. Sin √ Aq—Cq ⊏ A, erit √ Dq—Eq ⊏ Eq.

d 12. 6.
e *constr.*
f *sch.* 12. 10
g 22. 10.
h 10. 10.
k 24. 10.
l 22. 10.
m 16. 6.
n 15. 10.

In numeris, sit A, 8; C, √ 48; B, √ 28. erit D *v* √ 3072; & E *v* √ 588. quare D E :: 2 √ 3. & DE = √ 1344.

PROP. XXXIV.

Invenire duas rectas lineas AF, BF *potentia incommensurabiles, quæ faciant compositum quidem ex ipsarum quadratis rationale, rectangulum vero sub ipsis contentum, medium.*

a 31. 10.
b 10. 1.
c 28. 6.

a Reperiantur AB, CD ρ̊ ⊐; ita ut √ ABq—CDq ⊏ AB. *b* biseca CD in G. *c* fac rectang. AEB = GCq. Super AB diametrum duc semicirculum AFB. erige perpendicularem EF. duc AF, BF. Hæ sunt quæ indagandæ erant.

d 12. 6.
e *cor.* 8. 6. & 17. 6.
f 7. 5.
g 19. 10.
h 10. 10.

Nam AE. BE *d* :: BA × AE. AB × BE. Sed BA × AE *e* = AFq; *e* & AB × BE = FBq. *f* ergo AE. EB :: AFq. FBq. ergo cum AE *g* ⊏ EB, *h* erit AFq ⊏ FBq. Quinetiam ABq (*k* AFq + FBq) *l* est ρ̊ν. denique EFq *l* = AEB *l* = CGq. *m* ergo EF = CG. ergo CD × AB = 2 EF × AB. atqui CD × AB *n* est μν. *o* ergo AB × EF, *p* vel AF × FB, est μν. Q. E. D.

k 31. 3. & 47. 1.
l *constr.*
m 1. *ax* 1.
n 22. 10.
o 24. 10.
p *sch* 22. 6.

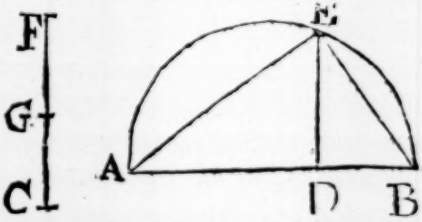
Ex-

Explicatio per numeros.

Sit AB, 6. CD, $\sqrt{12}$. quare CG $= \sqrt{12\frac{1}{4}} = \sqrt{3}$. Eſt vero AE $= 3 + \sqrt{6}$. & EB $= 3 - \sqrt{6}$. & unde AF erit $\sqrt{: 18 + \sqrt{216}}$. Et FB, $\sqrt{: 18 - \sqrt{216}}$. item AFq + FBq eſt 36, & AF × FB $= \sqrt{108}$.

Cæterum AE invenitur ſic. Quia BA (6.) AF :: AF. AE; erit 6 AE $=$ AFq $=$ AEq + 3 (EFq.) ergo 6 AE $-$ AEq $= 3$. pone $3 + e =$ AE. ergo $18 + 6e - 9 - 6e - ee$, hoc eſt $9 - ee = 3$. vel $ee = 6$. quare $e = \sqrt{6}$. proinde AE $= 3 + \sqrt{6}$.

PROP. XXXV.



Invenire duas rectas lineas AE, EB *potentia incommenſurabiles, quæ faciant compoſitum quidem ex ipſarum quadratis medium, rectangulum vero ſub ipſis contentum, rationale.*

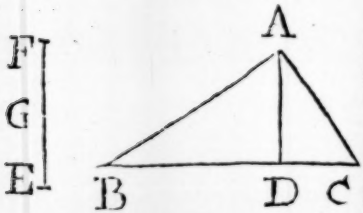
a Sume AB, & CF μ ⌐, ita ut AB × CF ſit $\dot{\rho}\nu$, atque $\sqrt{}$ ABq $-$ CFq ⌐ AB. & reliqua fiant, ut in præcedenti. erunt AE, EB, quas petis. *a* 32. 10.

Nam, ut iſthic oſtenſum eſt, AEq ⌐ EBq: item ABq (AEq + EBq) eſt $\mu\nu$. & denique AB × CF *b* eſt $\dot{\rho}\nu$, idcirco & *c* AB × DE, *d* hoc eſt, AE × EB, eſt $\dot{\rho}\nu$. ergo, &c.

b conſtr.
c ſchol. 12. 10.
d ſch. 22. 6.

PROP.

PROP. XXXVI.



Invenire duas rectas lineas BA, AC *potentia incommensurabiles, quæ faciant & compositum ex ipsarum quadratis medium, & rectangulum sub ipsis comprehensum medium, incommensurabileque composito ex ipsarum quadratis.*

a 33. 10. *a* Accipe BC & EF μ ▭ ; ita ut BC × EF sit $\mu\nu$. & $\sqrt{}$ BCq − EFq ▭ BC. & reliqua fiant, ut in præcedentibus. Erunt BA, AC exoptata. Nam, ut prius, BAq ▭ ACq ; item BAq + ACq est $\mu\nu$. & BA × AC est $\mu\nu$. Denique BC

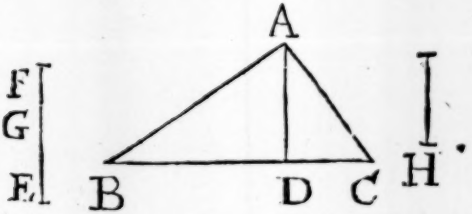
b *constr.* *b* ▭ EF, atque *c* ideo BC ▭ EG; estque BC.

c 13. 10. EG *d* :: BCq. BC × EG, (BC × AD, vel BA

d 1. 6. × AC) *e* ergo BCq (ABq + ACq) ▭ BA ×

e 14. 10. AC, ergo, &c.

Schol.



Invenire duas medias longitudine & potentia incommensurabiles.

a 36. 10. *a* Sume BC μ. sitque BA × AC $\mu\nu$, & ▭

b 13. 6. B Cq (B Aq + A Cq.) *b* Fac B A. H :: H

AC. Sunt BC, & H μ ▭. Nam BC est μ.

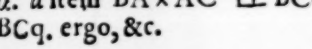
c 17. 6. *a* & BA × AC (*c* Hq) est $\mu\nu$. quare H est etiam

μ.

μ. *d* item BA × AC ▭ BCq; ergo Hq ▭ BCq. ergo, &c. d 14. 10.

Principium ſenariorum per compoſitionem.

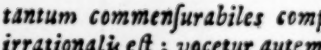
PROP. XXXVII.



Si duæ rationales AB, BC *potentia tantum commenſurabiles componantur, tota* AC *irrationalis eſt; vocetur autem ex binis nominibus.*

Nam quia AB *a* ▭ BC, *b* erit ACq ▭ ABq. Sed AB *a* eſt $\dot{\rho}$. *c* ergo AC eſt $\dot{\rho}$. Q.E.D. a *hyp.* b *lem.* 25. 10. c 11. *def.* 10

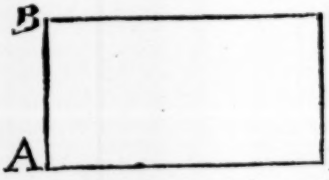
PROP. XXXVIII.



Si duæ mediæ AB, BC *potentia tantum commenſurabiles componantur; quæ rationale contineant, tota* AC *irrationalis eſt; vocetur autem ex binis mediis prima.*

Nam quoniam AB *a* ▭ BC, *b* erit ACq ▭ AB × BC, $\dot{\rho}\nu$. *c* ergo AC eſt $\dot{\rho}$. Q.E.D. a *hyp.* b *lem.* 26. 10. c 11. *def.* 10

LEMMA.

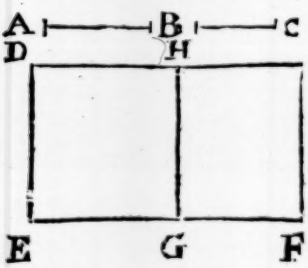


Quod ſub linea rationali AB, *& irrationali* BC *continetur rectangulum* AC, *irrationale eſt.*

Nam ſi rectang. AC dicatur $\dot{\rho}\nu$; quum AB ſit $\dot{\rho}$; *b* erit latitudo BC etiam $\dot{\rho}^{s}$. contra Hyp. a *hyp.* b 21. 10.

PROP.

PROP. XXXIX.



Si duæ mediæ AB, BC *potentia tantum commensurabiles componantur, quæ medium contineant, tota* AC *irrationalis erit; vocetur autem ex binis mediis secunda.*

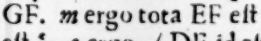
Ad expositam DE $\dot{\rho}$ *a* fac rectang. DF = ACq; *b* & DG = ABq + BCq.

a *cor.* 16. 6.
b 47. 1. & 11. 6.

Quoniam ABq *c* $\sqsupset$ BCq, *d* erit ABq + BCq, hoc est DG $\sqsupset$ ABq; sed ABq *e* est $\mu\nu$. *e* ergo DG est $\mu\nu$. verum rectang. ABC ponitur $\mu\nu$; *e* ideoque 2 ABC (*f* HF) est $\mu\nu$; *g* ergo EG, & GF sunt $\dot{\rho}$ quia vero DG *h* $\sqsupset$ HF; atque DG. HF :: *k* EG. GF *l* erit EG $\sqsupset$ GF. *m* ergo tota EF est $\dot{\rho}$. *n* quare rectang. DF est $\dot{\rho}\nu$. *o* ergo $\surd$ DF, id est AC, est $\dot{\rho}$. Q. E. D.

c *hyp.*
d 16. 10.
e 24. 10.
f 4. 2.
g 23. 10.
h *lem.* 26. 1.
k 1. 6.
l 10. 10.
m 37. 10.
n *lem.* 38. 10.
o 11. *def.* 10

PROP. XL.



Si duæ rectæ lineæ AB, BC *potentia tantum commensurabiles componantur, quæ faciant compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur, medium; tota recta linea* AC, *irrationalis erit: vocetur autem major.*

Nam quia ABq + BCq *a* est $\dot{\rho}\nu$, & *b* $\sqsupset$ 2 ABC *c* $\mu\nu$, & proinde ACq (*d* ABq + BCq + 2 ABC) *e* $\sqsupset$ ABq + BCq $\dot{\rho}\nu$, *f* erit AC $\dot{\rho}$. Q. E. D.

a *hyp.*
b *sch.* 12. 10
c *hyp.* & 24. 10.
d 4. 2.
e 17. 10.
f 11. *def.* 10

PROP.

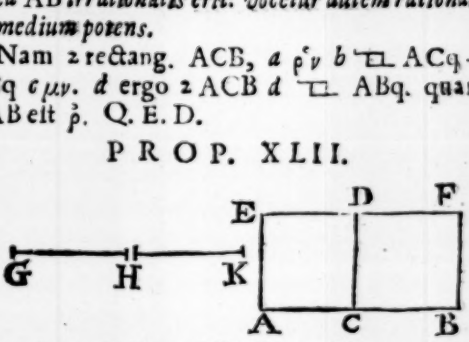
PROP. XLI.

Si duæ rectæ lineæ AC, CB *potentia incommensurabiles componantur, quæ faciant compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; tota recta linea* AB *irrationalis erit: vocetur autem rationale ac medium potens.*

Nam 2 rectang. ACB, *a* ρ^v *b* ▭ ACq + CBq *c* μν. *d* ergo 2 ACB *d* ▭ ABq. quare *e* AB est ῥ. Q. E. D.

a *hyp.* & *ſch.* 12. 10.
b *ſch.* 12. 10
c *hyp.*
d 17. 10.
e 11. *def.* 10

PROP. XLII.



Si duæ rectæ lineæ GH, HK *potentia incommensurabiles componantur, quæ faciant & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur medium; incommensurabileque composito ex quadratis ipsarum; tota recta linea* GK *irrationalis erit: vocetur autem bina media potens*

Ad expositam FB ρ^v, fiant rectang. AF = GKq, & CF = GHq + HKq Quoniam GHq + HKq (GF) *a* est μν; latitudo CB *b* erit ρ^v. Item quia 2 rectang. GHK (*c* AD) *a* est μν, etiam AC *b* erit ρ^v. Porro quia rectang AD *a* ▭ CF, *d* atque AD. CF :: AC. CB, *e* erit AC ▭ CB *f* Quare AB est *g* ῥ. ergo rectang. AF, id est, GKq est ῥν. *h* proinde GK est ῥ. Q. E. D.

a *hyp.*
b 23. 18.
c 4. 2.
d 2. 6.
e 10. 10.
f 37. 10.
g *lem.* 38. 10.
h 11. *def.* 10

PROP.

PROP. XLIII.

Quæ ex binis nominibus AB, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Si fieri poteſt, binomium AB alibi in E ſecetur in alia nomina AE, EB. Liquet AB ſecari utrobique inæqualiter, quia AD $\square$ DB, & AE $\square$ EB.

Quoniam rectangula ADB, AEB *a* ſunt $\mu\alpha$;
a 37. 10. *a* & ſingula ADq, DBq, AEq, EBq ſunt $\rho^{c}\alpha$;
b ſch. 27. 10 *b* adeoque ADq + DBq, *b* & AEq + EBq etiam $\rho^{c}\alpha$, *b* idcirco ADq + DBq —: AEq + EBq.
c ſch. 5. 2. *c* hoc eſt, 2 AEB — 2 ADB eſt $\rho^{c}\nu$. *d* ergo AEB
d ſch. 12. 10 — ADB $\rho^{c}\nu$. ergo $\mu\nu$ ſuperat $\mu\nu$ per $\rho^{c}\nu$. *e* Q.E.A.
e 27. 10.

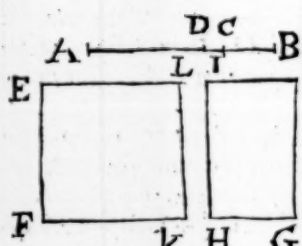
PROP. XLIV.

Quæ ex binis mediis prima AB, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Puta AB dividi in alia nomina AE, EB. quo
a 38. 10. poſito, ſingula ADq, DBq, EBq, *a* ſunt $\mu\alpha$; *a* &
b ſch. 27. 10 rectangula ADB, AEB, eorumque dupla, ſunt
c ſch. 5. 2. $\rho^{c}\alpha$. *b* ergo 2 AEB — 2 ADB, *c* hoc eſt ADq
d 27. 10. + DBq —: AEq + EBq eſt $\rho^{c}\nu$. *d* Q.E.A.

PROP.

PROP. XLV.



Quæ ex binis mediis secunda AB, *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Dic alia esse nomina AD, DB. Ad expositam EF ῥ, fac rectang. EG = ABq. & EH = ACq + CBq; item EK = ADq + DBq.

Quoniam ACq, CBq *a* sunt μα ⊏; *b* erit ACq + CBq (EH) μν. *c* ergo latitudo FH est ῥ *a* quin & rectang. ACB, *d* ideoque 2 ACB *e* (IG) est μν: *c* ergo HG, est etiam ῥ. Cum igitur EH *f* ⊏ IG, *g* atque EH. IG :: FH. HG; *h* erunt FH, HG ⊏. *k* ergo FG est binomium; cujus nomina FH, HG. Simili argumento FG est bin. cujus nomina FK, KG, contra 43. hujus.

a 39. 10.
b 16. & 24.
c 23. 10.
d 24. 10.
e 4. 2.
f *lem.* 26. 10
g 1. 6.
h 10. 10.
k 37. 10.

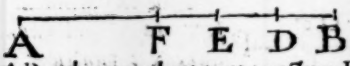
PROP. XLVI.

Major AB *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

Concipe alia nomina AE, EB. quo posito rectangula ADB, AEB *a* μα; *a* & tam ADq + DBq, quam AEq + EBq sunt ῥα. *b* erg. ADq + DBq — : AEq + EBq, *c* hoc est, 2 AEB — 2 ADB est ῥν. *d* Q. F. N.

a 40. 10.
b *sch.* 27. 10
c *sch.* 5. 2.
d 27. 10.

PROP. XLVII.

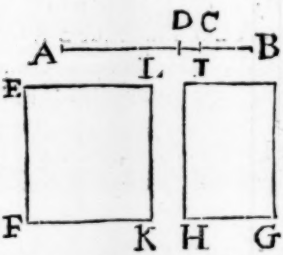


Rationale ac medium potens AB, *ad unum duntaxat punctum* D *dividitur in nomina* AD, DB.

a 41. 10. Dic alia nomina AE, EB. *a* ergo tam AEq + EBq, quam ADq + DBq sunt $\mu\alpha$. *a* & rectangula AEB, ADB, sunt $\rho^c\alpha$. *b* ergo 2 AEB — 2 ADB, *c* hoc est, ADq + DBq —: AEq + EBq est $\rho^c\nu$. Q. E. A.

b *sch.* 27. 10.
c *sch.* 5. 2.
d 27. 10.

PROP. XLVIII.



Bina media potens AB, *ad unum duntaxat punctum* C *dividitur in nomina* AC, CB.

Vis AB dividi in alia nomina AD, DB. Ad expositam EF ρ^c, fiant rectang. EG = ABq, & EH = ACq + CBq, & EK = ADq + DBq. Quoniam ACq + CBq, nempe EH, *a* est $\mu\nu$, *b* erit latitudo FH ρ^c. Item quia 2 ACB, *c* hoc est, IG, est *a* $\mu\nu$, *b* erit HG etiam ρ^c. Ergo cum EH *a* ⊏⊐ IG, sitque EH. IG *d* :: FH. HG, *e* erit FH ⊏⊐ HG. *f* ergo FG est bin. cujus nomina FH. HG. Eodem modo ejusdem nomina erunt FK, KG; contra 43 hujus.

a 42. 10.
b 23. 10.
c 4. 2.
d 1. 6.
e 10. 10.
f 37. 10.

Definitiones secundæ.

EXposita rationali, & quæ ex binis nominibus, divisa in nomina; cujus majus nomen plus possit quam minus, quadrato rectæ lineæ sibi longitudine commensurabilis.

I. Siquidem majus nomen expositæ rationali com-

commenſurabile ſit longitudine, vocetur tota ex binis nominibus prima.

II. Si vero minus nomen expoſitæ rationali longitudine ſit commenſurabile, vocetur ex binis nominibus ſecunda.

III. Quod ſi neutrum ipſorum nominum ſit longitudine commenſurabile expoſitæ rationali, vocetur ex binis nominibus tertia.

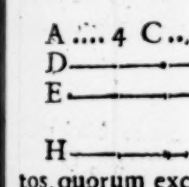
Rurſus, ſi majus nomen plus poſſit quam minus, quadrato rectæ lineæ ſibi longitudine incommenſurabilis;

IV. Si quidem majus nomen expoſitæ rationali commenſurabile ſit longitudine, vocetur ex binis nominibus quarta.

V. Si vero minus nomen, vocetur quinta.

VI. Quod ſi neutrum ipſorum nominum, vocetur ſexta.

PROP. XLIX.



Invenire ex binis nominibus primam, EG.

a Sume AB, AC numeros quadratos, quorum exceſſus CB non Q. exponatur D ῥ. *b* accipe quamvis EF ▭ D. *c* fac AB. CB :: EFq. FGq. erit EG bin. 1.

Nam EF *d* ▭ D. *e* ergo EF ῥ. *f* item EFq ▭ FGq. *g* ergo FG eſt etiam ῥ. item *d* quia EFq. FGq :: AB. CB :: Q. non Q. *b* erit EF ▭ FG. denique quia per converſionem rationis EFq. EFq — FGq :: AB. AC :: Q. Q. *k* erit EF ▭ √ EFq — FGq. *l* ergo EG eſt bin. 1. Q. E. F.

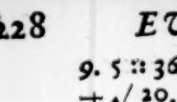
a ſch. 29. 10.
b 2. *lem.* 10. 10.
c 3. *lem.* 10. 10.
d *conſtr.*
e 6. *def.* 10.
f 6. 10.
g ſch. 12. 10.
h 9. 10.
k 9. 10.
l 1. *def.* 48. 10.

Explicatio per numeros.

Sit D, 8. EF, 6. AB, 9. CB, 5. quare cum

9. 5 :: 36. 20. erit FG, $\sqrt{20}$. proinde EG est $6 + \sqrt{20}$.

PROP. L.



Invenire ex binis nominibus secundam, EG. Accipe AB, & AC numeros quadratos, quorum excessus CB sit non Q. Sit D exposita ρ^e. sume FG $\square$ D. Fac CB. AB :: FGq. EFq. Erit EG quæsita.

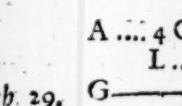
Probatur præcedentem.

Nam FG $\square$ D, quare FG est ρ^e. item EFq $\square$ FGq. ergo EF est etiam ρ. item quia FGq. EFq :: CB. AB :: non Q. Q. est FG $\square$ EF. denique quia CB. AB :: FGq. EFq, inverseque AB. CB :: EFq. FGq, erit ut in præcedenti, EF $\square$ $\sqrt{}$ EFq — FGq. *a* è quibus EG est bin. 2. Q. E. F.

a 2. def. 48. 10.

In numeris, sit D, 8; FG, 10; AB, 9; CB, 5. erit EF, $\sqrt{180}$. quare EG est $10 + \sqrt{180}$.

PROP. LI.



Invenire ex binis nominibus tertiam, DF. *a* Sume numeros AB, AC quadratos, quorum excessus CB non Q. sitq; L numerus non Q, proxime major quam CB, nempe unitate, vel binario. sit G exposita ρ. *b* Fac L. AB :: Gq. DEq. *b* & AB. CB :: DEq. EFq. erit DF bin 3.

a 9. 10.
b 3. lem 10 10.
c constr. 6. 10.
d sch. 12. 10
e 6. 10.
f 9. 10.

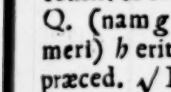
Nam quia DEq *c* $\square$ Gq, *d* est DE ρ. item Gq. DEq :: L. AB :: non Q. Q. *e* ergo G $\square$ DE. item quia DEq *e* $\square$ EFq, *d* etiam EF est ρ quinetiam quia DEq. EFq :: AB. CB :: non Q. *f* est DE $\square$ EF. porro, quia per constr.

conſtr. & ex æquali Gq. EFq :: L. CB :: non Q. Q. (nam *g* L, & CB non ſunt ſimiles plani numeri) *h* erit G etiam ⌐ EF. denique ut in præced. √ DEq — EFq ⌐ DE. *k* ergo DF eſt bin. 3. Q. E. F.

g *ſch.* 27. 8.
h 9. 10.
k 3. *def.* 48. 10.

In numeris, ſit AB, 9; CB, 5; L, 6; G, 8. erit DE, √ 96 & EF, √ $\frac{480}{9}$. quare DF = √ 96 + √ $\frac{480}{9}$.

PROP. LII.



Invenire ex binis nominibus quartam, DF.

a Sume quemvis numerum quadratum AB, *a* quę divide in AC, CB non quadratos. ſit G expoſita ρ^c. *b* accipe DE ⌐ G. *c* Fac AB. CB :: DEq. EFq. erit DF bin. 4.

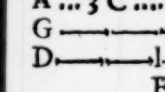
a *ſch.* 29. 10
b 2. *lem.* 10
c 3. *lem.* 10.

Nam ut in 49. hujus, DF oſtendetur bin. item, quia per conſtr. & converſionem rationis DEq. DEq — EFq :: AB. AC :: Q. non Q. *d* erit DE ⌐ √ DEq — EFq. *e* ergo DF eſt bin. 4. Q. E. F.

d 9. 10.
e 4. *def.* 48 10.

In numeris, ſit G, 8; DE, 6. erit EF √ 24. ergo DF eſt 6 + √ 24.

PROP. LIII.



Invenire ex binis nominibus quintam, DF.

Accipe quemvis numerum quadratum AB, cujus ſegmenta AC, CB ſint non Q. ſit G expoſita ρ^c. ſume EF ⌐ G. fac CB. AB :: EFq. DEq. erit DF bin. 5.

Nam ut in 50. hujus, erit DF bin. & quia per conſtr. & invertendo DEq. EFq :: AB. CB, ideoque per converſionem rationis DEq. DEq — EFq :: AB. AC :: Q. non Q. *a* erit

a 9. 10.
b 5. *def.* 48 10.

DE ⊏ √ DEq — EFq. *b* ergo DF est bin. 4. Q. E. F.

In numeris, sit G, 7; EF, 6. erit DE √ 54. quare DF est 6 + √ 54.

PROP. LIV.

A ..... 5 C ..... 7 B
L 9
G ——————
D ——————— F
E
H ——————

Invenire ex binis nominibus sextam.

Accipe AC, CB primos numeros utcunque, sic ut AC + CB (AB) sit non Q. sume etiam quemvis L num. Q. sit G expol. ꝑ. *a* fiatque L. AB :: Gq. DEq. atque AB. CB :: DEq. EFq. erit DF. bin. 6.

a 3. *lem.* 10. 10.

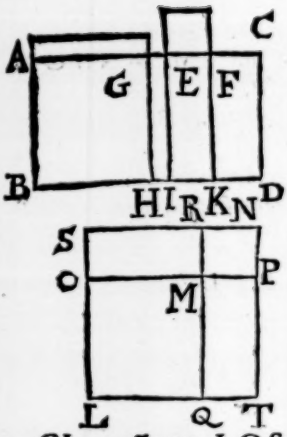
Nam ut in 51. hujus, DF ostendetur bin. item quod DE, & EF ⊏ G. denique igitur quia per constr. & conversionem rationis DEq. DEq — EFq :: AB. AC :: non Q. Q. (Nam AB primus est ad AC, *b* ideoque ei dissimilis) *c* ergo DE ⊏ √ DEq — EFq. *d* ergo DF est bin. 6. Q. E. F.

b *sch* 27. 8. *c* 9. 10. *d* 6. *def.* 48. 10.

In numeris, sit G, 6; DE √ 48. erit EF √ 28. quare DF est √ 48 + √ 28.

LEM-

LEMMA.



Sit AD *rectangulum, cujus latus* AC *ſecetur inæqualiter in* E; *biſectumq;* *ſit ſegmentum minus* EC *in* F; *atque ad* AE, *a fiat rectang.* AGE = EFq; *perque* G, E, F *b ducantur ad* AB *parallela* GH, EI, FK. *c Fiat autem quadratum* LM = *rectang.* AH, *atque ad* OMP *productam c fiat quadratum* MN = GI; *rectæque* LOS, LQT, NRS, NPT *producantur.*

a 28. 6.
b 31. 1.
c 14. 2.

Dico 1. MS, MT *ſunt rectangula.* Nam ob quadratorum angulos OMQ, RMP rectos, *a* erit QMR recta linea. *b* ergo anguli RMO, QMP recti ſunt. quare Pgra MS, MT ſunt rectangula.

a *ſch.* 15. 1.
b 13. 1.

2. *Hinc patet* LS *e* = LT; *& proinde* LN *eſſe quadratum.*

c 2. *ax.* 2.

3. *Rectangula* SM, MT, EK, FD *æqualia ſunt.* Nam quia rectang. AGE *d* = EFq, *e* erit AE. EF :: EF. GE. *f* ideoque AH. EK :: EK. GI. hoc eſt per contr. LM. EK :: EK. MN. *g* verum LM. SM :: SM. MN. ergo EK *h* = SM *k* = FD *l* = MT.

d *hyp.*
e 17. 6.
f 1. 6.
g *ſch.* 22 6.
h 9. 5.
k 36. 1.
l 43. 1.

4. *Hinc* LN *m* = AD.

m 2. *ax.* 1.

5. *Quia* EC *biſecta eſt in* F, *n patet* EF, FC, EC ▭ *eſſe.*

n 16. 10.

6. Si AE ▭ EC, & AE ▭ √ AEq − ECq, *o* erunt AG, GE, AE ▭. item, quia

o 18. & 16. 10.

p 10. 10. AG. GE :: AH. GI *p* erunt AH, GI; hoc eſt LM, MN □. item iiſdem poſitis,

7. OM □ MP. Nam per Hyp. AE, □ EC, *q* ergo EC □ GE. *q* quare EF □ GE.

q 14. 10. r 10. 10. ſed EF. GE :: EK. GI. *r* ergo EK □ GI, hoc eſt SM □ MN. atqui SM. MN :: OM. MP. *r* ergo OM □ MP.

8. Sin ponatur AE □ $\sqrt{AEq - ECq}$, ſ patet AG, GE, AE eſſe □. unde LM □ MN. nam AG. GE :: AH. GI :: LM. MN.

ſ 19. & 17. 10.

His bene perſpectis, facile ſex ſequentes Propoſitiones expediemus.

PROP. LV.

Si ſpatium AD *contineatur ſub rationali* AB, *& ex binis nominibus prima* AC, (AE + EC;) *recta linea* OP *ſpatium potens irrationalis eſt, quæ ex binis nominibus appellatur.*

Suppoſitis iis, quæ in lemmate proxime præcedenti deſcripta, & demonſtrata ſunt, liquet rectam OP poſſe ſpatium AD. *a* item AG, GE, AE ſunt □. ergo cum AE *b* ſit ρ^c □ AB, *c* erunt AG, & GE, ρ^c □ AB. *d* ergo rectangula AH, GI, hoc eſt quadrata LM, MN ſunt $\rho^c a$. ergo OM, MP ſunt $\rho^c e$ □. *f* proinde OP eſt bin. Q. E. D.

a *hyp.* & *lem.* 54. 10
b *hyp.*
c ſch. 12. 10
d 20. 10.
e *lem.* 54. 10.

In numeris, ſit AB, 5; AC, $4 + \sqrt{12}$. quare rectang. AD = $20 + \sqrt{300}$ = quadr. LN. ergo OP eſt $\sqrt{15} + \sqrt{5}$; nempe bin. 6.

f 37. 10.

PROP.

PROP. LVI.

Si ſpatium AD *contineatur ſub rationali* AB, *& ex binis nominibus ſecunda* AC (AE+EC;) *recta linea* OP *ſpatium* AD *potens, irrationalis eſt, quæ ex binis mediis prima appellatur.*

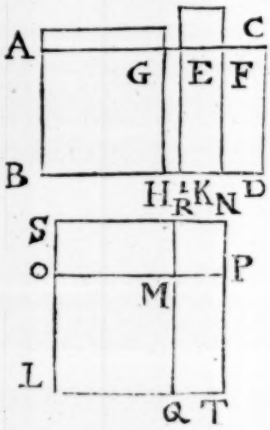
Rurſus adhibito lemmate ad 54. hujus, erit OP = √AD. *a* item AE, AG, GE ſunt □. ergo quum AE *b* ſit ρ, □ AB, *c* erunt AG, GE etiam ρ^c □ AB. ergo rectangula AH, GI; hoc eſt OMq, MPq *d* ſunt μα. *e* quinetiam OM □ MP. denique EF □ EC, & EC *f* □ AB. *f* quare EF eſt ρ^c □ AB. *g* ergo EK; hoc eſt SM, vel OMP eſt ρ^c v. *h* Proinde OP eſt 2 μ prima. Q. E. D.

In numeris, ſit AB, 5; & AC, √48: +6. ergo rectang. AD = √1200: +30 = OPq. ergo OP eſt *v*√675+*v*√75; nempe bimed. 1.

Vide Schem. 57.

a *hyp.* & *lem.* 54. 10
b *hyp.*
c *ſch.* 12. 10
d 22. 10.
e *lem.* 54. 10.
f *hyp.* 12. 10.
g 20. 10.
h 38. 10.

PROP. LVII.



Si ſpatium AD *contineatur ſub rationali* AB, *& ex binis nominibus tertia* AC (AE+EC;) *recta linea* OP *ſpatium* AD *potens, irrationalis eſt, quæ ex binis mediis ſecunda dicitur.*

Ut prius, OPq = AD. item rectangula AH, GI, hoc eſt OMq, MPq ſunt μα. *a* item EK, vel OMP eſt μv. *b* ergo OP eſt bimed. 2.

a *hyp.* & 22. 10.
b 39. 10.

In

In numeris, sit AB,5; AC,$\sqrt{32}+\sqrt{24}$.quare AD est $\sqrt{800}+\sqrt{600}$=OPq.proinde OP est $v\sqrt{450}+v\sqrt{50}$; hoc est bimed. 2.

PROP. LVIII.

Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus quarta* AC (AE+EC;) *recta linea* OP *spatium potens, irrationalis est, quæ vocatur major.*

Nam iterum OMq *a* □ MPq.rectang. vero AI, hoc est OMq+MPq *b* est ρ′*v*. *c* item EK, vel OMP est μ*v*. *d* ergo OP ($\sqrt{AD}$) est major. Q. E. D.

a *lem.* 54. 10.
b *hyp. &* 20. 10.
c *hyp. &* 22. 10.
d 40. 10.

In numeris, sit AB 5; & AC, $4+\sqrt{8}$. ergo rectang. AD est $20+\sqrt{200}$. quare OP est $\sqrt{:}\ 20+\sqrt{200}$.

PROP. LIX.

Si spatium AD *contineatur sub rationali* AB, *& ex binis nominibus quinta* AC; *recta linea* OP *spatium* AD *potens, irrationalis est, quæ rationale & medium potens appellatur.*

Rursus OMP □ MPq. rectang. vero AI, vel OMq+MPq est μ*v*. *a* item rectang. EK, vel OMP est ρ′*v*. *b* ergo OP ($\sqrt{AD}$) est potens ρ′*v*, & μ*v*. Q. E. D.

a *ut in præc.*
b 41. 10.

In numeris, sit AB, 5; & AC $2+\sqrt{8}$. ergo rectang. AD $=10+\sqrt{200}$=OPq.quare OP est $\sqrt{:}\ 10+\sqrt{200}$.

PROP.

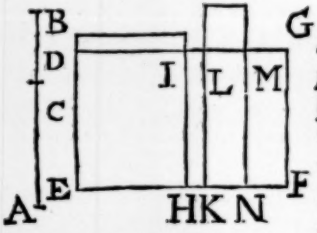
PROP. LX.

Si ſpatium AD *contineatur ſub rationali* AB, *& ex binis nominibus ſexta* BC (AE + EC;) *recta linea* OP *ſpatium* AD *potens, irrationalis eſt, quæ bina media potens appellatur.*

Ut ſæpe prius, OMq ▭ MPq. & OMq + MPq eſt $\mu\nu$. & rectang (EK) OMP etiam $\mu\nu$. *a* ergo OP = $\sqrt{}$ AD eſt potens $2\mu\alpha$. Q. E. D. a 42. 10.

In numeris, ſit AB, 5; AC, $\sqrt{12} + \sqrt{8}$; ergo rectang. AD, vel OPq eſt $\sqrt{300} + \sqrt{200}$. proinde OP eſt $\sqrt{}$: $\sqrt{300} + \sqrt{200}$.

LEMMA.



Sit recta AB *inæqualiter ſecta in* C, *ſitque* AC *majus ſegmentum; & cuivis* DE *applicentur rectangula,* DF = ABq, & DH = ACq, & IK = CBq. *ſitque* LG *biſecta in* M, *ducaturque* MN *parall.* GF.

Dico 1. *Rectang.* ACB = LN, vel MF. *a* Nam 2 ACB = LF. a 4. 2. & 3.

2. DL ▭ LG. nam DK (ACq + CBq) *ax.* 1. *b* ▭ LF (2 ACB) ergo cum DK, LF ſint æque b 7. 2. alta, *c* erit DL ▭ LG. c 1. 6.

3. Si AC ⊓ CB, *d* erit rectang. DK ▭ d 16. 10. ACq, & CBq.

4 *Item,* DL ▭ LG. nam ACq + CBq *e* ▭ 2 ACB: hoc eſt DK ▭ LF. ſed DK. LF *e* :: DL. LG. *f* ergo DL ▭ LG. e *lem.* 26. 10.

5. *Ad hæc,* DL ▭ $\sqrt{}$ DLq − LGq. Nam ACq. ACB *g* :: ACB. CBq. hoc eſt DH. LN :: f 10. 10. g 1. 6.

LN :: LN. IK. *c* quare DI. LM :: LM. IL.
h 17. 6. *b* ergo DI × IL = LMq. ergo cum ACq *k* □
k hyp. CBq. hoc est DH □ IK, & *l* proinde DI □
l 10. 10. IL, *m* erit DL □ √DLq—LGq. Q. E. D.
m 18. 10. 6. *Sin ponatur* ACq □ CBq, *n erit* DL □
n 19. 10. √DLq—LGq.

Hoc lemma præparationis vicem ſubeat pro 6. ſequentibus propoſitionibus.

PROP. LXI.

Quadratum ejus quæ ex binis nominibus (AC +CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus primam.*

Suppoſitis iis, quæ in lemmate proxime antecedenti deſcripta & demonſtrata ſunt. Quoniam AC, CB *a* ſunt $\dot{\rho}$ □, *b* erit rectang. DK
a hyp. b lem. 60. □ ACq; *c* ergo DK eſt $\dot{\rho}\nu$. *d* ergo DL
10. DE $\dot{\rho}$. rectang. vero ACB, ideoque 2 ACB
c ſch. 12. 10 (LF) *e* eſt $\mu\nu$. *f* ergo latitudo LG eſt $\dot{\rho}$ □
d 21. 10. DE *g* ergo etiam DL □ LG. *h* item DL □
e 22. & 24. √DLq—LGq. ex quibus *k* ſequitur DG eſſe
10. bin. 1. Q. E. D.
f 23. 10.
g 13. 10.

PROP. LXII.

h lem. 60. *Quadratum ejus, quæ ex binis mediis prima*
10. (AC+CB) *ad rationalem* DE *applicatum, fa-*
k 1. def 48. *cit latitudinem* DG *ex binis nominibus ſecundam.*
10. Rurſus adhibito lemmate proxime præce-
a 24. 10. denti; Rectang. DK □ ACq. *a* ergo DK eſt
b 23. 10. $\mu\nu$ *b* ergo latitudo DK eſt $\dot{\rho}$ □ DE. Quia ve-
c hyp. & ro rectang. ACB, ideoque LF (2 ACB)
ſch. 12 10. *c* eſt $\dot{\rho}\nu$, *d* erit LG $\dot{\rho}$ □ DE. *e* ergo DL,
d 21. 10. LG ſunt □. *f* item DL □ √DLq—
e 13. 10. LGq. *g* ex quibus patet DG eſſe bin. 2. Q. E. D.
f lem. 60.
10.
g 2. def. 48.
81.

PROP.

PROP. LXIII.

Quadratum ejus, quæ ex binis mediis ſecunda (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus tertiam.*

Ut in præced. DL eſt ρ ▭ DE. porro quia rectang. ACB, ideoque LF (2 ACB) *a* eſt $\mu\nu$, *b* erit LG ρ ▭ DE. *c* quinetiam DL ▭ LG. *c* itemque DL ▭ √ DLq — LGq. *d* ergo DG eſt bin. 3. Q. E. D.

a *hyp* & 24. 10.
b 23. 10.
c *lem.* 60. 10.
d 3. *def.* 48. 10.

PROP. LXIV.

Quadratum Majoris (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus quartam.*

Rurſus ACq + CBq, hoc eſt DK *a* eſt $\rho^e\nu$. *b* ergo DL eſt ρ^e ▭ DE. item ACB, ideoque LF (2 ACB) *c* eſt $\mu\nu$. *d* ergo LG eſt ρ^e ▭ DE. *e* proinde etiam DL ▭ LG. denique quia AC ▭ BC, *f* erit DL ▭ DLq — LGq. *g* unde DG. eſt bin. 4. Q. E. D.

a *hyp.* & *ſch.* 12. 10.
b 21. 10.
c *hyp.* & 24. 10.
d 23. 10.
e 13. 10.
f *lem.* 60. 10.
g 4. *def.* 48. 10.

PROP. LXV.

Quadratum ejus, quæ rationale ac medium poteſt, (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus quintam.*

Iterum, DK eſt $\mu\nu$. *a* ergo DL eſt ρ ▭ DE. item LF eſt $\rho^e\nu$. *b* ergo LG eſt ρ^e ▭ DE. *c* ergo DL ▭ LG. *d* item DL ▭ √ DLq — LGq. *e* proinde DG eſt bin. 5.

a 13. 10.
b 21. 10.
c 13. 10.
d *lem.* 60. 10.
e 5. *def.* 48. 10.

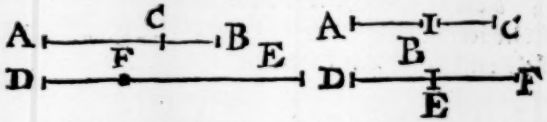
PROP. LXVI.

Quadratum ejus, quæ bina media poteſt (AC + CB) *ad rationalem* DE *applicatum, facit latitudinem* DG *ex binis nominibus ſextam.*

Ut

a *hyp.* Ut prius, DL & LG sunt p^e ⊏⊐ DE.
b 14. 10. Quia vero ACq + CBq (DK) *a* ⊏⊐ ACB,
c 1. 6. *b* ideoque DK ⊏⊐ LF (2 ACB) estque DK.
d 10. 10. LF *c* :: DL. LG. *d* erit DL ⊏⊐ LG *e* denique
e *lem.* 60.10 DL ⊏⊐ $\sqrt{}$ DLq — LGq. *f* ex quibus liquet
f 6. *def.* 48. DG esse bin. 6. Q. E. D.
10.

LEMMA.



Sint AB, DE ⊏⊐; *fiatque* AB. DE :: AC DF.

Dico 1. AC ⊏⊐ DF. ut patet ex 10. 10.
a 19. 5. item CB ⊏⊐ FE. *a* quia AB. DE :: CB. FE.

2. AC. CB :: DF. FE. Nam AC. DF :: AB. DE :: CB. FE. ergo permutando AC. CB :: DF. FE.

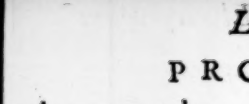
3. *Rectang.* ACB ⊏⊐ DFE. Nam ACq.
b 1. 6. ACB *b* :: AC. CB *c* :: DF. EF :: DFq. DFE.
c *prius.* quare permutando ACq. DFq :: ACB. DFE.
d 10. 10. ergo cum ACq ⊏⊐ DFq, *d* erit ACB ⊏⊐ DFE.

4. ACq + CBq ⊏⊐ DFq + FEq. Nam
e 22. 6. quia ACq. CBq *c* :: DFq. FEq. erit componendo ACq+CBq. CBq :: DFq + FEq. FEq. ergo
f 10.10. cum CBq ⊏⊐ FEq, *f* erit ACq + CBq ⊏⊐ DFq + FEq.

g 10. 10. 5. Hinc, si AC ⊐⊐, vel ⊐⊐ CB, *g* erit pariter DE ⊐⊐, vel ⊐⊐ EF.

PROP.

PROP. LXVII.



Ei, quæ ex binis nominibus (AC + CB) *longitudine commensurabilis* DE, *& ipsa ex binis nominibus est, atque ordine eadem.*

Fac AB. DE :: AC. DF. *a* sunt AC, DF '□; *a* & CB, FE '□. quare cum AC, & CB *b* sint ρ^c '□, *c* erunt DF, FE ρ^c '□. ergo DE est etiam bin. Quia vero AC. CB *a* :: DF. FE. si AC '□, vel □ √ ACq — BCq, *d* etiam similiter DF □, vel □ √ DFq — FEq. item si AC '□, vel □ ρ^c exposi. *e* erit similiter DF '□, vel □ ρ^c exposi. at si CB '□ vel □ ρ^c, *e* erit pariter FE '□ vel □ ρ^c. Sin vero utraque AC, CB □ ρ^c, erit utraque etiam DF, FE '□ ρ^c. *g* Hoc est, quodcunque binomium fuerit AB, erit DE ejusdem ordinis. Q. E. D.

a *lem.* 66. 10.
b *hyp.*
c *lem.* 66. 10. *& scb.*
d 12. 10.
e 15. 10.
e 12. 10. & 14. 10.
g *Per def.* 48. 10.

PROP. LXVIII.

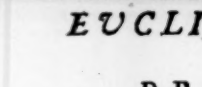
Ei, quæ ex binis mediis (AC + CB) *longitudine commensurabilis* DE, *& ipsa ex binis mediis est, atque ordine eadem.*

a Fiat AB. DE :: AC. DF. *b* ergo AC □ DF, & CB □ FE. ergo cum AC & CB *c* sint μ, *d* etiam DF, & FE erunt μ. & cum AC *c* '□ CB, *e* erit FD '□ FE. *f* ergo DE est 2 μ. Si igitur rectang. ACB sit ρ^cν, quia DFE *b* □ ACB, *g* etiam DFE est ρ^cν; & si illud μν; *h* hoc etiam erit μν. *k* Id est, sive AB fit bimed. 1. sive bimed. 2. erit DF ejusdem ordinis. Q. E. D.

a 12. 6.
b *lem.* 66. 10.
c *hyp.*
d 24. 10.
e 10. 10.
f 38. 10.
g *scb.* 12. 10
h 14. 10.
k 38. vel 39. 10.

PROP.

PROP. LXIX.



Majori (AC + CB) *commenſurabilis* DE, & *ipſa major eſt.*

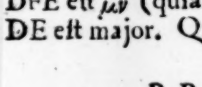
Fac AB. DE :: AC. DF. Quoniam AC
a *byp.* *a* ⊐ CB, *b* erit DF ⊐ FE. item A Cq +
b *lem.* 66. CBq *a* eſt $\rho^c v$; proinde cum DFq + FEq *b* ⊏
10. ACq + CBq, *c* etiam DFq + FEq eſt $\rho^c v$. de-
c *ſch.* 12. 10 nique rectang. ACB *a* eſt μv. *d* ergo rectang.
d 24. 10. DFE eſt μv (quia DFE *b* ⊏ ACB.) *e* Quare
e 40. 10. DE eſt major. Q. E. D.

PROP. LXX.

Rationale ac medium potenti (A C + C B) *commenſurabilis* DE, *& ipſa rationale ac medium potens eſt.*

Iterum fac AB. DE :: AC. DF. Quia AC
a *byp.* *a* ⊐ CB, *b* etiam D F ⊐ F E. item quia
b *lem.* 66. A Cq + C Bq *a* eſt μv, *c* erit D Fq + F Eq μv.
10. denique quia rectang. ACB *c* eſt $\rho^c v$, *d* etiam
c 24 10 DFE eſt $\rho^c v$. *e* ergo D E eſt potens $\rho^c v$, ac μv.
d *ſch.* 12. 10 Q. E. D.
e 41. 10.

PROP. LXXI.



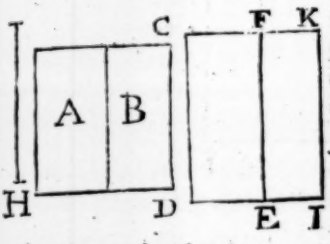
Bina media potenti (AC + CB) *commenſurabilis* DE, *& ipſa bina media potens eſt.*

Divide DE, ut in præced. Quia ACq *a* ⊏
b *lem.* 66. CBq, *b* erit DFq ⊏ FEq. item quia A Cq
10. + CBq *a* eſt μv, *c* erit DFq + FEq etiam μv.
c 24. 10. pariterque quia ACB *a* eſt μv. *d* etiam DFE eſt
d 24. 10. μv. denique quia A Cq + C Bq ⊏ A C B,
e erit

e erit DFq + FEq ▭ DFE. *f* è quibus ſequitur DE eſſe potentem 2 $\mu\alpha$. Q. E. D.

e 14. 10.
f 42. 10.

PROP. LXXII.

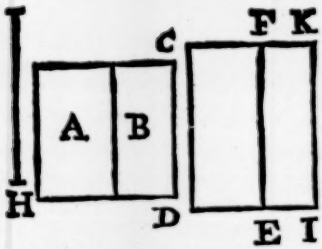


Si rationale A, *& medium* B *componantur, quatuor irrationales fiunt; vel ea quæ ex binis nominibus, vel quæ ex binis mediis prima, vel major, vel rationale ac medium potens.*

Nimirum ſi Hq = A + B, erit H una 4 linearum, quas theorema deſignat. Nam ad CD expoſitum ρ, *a* fiat rectang. CE = A; item FI = B; *b* ideoque CI = Hq. Quoniam igitur A eſt $\rho\nu$, etiam CE eſt $\rho\nu$. *c* ergo latitudo CF eſt ρ ▭ CD. & quia B eſt $\mu\nu$, erit FI $\mu\nu$. *d* ergo FK eſt ρ' ▭ CD. *e* ergo CF, FK ſunt ρ' ▭. Tota igitur CK *f* eſt bin. Si igitur A ⌐ B, hoc eſt CE ⌐ FI, *g* erit CF ⌐ FK. ergo ſi CF ▭ √CFq − FKq, *b* erit CK bin. 1. & proinde H = √CI *k* eſt bin. Si ponatur CF ▭ √CFq − FKq, *l* erit CK bin. 4 quare H (√CI) *m* eſt major. Sin A ¬ B; *n* erit CF ¬ FK; proinde ſi FK ▭ √FKq − CFq, *n* erit CK bin. 2. *o* quare H eſt 2 μ prima. denique ſi FK ▭ √FKq − CFq, *p* erit CK bin. 5. *q* unde H erit potens $\rho'\nu$ ac $\mu\nu$. Q. E. D.

a *cor.* 16. 6.
b 2. *ax.* 1.
c 21. 10.
d 23. 10.
e 13. 10.
f 37. 10.
g 1. 6.
h 1. *def.* 48. 10.
k 55. 10.
l 4. *def.* 48. 10.
m 58. 10.
n 2. *def.* 48. 10.
o 56. 10.
p 5. *def.* 48. 10.
q 59. 10.

PROP. LXXIII.



Si duo media A, B, *inter se incommensurabilia componantur, duæ reliquæ irrationales fiunt; vel ex binis mediis secunda, vel bina media potens.*

Nempe H potens A+B est una dictarum irrationalium. Nam ad CD expos. ρ, fac rectang. CE=A, & FI=B. unde Hq=CI. Quoniam igitur CE, & FI *a* sunt μα, *b* erunt latitudines CF, FK ρ ⊏⊐ CD. item quia CE ⊏⊐ FI; estque CE. FI *c* :: CF. FK, *d* erit CF ⊏⊐ FK *e* ergo CK est bin. 3. nempe, si CF ⊏⊐ √ CFq—FKq. unde H = √ CI *f* erit 2 μ 2a. Sin vero CF ⊏⊐ √ CFq—FKq, *g* erit CK bin. 6. & *b* proinde H est potens 2 μα. Q. E. D.

a *hyp.*
b 23. 10.
c 1. 6.
d 10. 10.
e 3. *def.* 48. 10.
f 57. 10.
g 6. *def.* 48. 10.
h 60. 10.

Principium Senariorum per detractionem.

PROP. LXXIV.

D E F

Si à rationali DF *rationalis* DE *auferatur, potentia tantum commensurabilis existens toti* DF; *reliqua* EF *irrationalis est: vocetur autem apotome.*

Nam EFq *a* ⊏⊐ DEq; sed DEq *b* est ρ ν. *c* ergo EF est ρ. Q. E. D.

In numeris, fit DF, 2; DE, √3. EF erit 2—√3.

a *lem.* 26. 10.
b *hyp.*
c 10. & 11. *def.* 10.

PROP.

PROP. LXXV.

D E F

Si à media DF *media* DE *auferatur, potentia tantum commensurabilis existens toti* DF, *quæ cum tota* DF *rationale contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome prima.*

Nam EFq *a* □ rectang. FDE. ergo cum FDE *b* sit $\rho^{c}v$, *c* erit EF $\dot{\rho}$. Q. E. D.

In numeris, sit DF $v\sqrt{54}$; & DE $v\sqrt{24}$. ergo EF est $v\sqrt{54} - v\sqrt{24}$.

a *sch.* 26. 10.
b *hyp.*
c 20. & 11. *def.* 10.

PROP. LXXVI.

D E F

Si à media DF *media* DE *auferatur, potentia tantum commensurabilis existens toti* DF, *quæ cum tota* DF *medium contineat; reliqua* EF *irrationalis est; vocetur autem mediæ apotome secunda.*

Quia DFq, & DEq, *a* sunt $\mu\alpha$ □, *b* erit DFq + DEq □ DEq *c* quare DFq + DEq est μv. item rectang. FDE, *e* ideoque 2 FDE *a* est μv. ergo EFq (*d* DFq + DEq − 2 FDE) *e* est $\dot{\rho}v$. quare EF est $\dot{\rho}$. Q. E. D.

a *hyp.*
b 16. 10.
c 24. 10.
d *cor.* 7. 2.
e 27. 10.

In numeris, sit DF, $v\sqrt{18}$; & DE, $v\sqrt{8}$. erit EF $v\sqrt{18} - v\sqrt{8}$.

PROP. LXXVII.

A B C

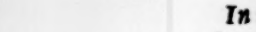
Si à recta linea AC *recta auferatur* AB, *potentia incommensurabilis existens toti* BC, *quæ cum tota* AC *faciat compositum quidem ex ipsarum quadratis rationale, quod autem sub ipsis continetur medium; reliqua* BC *irrationalis est: vocetur autem minor.*

Nam ACq + ABq *a* est $\rho^{c}v$. at rectang. ACB *a* est μv. *b* ergo 2 CAB □ ACq + ABq (*c* 2 CAB + BCq;) *d* ergo ACq + ABq □ BCq. *e* ergo BC est $\dot{\rho}$. Q. E. D.

a *hyp.*
b *sch.* 12. 10
c 7. 2.
d 17. 10.
e 11. *def.* 10

In numeris, sit AC, $\sqrt{:18+\sqrt{108}}$. AB $\sqrt{:18-\sqrt{108}}$. ergo BC est $\sqrt{:18+\sqrt{108}}-\sqrt{:18-\sqrt{108}}$.

PROP. LXXVIII.



Si à recta linea DF *recta auferatur* DE *potentia incommensurabilis existens toti* DF, *quæ cum tota* DF *faciat compositum quidem ex ipsarum quadratis medium, quod autem sub ipsis continetur, rationale; reliqua* EF *irrationalis est: vocetur autem cum rationali medium totum efficiens.*

a *hyp.* & *sch.* 12. 10. b *hyp.* c *sch.* 12. 10 d 7. 2. e *sch* 12. 10 & 11. *def.* 10.

Nam 2 FDE *a* est ρv. *b* & DFq + DEq est μv. *c* ergo 2 FDE $\sqcap$ DFq + DEq *d* (2 FDE + EFq) *c* ergo EF est $\dot{\rho}$. Q. E. D.

In numeris, sit DF, $\sqrt{:\sqrt{216}+\sqrt{72}}$. DE, $\sqrt{:\sqrt{216}-\sqrt{72}}$. ergo EF est $\sqrt{:\sqrt{216}+\sqrt{72}}-\sqrt{:\sqrt{216}-\sqrt{72}}$.

PROP. LXXIX.

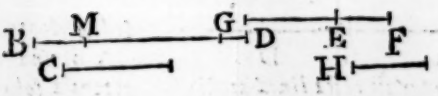
Si à recta DF *recta auferatur* DE, *potentia incommensurabilis existens toti* DF, *quæ cum tota faciat & compositum ex ipsarum quadratis, medium; & quod sub ipsis continetur, medium, incommensurabileque composito ex quadratis ipsarum, reliqua irrationalis est: vocetur autem cum medio medium totum efficiens.*

a *hyp.* & 24. 10. b 27 10. c *cor.* 7. 2. d 11 *def.* 10

Nam 2 FDE, & DFq + DEq *a* sunt $\mu\alpha$; *b* ergo EFq (*c* DFq + DEq − 2 FDE) est $\dot{\rho}v$. *d* proinde EF est $\dot{\rho}$. Q. E. D.

Exempl. gr. sit DF, $\sqrt{:\sqrt{180}+\sqrt{60}}$. DE, $\sqrt{:\sqrt{180}-\sqrt{60}}$. EF erit $\sqrt{:\sqrt{180}+\sqrt{60}}-\sqrt{:\sqrt{180}-\sqrt{60}}$.

LEMMA.



Si idem ſit exceſſus inter primam magnitudinem BG, *& ſecundam* C (MG) *qui inter tertiam magnitudinem* DF, *& quartam* H(EF;) *erit & viciſſim idem exceſſus inter primam magnitudinem* BG, *& tertiam* DF, *qui inter ſecundam* C, *& quartam* H.

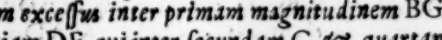
Nam quia *a* æqualibus BM, DE adjectæ ſunt inæquales MG, EF, *a* hoc eſt C, H; erit exceſſus totorum BG, DF, *b* æqualis exceſſui adjectorum, C, H. Q. E. D.

a *hyp.*
b 15. *ax.* 1.

Coroll.

Hinc, quatuor magnitudines Arithmetice proportionales, viciſſim erunt Arithmetice proportionales.

PROP. LXXX.



Apotomæ AB *una tantum congruit recta linea rationalis* BC, *potentia tantum commenſurabilis exiſtens toti* AB.

Si fieri poteſt, alia BD congruat. *a* ergo rectangula ACB, ADB; *b* ideoque eorum dupla ſunt $\mu\alpha$. cum igitur ACq + BCq — 2 ACB *c* = ABq *c* = ADq + DBq — 2 ADB, ergo viciſſim ACq + BCq —: ADq + BDq *d* = 2 ACB —: 2 ADB. Sed ACq + BCq —: ADq + BDq *e* eſt $\rho\nu$. *f* ergo 2 ACB —: 2 ADB eſt $\rho\nu$. Q. E. D.

a 22. 10.
b 24. 10.
c *cor.* 7. 2.
d *lem.* 79. 10.
e *hyp.* & 27. 10.
f *ſch.* 12. 10
g 27. 10.

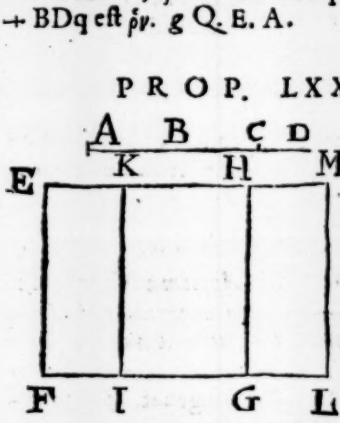
PROP. LXXXI.

Mediæ Apotomæ primæ AB *una tantum congruit recta linea media* BC, *potentia ſolum commenſurabilis exiſtens toti, & cum tota rationale continens.*

Dic etiam BD congruere. igitur quoniam tam ACq, & BCq; quam ADq, & BDq *a* ſunt $\mu\alpha$ ▭. *b* etiam ACq+BCq, & ADq+BDq erunt $\mu\alpha$ *c* ſed rectangula ACB, ADB; *d* adeoq; 2 ACB, & 2 ADB ſunt $\rho\alpha$. *e* ergo 2 ACB —: 2 ADB; *f* hoc eſt ACq+BCq—: ADq+BDq eſt $\rho\nu$. *g* Q. E. A.

a *hyp.*
b 16. & 24. 10.
c *hyp.*
d *ſch.* 12. 10.
e *ſch.* 27. 10.
f 7. 2. & *lem.* 79. 10.
g 27. 10.

PROP. LXXXII.



Mediæ Apotomæ ſecundæ AB *una tantum congruit recta linea media* BC, *potentia ſolum commenſurabilis exiſtens toti, & cum tota medium continens.*

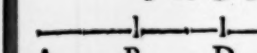
Si fieri poteſt, congruat alia BD. Ad EF ρ fiant rectang. EG=ACq+BCq; item rectang. EL=ADq+BDq. Item EI=ABq. Jam 2 ACB+ABq=ACq+BCq=EG, ergo cum EI=ABq, *a* erit KG=2 ACB. porro ACq, & BCq *b* ſunt $\mu\alpha$ ▭. *c* Ergo EG (ACq+BCq) eſt $\mu\nu$. *d* ergo latitudo EH ρ ▭ EF. *e* Quineriam rectang. ACB; *f* ideoque 2 ACB (KG) eſt $\mu\nu$. *d* ergo KH eſt etiam ρ ▭ EF. denique quia ACq+BCq, id eſt, EG, *g* ▭ 2 ACB (KG) eſtque EG.

a 4. 2. & 3. *ax.* 1.
b *hyp.*
c 24. 10.
d 23. 10.
e *hyp.*
f 24. 10.
g *lem.* 26. 10.

EG. KG :: *b* EH. KH *k* erit EH □ KH. *l* ergo EK eſt apotome, cujus congruens KH. ſimili argumento erit KM ejuſdem EK congruens; contra 80 hujus.

h 1. 6.
k 10. 10.
l 74. 10.

PROP. LXXXIII.

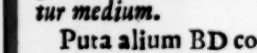


Minori AB, *una tantum congruit recta linea* (BC) *potentia incommenſurabilis exiſtens toti, & cum tota faciens compoſitum quidem ex ipſarum quadratis rationale; quod autem ſub ipſis continetur medium.*

Puta alium BD congruere. Cum igitur ACq +B Cq, & A Dq + B Dq *a* ſint ῥα, eorum exceſſus (2 *b* ACB —: 2 ADB) *c* eſt ῥν, *d* Q.E.A; quia ACB, & ADB ſunt μα per hypoth.

a *hyp.*
b *lem.* 97. 10.
c *ſch.* 27. 10
d 27. 10.

PROP. LXXXIV.

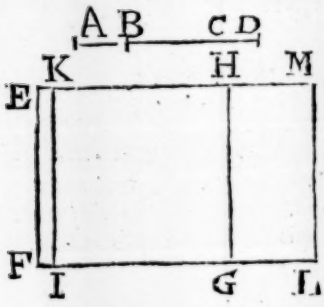


Ei (AB,) *quæ cum rationali medium totum facit, una tantum congruit recta linea* BC, *potentia incommenſurabilis exiſtens toti, & cum tota faciens compoſitum quidem ex ipſarum quadratis medium; quod autem ſub ipſis continetur, rationale.*

Dic aliam BD etiam congruere. *a* ergo rectangula ACB, ADB. *b* ideoque 2 ACB, & 2 ADB ſunt ῥα. ergo 2 ACB —: 2 ADB; *c* hoc eſt, A Cq + B Cq —: A Dq + B Dq *d* eſt ῥν. Q.E.A: quum A Cq + B Cq, & A Dq + BDq ſint μα per hypoth.

a *hyp.*
b *ſch.* 12. 10
c *lem.* 79.
d *ſch.* 27. 10

PROP. LXXXV.



Ei (AB,) *quæ cum medio medium totum facit una tantum congruit recta linea* BC *potentia incommensurabilis existens toti, & cum tota faciens & compositum ex ipsarum quadratis medium, & quod sub ipsis continetur, medium, incommensurabileque composito ex ipsarum quadratis.*

Suppositis iis quæ facta & ostensa sunt in 82 hujus; liquet EH, & KH esse ρ^{t} $\square$. EF. Porro igitur quia ACq+CBq, hoc est, rectang. EG
a *hyp.* *a* $\square$. ACB, *b* ideoque EG $\square$. 2 ACB (KG)
b 14. 10. estque EG. KG :: *c* EH. KH; erit EH $\square$
c 1. 6. KH. ergo EK est apotome, cujus congruens KH. Haud aliter KM eidem apotomæ EK. congruere ostendetur; contra 80 hujus.

Definitiones tertiæ.

EXposita rationali, & apotoma, si tota plus possit quam congruens quadrato rectæ lineæ sibi longitudine commensurabilis;

I. Si quidem tota expositæ rationali longitudine sit commensurabilis, vocetur apotome prima.

II. Si vero congruens expositæ rationali longitudine sit commensurabilis, vocetur apotome secunda.

III. Quod si neque tota, neque congruens expositæ rationali sit longitudine commensurabilis, vocetur apotome tertia.

Rur-

Rurſus, ſi tota plus poſſit quam congruens quadrato rectæ ſibi longitudine incommenſurabilis;

IV. Si quidem tota expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome quarta.

V. Si vero congruens expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome quinta.

VI. Quod ſi neque tota, neque congruens, expoſitæ rationali ſit longitudine commenſurabilis, vocetur apotome ſexta.

PROP. LXXXVI, 87, 88, 89, 90, 91.

A....4 C.....5 B
D———————
E———————F
G
H———

Invenire apotomen primam, ſecundam, tertiam, quartam, quintam, ſextam.

Apotomæ inveniuntur, ſubductis minoribus binomiorum nominibus ex majoribus. Exemp. gr. Sit $6+\sqrt{20}$, bin. 1. erit $6-\sqrt{20}$, apot. 1. &c. Quare de earum inventione plura repetere nihil eſt neceſſe.

LEMMA.

Si rectangulum AC *ſub rectis* AB, AD. *producatur* AD *ad* E, *& biſecetur* DE *in* F. *ſitque rectang.* AGE = FEq. *& compleantur rectangula* AI, DK, FH. *Fiant vero quadratum* LM = AH, *& quadratum* NO = GI, *producanturque* NSR, OST.

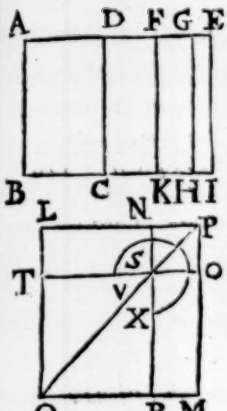
Dico primo, rectangul. AI = LM + NO = TOq + SOq. ut patet ex conſtr.

Se-

Secundo, *Rectang.* DK = LO. Nam quia
a *constr.* rectang. AGE *a* = FEq, *b* sunt AG, FE, GE
b 17. 6. ∺, *c* adeoque AH, FI, GI ∺ ; *a* hoc est, LM,
c 1. 6. FI, NO ∺. atqui LM, LO, NO *d* sunt ∺ ; ergo
d *Sch.* 22. 6. FI = *e* LO *f* = DK = *g* NM.
e 9. 5. Tertio, *Hinc*, AC = AI − DK − FI =
f 35. 1. LM + NO − LO − NM = TR.
g 43. 1. Quarto, *b Liquet* DF, FE, DE esse □.
h 16. 10. Quinto, *si* AE ⊐ DE, & AE □ √ AEq
k 18. 10. & − DEq, *k* erunt AG, GE, AE □.
10. 10. Sexto, *Item, quia* AE *l* □ DE, *m erunt* AE,
l *hyp.* FE □. *n ideoque* AI, FI ; *hoc est*, LM + NO
m 13. 10. & LO *sunt* □.
n 1. 6. & Septimo, *Item quia* AG * □ GE, *n erunt* AH
10. 10. GI, *hoc est*, LM, NO □.
* prius. Octavo, *Sed quia* AE *l* □ DE, *o erunt* FE,
o 14. 10. GE □. *n ideoque rectang* FI □ GI, *hoc est* LO
p 2. 6. □ NO. *quare cum* LO. NO *p* :: TS. SO *q erunt*
q 10. 10. TS, SO □.
r 19. 10. Nono, *Sin ponatur* AE □ √ AEq − DEq ;
& 17. 10. *r erunt* AG, GE, AE □.
s 1. 6. & 10 Decimo, *s Quare rectang.* AH, GI, *hoc est*
10. TOq, SOq *erunt* □.

PROP.

PROP. XCII.



Si ſpatium AC *contineatur ſub rationali* AB, *& Apotoma prima* AD (AE—DE;) *recta linea* TS *ſpatium* AC *potens, apotome eſt.*

Adhibe lemma proxime antecedens pro præparatione ad demonſtrationem hujus. Igitur TS = √ AC. item AG, GE, AE ſunt □; ergo cum AE □ *a* AB ρ̂; *b* erunt AG, & GE □ AB. *c* ergo rectangula AH & GI, hoc eſt TOq & SOq ſunt ρ̂ *a*. *d* item TO, SO ſunt ρ̂ ⌐, *e* proinde TS eſt apotome. Q. E. D.

a *hyp.*
b 12. 10.
c 20. 10.
d *lem.* 91. 10.
e 74. 10.

PROP. XCIII.

Vide Schem. præced.

Si ſpatium AC *contineatur ſub rationali* AB, *& apotoma ſecunda* AD (AE—DE;) *recta linea* TS *ſpatium* AC *potens; mediæ eſt apotome prima.*

Rurſus juxta lemma antecedens, AG, GE, AE ſunt □. cum igitur AE *a* ſit ρ̂ □ AB, *b* erunt AE, GE etiam ρ̂ □ AB. *c* ergo rectangula AH, GI, hoc eſt TOq, SOq, ſunt μα; *d* item TO ⌐ SO. Denique quia DE *e* □ AB. ρ̂. *f* erit rectang. DI, ejuſque ſemiſſis DK, vel LO, hoc eſt TOS ρ̂ν *g* è quibus ſequitur TS (√ AC) eſſe mediæ apot. 1. Q. E. D.

a *hyp.*
b 13. 10.
c 22. 10.
d *lem.* 74. 10.
e *hyp.*
f 20. 10.
g 75. 10.

PROP.

PROP. XCIV.

Vide idem.

Si ſpatium AC *contineatur ſub rationali* AB, *& apotomæ tertia* AD (AE — DE;) *recta linea* TS *ſpatium* AC *potens, mediæ eſt apotome ſecunda.*

Ut in præcedenti TO, & SO ſunt μ. Quoniam igitur DE *a* eſt ρ^e ⊓ AB, *b* erit rectang. DI, *c* ideoque DK; vel TOS $\mu\nu$. *d* ergo TS = √AC eſt mediæ apot. 2. Q. E. D.

a *byp.*
b 22. 10.
c 24. 10.
d 76. 10.

PROP. XCV.

Vide idem.

Si ſpatium AC *contineatur ſub rationali* AB, *& apotomæ quartæ* AD (AE — DE) *recta linea* TS *ſpatium* AC *potens, minor eſt.*

Rurſus TO *a* ⊓ SO. Quoniam igitur AE *b* eſt ρ^e ⊓ AB, *c* erit AI, (TOq+SOq) $\rho^e\nu$. atqui ut prius rectang. TOS eſt $\mu\nu$. *d* ergo TS = √AC eſt minor. Q. E. D.

a *lem.* 91. 10.
b *byp.*
c 20. 10.
d 77. 10.

PROP. XCVI.

Vide idem.

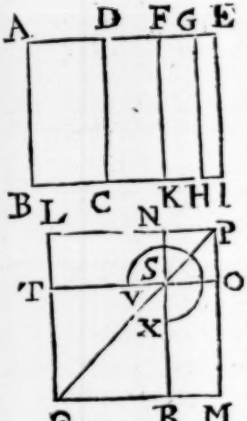
Si ſpatium AC *contineatur ſub rationali* AB, *& apotomæ quintæ* AD (AE — DE;) *recta linea* TS *ſpatium* AC *potens, eſt quæ cum rationali medium totum efficit.*

Rurſus enim TO ⊓ SO. itaque cum AE *a* ſit ρ^e ⊓ AB, *b* erit AI, hoc eſt TOq+SOq $\mu\nu$. Sed prout in 93 rectang. TOS eſt $\rho^e\nu$. *c* proinde TS = √AC eſt quæ cum $\rho^e\nu$ facit totum $\mu\nu$. Q. E. D.

a *byp.*
b 22. 10.
c 78. 10.

PROP.

PROP. XCVII.



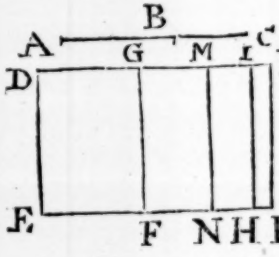
Si ſpatium AC *contineatur ſub rationali* AB, *& apotoma ſexta* AD (AE − DE;) *recta linea* TS *ſpatium* AC *potens, eſt quæ cum medio medium totum efficit.*

Itidem, ut ſæpe prius, TO □ SO. item ut in 96, TOq + SOq eſt μν. rectang. vero TOS eſt ρ' ν, ut in 94. *a* deinde TOq + SOq 10. □ TOS. *b* ergo TS = √AC eſt quæ cum μν facit totum μν. Q. E. D.

a *lem.* 91.

b 79. 10.

LEMMA.



Ad rectam quamvis DE * *applicentur rectang.* DF = ABq, *&* DH = ACq, *&* IK = BCq; *& ſit* GL *biſecta in* M; *ducta que ſit* MN *parall.* GF.

* cor. 16. 6.

Exit primo, *Rectang.* DK = ACq + BCq, *ut conſtructio indicat.*

Secundo, *Rectang.* ACB = GN, *vel* MK. Nam DK *a* = ACq + BCq *b* = 2 ACB + ABq at ABq *a* = DF. ergo GK *c* = 2 ACB. & *d* proinde GN, vel MK = ACB.

a *conſtr.* b 7. 2. c 3. *ax.* 1.

Tertio, *Rectang.* DH = MLq. Nam quia *d* 7. *ax* 1. ACq. ACB *e* :: ACB. BCq; hoc eſt DH. *e* 1. 6.

MK

f 17. 6. MK :: MK. IK, *e* erit DI. ML :: ML. IL. *f* ergo DIL = MLq.

Quarto, *Si ponatur* AC ⊓ BC, *erit* DK ▭ ACq. Nam ACq + BCq (DK) *g* ▭ ACq.

g 16. 10.

Quinto, *Item*, DL ▭ √DLq − GLq.

h 10. 10. Nam quia DH (ACq) ▭ IK (BCq) *h* erit DI

k 18. 10. ▭ IL. *k* ergo √DLq − GLq − DL.

Sexto, *Item* DL ▭ GL. Nam ACq +

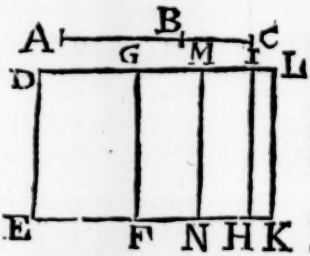
l *lem.* 26. BCq ▭ *l* 2 ACB; hoc est, DK ▭ GK. *m* ergo

10. DL ▭ GL.

m 10. 10. Septimo, *Sin ponatur* AC ⊓ BC, *n erit* DL

n 19. 10. ▭ √DLq − GLq.

PROP. XCVIII.



Quadratum apotomæ AB (AC − BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen primam.*

Fac ut in lemmate proxime præcedenti.

a *hyp.* Quoniam igitur AC, BC *a* sunt ῥ ⊓,

b *lem.* 97. *b* erit DK (ACq + BCq) ▭ ACq; *c* ergo

10. DK est ῥγ. *d* quare DL est ῥ ▭ DE. *e* item

c *sch.* 12 10 rectang. GK (2 ACB) est μγ. *f* ergo GL est ῥ

d 21. 10. ▭ DE. *g* proinde DL ▭ GL; *h* sed DLq

e 22. & 24. ▭ GLq. *k* ergo DG est apotome, & *l* quidem

10. prima (quia *m* AC ⊓ BC, & propterea DL

f 23. 10. ▭ √DLq − GLq.) Q. E. D.

g 13 10.

h *sch.* 12. 10

k 74. 10.

l 1. *def.* 85.

10.

m *lem.* 97.

10.

PROP.

PROP. XCIX.

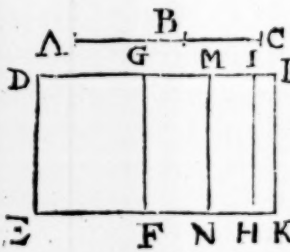
Vide Schema subsequens.

Quadratum mediæ apotomæ primæ AB (AC−BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen secundam.*

Rursus (supposito lemmate præcedenti) quia AC, & BC *a* sunt μ ⊐ *b*, erit DK (ACq+BCq) ⊏ ACq; *c* quare DK est $\mu\nu$. *d* ergo DL est ρ^e ⊏ DE. *e* item GK (2. ACB) est $\rho^e\nu$ *f* ergo GL est ρ^e ⊏ DE; *g* quare DL ⊏ GL. *h* Sed DLq ⊏ GLq. *k* ergo DG est apotome. quia vero DL *l* ⊏ √ DLq−GLq, *m* erit DG apotome secunda. Q.E.D.

a *hyp.*
b *lem.* 97. 10.
c 24. 10.
d 23. 10.
e *hyp.* & *sch.* 12. 10.
f 21. 10.
g 13. 10.
h *sch.* 12. 10.
k 74. 10.
l *lem.* 97. 10.
m 2. *def.* 85. 10.

PROP. C.



Quadratum mediæ apotomæ secundæ AB (AC−BC) *ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen tertiam.*

Iterum DK est $\mu\nu$, *a* quare DL est ρ^e ⊏ DE. item GK est $\mu\nu$. *a* unde GL est ρ^e ⊏ DE; *b* item DK ⊏ GK, *c* quare DL ⊏ GL; *d* at DLq ⊏ GLq. *e* ergo DG est apot. & quidem *f* 3a. *g* quia DL ⊏ √ DLq−GLq. Q. E. D.

a 23. 10.
b *lem.* 26. 10.
c 1. 6. & 10. 10.
d *sch.* 12. 10.
e 74. 10.
f 3. *def.* 85. 10.
g *lem* 97. 10.

PROP. CI.

Vide Schema præced.

Quadratum minoris AB (AC−BC) *ad rationalem*

tionalem DE *applicatum, facit latitudinem* DG *apotomen quartam.*

Ut prius, ACq + BCq, hoc eſt DK eſt $\rho^c v$;
a 21. 10. *a* ergo DL eſt ρ^c □ DE. at rectang. ACB, ideoque GK (2 ACB) * eſt μv, *b* quare GL eſt ρ^c
* *hyp.*
b 23. 10. □ DE. *c* ergo DL □ GL. *d* at DLq □ GLq quia vero * ACq □ BCq, *e* erit DL □
c 13. 10. √ DLq − GLq: *f* ergo DG conditiones habet
d *ſch.* 12. 10 apotomæ quartæ. Q. E. D.
e *lem.* 97. 10.
f 4. *def.* 85. 10.

PROP. CII.

Vide Schem. præced.

Quadratum ejus AB (AC − BC,) *quæ cum rationali medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen quintam.*

a 23 10. Rurſus enim, DK eſt μv, *a* quare DL eſt ρ^c
b 21. 10. □ DE. item GK eſt $\rho^c v$, *b* unde GL eſt ρ^c. □
c 13. 10. DE. *c* ergo DL □ GL, *d* ſed DLq □ GLq.
d *ſch.* 12 10 porro, DL *e* □ √ DLq − GLq. ex quibus,
e *lem.* 97. 10. DG *f* eſt apot. quinta. Q. E. D.
f 5. *def.* 85. 10.

PROP. CIII.

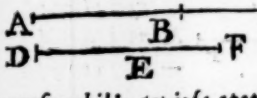
Vide Schema idem.

Quadratum ejus AB (AC − BC,) *quæ cum medio medium totum efficit, ad rationalem* DE *applicatum, facit latitudinem* DG *apotomen ſextam.*

Haud aliter, quam antea, DK, & GK ſunt
a 23. 10. $\mu\alpha$; *a* quare DL & GL ſunt ρ^c □. DE. item
b *hyp.* & DK *b* □ GK, *c* quare DL □ GL. *d* ergo
lem. 97. 10. DG eſt apot. *b* cum igitur ACq □ BCq, ideo-
c 10. 10. que DL □ √ DLq − GLq, *e* erit DG. apot.
d 74. 10. ſexta. Q. E. D.
e 6. *def.* 85. 10.

PROP.

PROP. CIV.



Recta linea DE *apotomæ* AB (AC—BC) *longitudine commensurabilis, & ipsa apotome est, atque ordine eadem.*

LEMMA.

Sit AB. DE :: AC. DF. *&* AB □ DE.

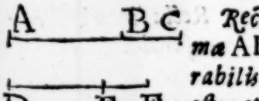
Dico AC + BC □ DF + EF.

Nam AC. BC *a* :: DF. EF. ergo componendo AC+BC. BC :: DF+EF. EF. ergo permutando AC + BC. DF+EF :: BC. EF. *a* at BC □ EF. *b* ergo AC+BC □ DF+EF. Q.E.D.

a Fac AB. DE :: AC. DF. *b* igitur AC + BC □ DF+EF. ergo cum AC+BC *c* binomium sit, *d* erit DF+EF ejusdem ordinis binomium: *e* quare DF—EF ejusdem ordinis apotome est, cujus AC—BC. Q.E.D.

a *lem.* 66. 10.
b 10. 10.
a 12. 6.
b *lem.* 103. 10.
c *hyp.*
d 61. 10.
e *Per definitiones ad* 85. 10.

PROP. CV.

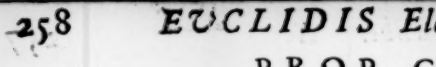


Recta linea DE *mediæ apotomæ* AB (AC—BC) *commensurabilis, & ipsa mediæ apotome est, atque ordine eadem.*

Iterum *a* fac AB. DE :: AC. DF. *b* quare AC + BC □ DF + EF. *c* ergo DF + EF est bimed. ejusdem ordinis, cujus AC + BC. *d* proinde & DF—EF mediæ apotome erit ejusdem classis, cujus AC—BC. Q.E.D.

a 12. 6.
b *lem.* 103. 10.
c 68. 10.
d 75 & 76. 10.

PROP. CVI.

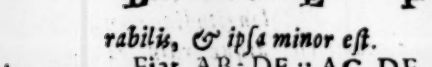


Recta linea DE *Minori* AB (AC—BC) *commensurabilis, & ipsa minor est.*

Fiat AB. DE :: AC. DF. *a* estque AC+BC □ DF+EF. atqui AC+BC *b* est Major, *c* ergo DF+EF quoque Major est. *d* & proinde DF—EF est Minor. Q. E. D.

a *lem.* 103. 10
b *hyp.*
c 69. 10.
d 77. 10.

PROP. CVII.

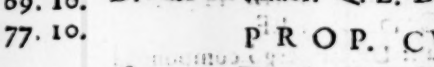


Recta linea DE *commensurabilis ei* AB (AC—BC) *quæ cum rationali medium totum efficit, & ipsa cum rationali medium totum efficiens est.*

Nam ad modum præcedentium ostendemus DF+EF esse potentem $\rho\nu$, & $\mu\nu$. *a* ergo DF—EF est ut dicitur.

a 78. 10.

PROP. CVIII.



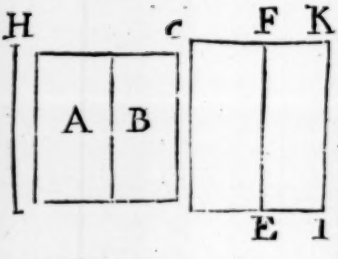
Recta linea DE *commensurabilis ei* AB (AC—BC) *quæ cum medio medium totum efficit, & ipsa cum medio medium totum efficiens est.*

Nam, ad normam præcedentium, erit DF+EF potens $2\mu\alpha$. *a* ergo DF—EF erit ut in propos.

a 79. 10.

PROP.

PROP. CIX.



Medio B *à rationali* A+B *detracto, recta linea* H, *quæ reliquum ſpatium* A *poteſt, unâ ex duabus irrationalibus fit, vel apotome, vel Minor.*

Ad CD $\overset{s}{\rho}$, fac rectang. CI = A+B; & FI = B. quare CE *a* = A: (Hq) Quoniam igitur CI *b* eſt $\overset{s}{\rho}_v$, *c* erit CK $\overset{s}{\rho}$ □ CD. ſed quia FI *b* eſt μ_v, *d* erit FK $\overset{s}{\rho}$ □ CD. *e* unde CK □ FK *f* ergo CF eſt apotome. Si igitur CK □ √ CKq — FKq. *g* erit CF apot. prima; *h* quare CE (H) eſt apotome. ſin CK □ √ CKq — FKq, *k* erit CF apot. quinta. & proinde H (√ CE) *l* erit Minor. Q. E. D.

a 3. *ax.* 1.
b *hyp.* & *conſtr.*
c 21. 10.
d 23. 10.
e 13. 10.
f 74. 10.
g 1. *def.* 85. 10.
h 91. 10.
k 4. *def.* 85. 10.
l 95. 10.

PROP. CX.

Vide Schem. præced.

Rationali B *à medio* A+B *detracto; aliæ duæ irrationales fiunt, vel mediæ apotome prima, vel cum rationali medium totum efficiens.*

Ad CD expoſ. $\overset{s}{\rho}$ fiant rectang. CI = A+B; & FI = B, *a* unde CE = A = Hq Quoniam igitur CI *b* eſt μ_v: *c* erit CK $\overset{s}{\rho}$ □ CD. ſed quia FI *b* eſt ρ'_v, *d* erit FK ρ' □ CD. *e* unde CK □ FK. *f* ergo CF eſt apot. *g* nempe ſecunda; ſi CK □ √ CKq — FKq, *b* quare H (√ CE) eſt mediæ apot. prima. Sin vero CK □ √ CKq — FKq, *k* erit CF apot. quinta. & proinde H (√ CE) *l* erit faciens μ_v cum ρ'_v. Q. E. D.

a 3. *ax.* 1.
b *hyp.* & *conſtr.*
c 23. 10.
d 21. 10.
e 13. 10.
f 74. 10.
g 2. *def.* 85. 10.
h 93. 10.
k 5. *def.* 85. 10.
l 6. 10.

PROP. CXI.

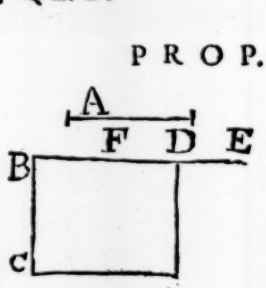
Vide Schema idem.

Media B *à medio* A+B *detracto quod ſit incommenſurabile toti* A+B; *reliquæ duæ irrationales fiunt, vel mediæ apotome ſecunda, vel cum medio medium totum efficiens.*

Ad C D $\dot{\rho}$ fiant rectang. C I = A + B; & F I = B, *a* quare C E = A = Hq. Quoniam igitur C I eſt $\mu\nu$. *b* erit C K $\dot{\rho}$ $\square$ C D. eodem modo erit F K $\dot{\rho}$ $\square$ C D. item quia C I *c* $\square$ F I, *d* erit C K $\square$ F K; *e* quare C F eſt apotome, *f* tertia ſcilicet, ſi C K $\square$ $\sqrt{\text{C K} - \text{F Kq}}$, *g* unde H ($\sqrt{\text{C E}}$) erit mediæ apot. ſecunda. verum ſi C K $\square$ $\sqrt{\text{C Kq} - \text{F Kq}}$, *b* erit C F apot. ſexta. *k* quare H erit faciens $\mu\nu$ cum μ. Q. E. D.

a 3. *ax.* 1.
b 23. 10.
c *hyp.*
d 10. 10.
e 74. 10.
f 3. *def.* 85. 10.
g 94. 10.
h 6. *def* 85. 10.
k 97. 10.

PROP. CXII.



Apotome A *non eſt eadem, quæ ex binis nominibus.*

Ad expoſ. B C $\dot{\rho}$, fiat rectang. C D = Aq. Ergo cum A ſit apotome, *a* erit B D apot. prima. ejus congruens ſit D E. *b* quare B E, D E ſunt $\dot{\rho}$ $\square$. *c* & B E $\square$ B C. Vis A eſſe bin. ergo B D eſt bin. 1. ejus nomina ſint B F, F D; ſitque B F $\sqsupset$ F D; *d* ergo B F, F D ſunt $\dot{\rho}$ $\square$; & B F *e* $\square$ B C. ergo cum B C $\square$ B E, *f* erit B E $\square$ F B. *g* ergo B E $\square$ F E. *h* ergo F B eſt $\dot{\rho}$ item quia B E $\square$ D E, *k* erit F E $\square$ D E. *l* quare F D eſt apotome, *l* adeoque F D eſt $\dot{\rho}$. ſed oſtenſa eſt $\dot{\rho}$. quæ repugnant. ergo A male dicitur binomium. Q. E. D.

a 98. 10.
b 74. 10
c 1 *def.* 85. 10.
d 37. 10.
e 1 *def* 48 bin. 10.
f 12. 10.
g *cor.* 16. 10
h *ſch* 12. 10
k 14. 10
l 74. 10.

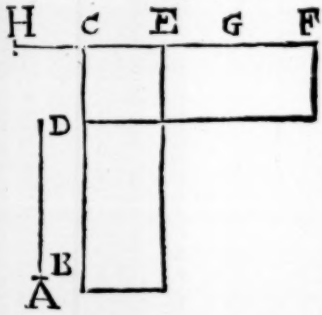
Nomi-

Nomina 13. *linearum irrationalium inter ſe differentium.*

1. Media.
2. Ex binis nominibus, cujus 6 ſpecies.
3. Ex binis mediis prima.
4. Ex binis mediis ſecunda.
5. Major.
6. Rationale ac medium potens.
7. Bina media potens.
8. Apotome, cujus etiam 6 ſpecies.
9. Mediæ apotome prima.
10. Mediæ apotome ſecunda.
11. Minor.
12. Cum rationali medium totum efficiens.
13. Cum medio medium totum efficiens.

Cum latitudinum differentiæ arguant differentias rectarum, quarum quadrata ſunt applicata ad aliquam rationalem, ſitque demonſtratum in præcedentibus, latitudines quæ oriuntur ex applicationibus quadratorum harum 13 *linearum inter ſe differre, perſpicue ſequitur has* 13 *lineas inter ſe differre.*

PROP. CXIII.



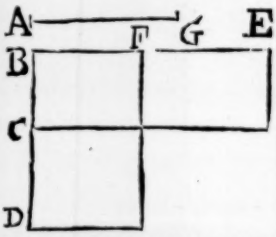
Quadratum rationalis A *ad eam, quæ ex binis nominibus* BC (BD + DC) *applicatum, latitudinem facit apotomen* EC, *cujus nomina* EH, CH *commenſurabilia ſunt nominibus* BD, DC *ejus, quæ ex binis nominibus*

& in eadem proportione (EH. BD :: CH. DC;) *& adhæc, apotome* EC *quæ fit, eundem habet ordinem, quem ea* BC, *quæ ex binis nominibus.*

a *cor.*16.6. Ad DG minus nomen *a* fac rectang. DF =
b 14. 6. Aq = BE. quare BC. CD *b* :: FC. CE. ergo dividendo BD. DC :: FE. EC. cum igitur BD
c *hyp.* *c* ⊏ DC. *d* erit FE ⊏ EC. ſume EG = EC;
d 14. 5. fiatque FG. GE :: EC. CH. Erunt EH, CH, nomina apotomæ EC; quibus conveniunt ea, quæ in theoremate propoſita ſunt. Nam componendo FE. GE. (EC) :: EH. CH. ergo
e 12. 5. FH. EH *e* :: EH. CH *f* :: FE. EC *f* :: BD.
f *Prius.* DC. quare cum BD *g* ⊔ DC, *h* erit EH ⊔
g *hyp.* CH; *h* & FHq ⊔ EHq. ergo, quia FHq.
h 10. 10. EHq *k* :: FH. CH. *h* erit FH ⊔ CH, *l* ideoque
k *cor.*20.6. FC ⊔ CH. Porro CD *g* eſt ρ^e, & DF (Aq)
l 16. 10. *g* eſt ρ^e *v*, *m* ergo FC eſt ρ^e ⊔ CD, quare etiam
m 21.10. CH eſt ρ^e ⊔ CD. *n* igitur EH, CH ſunt ρ^e, ac
n *ſch.*12.10 ⊔ ut prius. *o* ergo EC eſt apotome, cui congruit CH. porro EH. CH *f* :: BD. DC, ideo per-
o 74. 10. mutando EH. BD :: CH. DC. unde quia CH *f*
p 10. 10. ⊔ DC, *p* erit EH ⊔ BD. quinimo pone BD
q 15. 10. ⊔ $\sqrt{BDq-DCq}$; *q* erit ideo EH ⊔ $\sqrt{EHq-}$
r 12. 10. CHq. item ſi BD ⊔ ρ^e expoſ *r* erit EH ⊔ ei-
ſ 1.*def.*48. dem ρ^e; ſ hoc eſt ſi BC ſit bin. 1. *t* erit EC apot.
10. prima. Similiter ſi DC ⊔ ρ^e expoſ. *t* erit CH
t 1.*def.*85. ⊔ eidem ρ^e. *u* hoc eſt ſi BC ſit bin. 2. *x* erit
10. EC apot. 2. & ſi hæc bin. 3. illa erit apot. 3,
u 2.*def.*48. &c. Sin BD ⊔ $\sqrt{BDq-DCq}$, *y* erit EH ⊔
10. $\sqrt{EHq-CHq}$; ſi igitur BC ſit bin. 4, vel 5,
x 1.*def.*85. vel 6. erit EC ſimiliter apot. 4, vel 5, vel 6.
10. Q. E. D.
y 15. 10.

PROP.

PROP. CXIV.

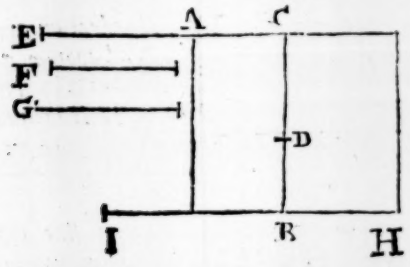


Quadratum rationalis A *ad apotomen* BC (BD — DC) *applicatum, facit latitudinem* BE *eam, quæ ex binis nominibus; cujus nomina* BE, GE *commensurabilia sint apotomæ* BC *nominibus* BD, DC, *& in eadem proportione; & adhuc, quæ ex binis nominibus fit* (BE,) *eundem habet ordinem, quem ipsa apotome* BC.

a Fac rectang. DF = Aq; & BE, FE *b* :: EG. GF. Quoniam igitur DF = Aq = GE, *c* erit BD. BC :: BE. BF. ergo per conversionem rationis BD. CD :: BE. FE :: EG. GF :: *d* BG. EG. sed BD *e* ⌐ CD. *f* ergo BG ⌐ GE. ergo quia BGq. GEq *g* :: BG. GF. *h* erit BG ⌑ GF. *k* ideoque BG ⌑ BF. porro BD *e* est ρ^c, & rectang. DF (Aq) *e* est $\rho^c\gamma$. *l* ergo BF est ρ^c ⌑ BD. *m* ergo etiam BG est ρ^c ⌑ BD. *n* ergo BG, GE sunt ρ^c ⌐. *o* quare BE est bin. denique igitur quia BD. CD :: BG. GE; & permutando BD. BG :: CD. GE; sitque BD ⌑ BG; *p* erit CD ⌑ GE. ergo si CB sit apot. prima; erit BE bin. 1, &c. ut in antecedenti. ergo, &c.

a *cor.* 16. 6.
b 12. 6.
c 14. 6.
d 19. 5.
e *hyp.*
f 10. 10.
g *cor.* 20. 6.
h 10. 10.
k *cor.* 16. 10.
l 21. 10.
m 12. 10.
n *sch.* 12. 10
o 37. 10.
p 10. 10.

PROP. CXV.



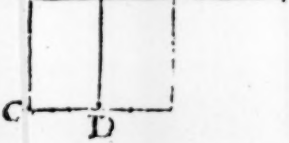
Si ſpatium A B *contineatur ſub apotoma* A C (CE—AE,) *& ea, quæ ex binis nominibus* CB; *cujus nomina* CD, DB *commenſurabilia ſint apotomæ nominibus* CE, AE, *& in eadem proportione* (CE. AE :: CD. DB;) *recta linea* F *ſpatium* AB *potens, eſt rationalis.*

Sit G quævis ρ̊; & fiat rectang. CH = Gq.
a 113. 10. *a* erit igitur BH (HI—IB) apotome; & HI
a □ CD *b* □ CE, *a* & BI □ DB; *a* atque
b *hyp.* HI. BI :: CD. DB *b* :: CE, EA. ergo permu-
c 19. 5. tando HI. CE :: BI. EA. *c* ergo BH. AC ::
d 12. 10. HI. CE :: BI. EA. ergo cum HI *d* □ CE,
e 10. 10. *e* erit B H □ A C. *f* ergo rectang. HC □
f 1. 6 & 10 BA. Sed HC (Gq) *b* eſt ρ̊ *v.* *g* ergo BA (Fq)
10. eſt ρ̊ *v.* proinde F eſt ρ̊. Q. E. D.
g *ſch.* 12. 10

Coroll.

Hinc, fieri poteſt, ut ſpatium rationale contineatur ſub duabus rectis irrationalibus.

PROP. CXVI.



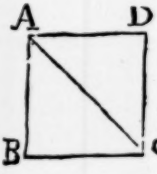
A media AB *ſiunt infinitæ irrationales* B E, E F, &c. *& nulla alicui antecedentium eſt eadem.*

Sit AC expoſ. ρ̊. fac-

ρ^c. ſitque AD ſpatium ſub AC, AB. *a* ergo AD eſt $\dot{\rho}v$. Sume BE $= \sqrt{AD}$. *b* ergo BE eſt $\dot{\rho}$, nulli priorum eadem. nullum enim quadratum alicujus priorum applicatum ad ρ^c, latitudinem efficit mediam. compleatur rectang. DE; *a* erit DE $\dot{\rho}v$, & *b* proinde EF ($\sqrt{DE}$) erit $\dot{\rho}$; & nulli priorum eadem. nullum enim priorum quadratum ad ρ^c applicatum, latitudinem efficit ipſam BE, ergo, &c.

a *lem.* 38. 10.
b 11. 10.

PROP. CXVII.



Propoſitum ſit nobis oſtendere, in quadratis figuris BD, *diametrum* AC *lateri* AB *incommenſurabilem eſſe.*

Nam ACq. ABq *a* :: 2. 1 *b* :: non Q. Q. *c* ergo AC $\sqcap$ AB. Q E.D.

a 47. 1.
b *cor.* 24. 8.
c 9. 10.

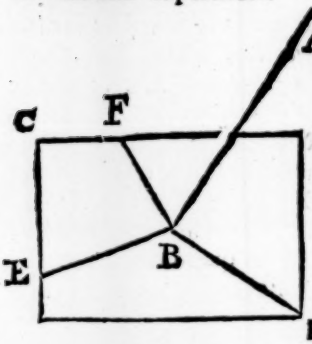
Celebratiſſimum eſt hoc theorema apud veteres philoſophos, adeo ut qui hoc neſciret, eum Plato non hominem eſſe, ſed pecudem diceret.

LIB. XI.

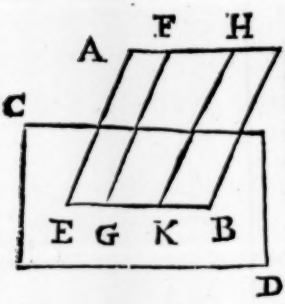
Definitiones.

I. SOlidum eſt, quod longitudinem, latitudinem, & craſſitudinem habet.

II. Solidi autem extremum eſt ſuperficies.

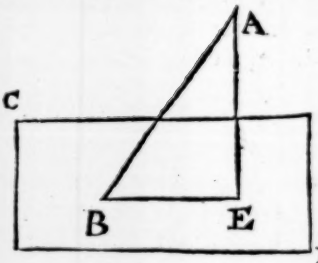


III. Linea recta AB eſt ad planum CD recta, cum ad rectas omnes lineas BD, BE, BF, à quibus illa tangitur, quæque in propoſito ſunt plano, rectos efficit angulos A B D, ABE, ABF.

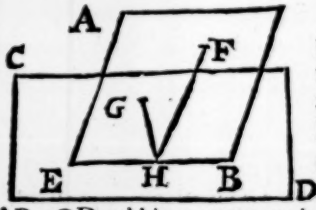


IV. Planum AB ad planum CD rectum eſt, cum rectæ lineæ FG, HK, quæ communi planorum ſectioni EB ad rectos angulos in uno plano AB ducuntur, alteri plano CD ad rectos ſunt angulos.

V. Rectæ



V. Rectæ lineæ AB ad planum CD inclinatio eſt, cum à ſublimi termino A rectæ alius lineæ AB ad planum CD deducta fuerit perpendicularis AE; atque à puncto E, quod perpendicularis AE in ipſo plano CD fecerit, ad propoſitæ illius lineæ extremum B, quod in eodem eſt plano, altera recta linea EB fuerit adjuncta: eſt, inquam, angulus acutus ABE inſiſtente linea AB, & adjuncta EB comprehenſus.



VI. Plani AB ad planum CD inclinatio, eſt angulus acutus FHG rectis lineis FH, GH contentus, quæ in utroque planorum AB, CD ad idem communis ſectionis BE punctum H ductæ, rectos cum ſectione BE efficiunt angulos FHB, GHB.

VII. Planum ad planum ſimiliter inclinatum eſſe dicitur, atque alterum ad alterum, cum dicti inclinationum anguli inter ſe fuerint æquales.

VIII. Parallela plana ſunt, quæ inter ſe non conveniunt.

IX. Similes ſolidæ figuræ ſunt, quæ ſimilibus planis continentur, multitudine æqualibus.

X. Æquales & ſimiles ſolidæ figuræ ſunt, quæ ſimilibus planis multitudine & magnitudine æqualibus continentur.

XI. Solidus

XI. Solidus angulus est plurium quam duarum linearum, quæ se mutuo contingunt, nec in eadem sunt superficie, ad omnes lineas inclinatio.

Aliter.

Solidus angulus est, qui pluribus quam duobus planis angulis in eodem non consistentibus plano, sed ad unum punctum constitutis, continetur.

XII. Pyramis est figura solida, planis comprehensa, quæ ab uno plano ad unum punctum constituuntur.

XIII. Prisma est figura solida, quæ planis continetur, quorum adversa duo sunt & æqualia, & similia, & parallela; alia vero parallelogramma.

XIV. Sphæra est, quando semicirculi manente diametro, circumductus semicirculus in seipsum rursus revolvitur unde moveri cœperat, circumassumpta figura.

Coroll.

Hinc radii omnes à centro ad superficiem sphæræ inter se sunt æquales.

XV. Axis autem sphæræ, est quiescens illa recta linea, circum quam semicirculus convertitur.

XVI. Centrum sphæræ est idem quod & semicirculi.

XVII. Diameter autem sphæræ, est recta quædam linea per centrum ducta, & utrinque à sphæræ superficie terminata.

XVIII. Conus est, quando rectanguli trianguli manente uno latere eorum, quæ circa rectum angulum, circumductum triangulum in seipsum rursus revolvitur unde moveri cœperat, circumassumpta figura. Atque si quiescens recta

linea

linea æqualis ſit reliquæ, quæ circa rectum angulum continetur, orthogonius erit conus; ſi vero minor, amblygonius; ſi vero major, oxygonius.

XIX. Axis autem coni, eſt quieſcens illa linea, circa quam triangulum vertitur.

XX. Baſis vero coni eſt circulus qui à circumducta recta linea deſcribitur.

XXI. Cylindrus eſt, quando rectanguli parallelogrammi manente uno latere eorum, quæ circa rectum angulum, circumductum parallelogrammum in ſeipſum rurſus revolvitur unde cœperat moveri, circumaſſumpta figura.

XXII. Axis autem cylindri, eſt quieſcens illa recta linea, circum quam parallelogrammum convertitur.

XXIII. Baſes vero cylindri ſunt circuli à duobus adverſis lateribus, quæ circumaguntur, deſcripti.

XXIV. Similes coni & cylindri ſunt, quorum & axes, & baſium diametri proportionales ſunt.

XXV. Cubus eſt figura ſolida ſub ſex quadratis æqualibus contenta.

XXVI. Tetraedrum eſt figura ſolida ſub quatuor triangulis æqualibus & æquilateris contenta.

XXVII. Octaedrum eſt figura ſolida ſub octo triangulis æqualibus & æquilateris contenta.

XXVIII. Dodecaedrum eſt figura ſolida ſub duodecim pentagonis æqualibus & æquilateris & æquiangulis contenta.

XXIX. Icoſaedrum eſt figura ſolida ſub viginti triangulis æqualibus & æquilateris contenta.

XXX. Parallelepipedum eſt figura ſolida ſex figuris quadrilateris, quarum quæ ex adverſo parallelæ ſunt, contenta.

XXXI. So-

XXXI. Solida figura in ſolida figura dicitur inſcribi, quando omnes anguli figuræ inſcriptæ conſtituuntur vel in angulis, vel in lateribus, vel denique in planis figuræ, cui inſcribitur.

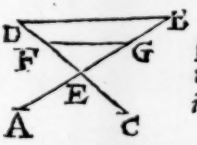
XXXII. Solida figura ſolidæ figuræ viciſſim circumſcribi dicitur, quando vel anguli, vel latera, vel denique plana figuræ circumſcriptæ tangunt omnes angulos figuræ, circum quam deſcribitur.

PROP. I.

Rectæ lineæ pars quædam AC *non eſt in ſubjecto plano, quædam vero* CB *in ſublimi.*

Producatur AC in ſubjecto plano uſque ad F. vis CB eſſe in directum ipſi AC; ergo duæ rectæ AB, AF habent commu-
a 10. dx 1. ne ſegmentum AC. *a* Q. F. N.

PROP. II.

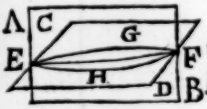


Si duæ rectæ lineæ AB, CD *ſe mutuo ſecent, in uno ſunt plano: atque triangulum omne* DEB *in uno eſt plano.*

Puta enim trianguli DEB partem EFG eſſe in uno plano, partem vero FDGB in altero. ergo rectæ ED pars EF eſt in ſubjecto plano, pars vero FD in ſublimi, *a* Q. E. A. ergo triangulum EDB in uno eſt plano;
2 1. 11. proinde & rectæ ED, EB; *a* quare & totæ AB, DC in uno plano exiſtunt. Q. E. D.

PROP.

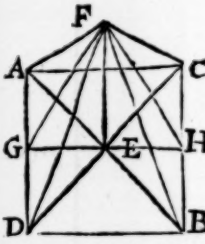
PROP. III.



Si duo plana AB, CD *ſe mutuo ſecent, communis eorum ſectio* EF *eſt recta linea.*

Si EF communis ſectio non eſt recta linea, *a* ducatur in plano AB recta EGF, *a* & in plano CD recta EHF. duæ igitur rectæ EGF, EHF claudunt ſpatium. *b* Q.E.A. a 1. *poſt.* 1. b 14. *ax.* 1.

PROP. IV.



Si recta linea EF *rectis duabus lineis* AB, CD *ſe mutuo ſecantibus in communi ſectione* E *ad rectos angulos inſiſtat: illa ducta etiam per ipſas plano* ACBD *ad angulos rectos erit.*

Accipe EA, EC, EB, ED æquales, & junge rectas AC, CB, BD, AD. per E ducatur quævis recta GH; junganturque FA, FC, FD, FB, FG, FH. Quoniam AE *a* = EB; a *conſtr.*
& DE *a* = EC; & ang. AED *b* = CEB, b 15. 1.
c erit AD = CB. *c* pariterque AC = DB. c 4. 1.
d ergo AD parall. CB. *d* & AC parall. d *ſch.* 34. 1.
DB. *e* quare ang. GAE = EBH. *e* & ang. e 29. 1.
AGE = EHB. ſed & AE *f* = EB *g* ergo GE f *conſtr.*
= EH, & *g* AG = BH. quare ob angulos rectos, g 26. 1.
ex hyp. & proinde pares ad E, *h* baſes FA, FC, h 4. 1.
FB, FD æquantur. Triangula igitur ADF, FBC ſibi mutuo æquilatera ſunt, *k* quare ang. k 8. 1.
DAF = CBF. ergo in triangulis AGF, FBH latera FG, FH *l* æquantur; & proinde etiam l 4. 1.
triangula FEG, FEH ſibi mutuo æquilatera ſunt. *m* ergo anguli FEG, FEH æquales ac m 8. 1.
n propterea recti ſunt. Eodem modo FE cum n 10. *def.* 1.

omni-

omnibus in plano ADBC per E ductis rectis li-
o 3. def. 11. neis rectos angulos constituit, o ideoque eidem plano recta est. Q. E. D.

PROP. V.

Si recta linea AB *rectis tribus lineis* AC, AD, AE *se mutuo tangentibus in communi sectione ad rectos angulos insistat; illæ tres rectæ in uno sunt plano.*

Nam AC, AD *a* sunt in
a 2. 11. uno plano FC. *a* item AB, AE sunt in uno plano BE. vis AE esse extra planum
b 3. 11. FC; sit igitur planorum intersectio *b* recta AG. Quoniam igitur BA ex hypoth. perpendicularis
c 4. 11. est rectis AC, AD, eadem *c* plano FC, *d* ideoque
d 3. def. 11. rectæ AG perpendicularis est. ergo (siquidem & *a* AB est in eodem cum AG, AE plano) anguli BAG, BAE recti, & proinde pares sunt, pars & totum. Q. E. A.

PROP. VI.

Si duæ rectæ lineæ AB, DC *eidem plano* EF *ad rectos sint angulos; parallelæ erunt illæ rectæ lineæ* AB, DC.

Ducatur AD, cui in plano EF perpendicularis sit DG = AB; junganturque BD, BG, AG. Quia in triangulis BAD,
a *hyp.* ADG anguli DAB, ADG *a* recti sunt; atque
b *constr.* AB *b* = DG; & AD communis est; *c* erit BD
c 4. 1. = AG; quare in triangulis AGB, BGD sibi
d 8. 1. mutuo æquilateris ang. BAG *d* = BDG; quorum BAG rectus cum sit, erit BDG etiam rectus. atqui ang. GDC rectus ponitur; ergo re-
e 5. 11. cta GD tribus DA, DB, CD recta est; *e* quæ
f 2. 11. idcoin uno sunt plano, *f* in quo AB existit;

cum

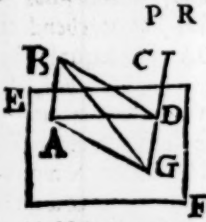
cum igitur AB, & CD sint in uno plano, & anguli interni BAD, CDA recti sint, *g* erunt AB, CD parallelæ. Q. E. D. g 28. 1.

PROP. VII.

Si duæ sint parallelæ rectæ lineæ AB, CD, *in quarum utraque sumpta sint quælibet puncta* E, F; *illa linea* EF, *quæ ad hæc puncta adjungitur, in eodem est cum parallelis plano* ABCD.

Planum in quo AB, CD, secet aliud planum per puncta E, F. si jam EF non est in plano ABCD, illa communis sectio non erit. Sit ergo EGF. *a* hæc igitur recta est linea. duæ ergo rectæ EF, EGF spatium claudunt. *b* Q. E. A. a 3. 11. b 14. ax. 1.

PROP. VIII.



Si duæ sint parallelæ rectæ lineæ AB, CD, *quarum altera* AB *ad rectos cuidam plano* EF *sit angulos; & reliqua* CD *eidem plano* EF *ad rectos angulos erit.*

Adscita præparatione & demonstratione sextæ hujus; anguli GDA & GDB recti sunt. *a* ergo GD recta est plano per AD, DB (*b* in quo etiam AB, CD existunt.) *c* ergo GD ipsi CD *el*perpendicularis; atqui ang. CDA etiam *d* rectus est, *e* ergo GD plano EF recta est. Q. E. D. a 4. 11. b 7. 11. c 3. *def.* 11. d 29. 1. e 5. 11.

PROP. IX.

Quæ (AB, CD) *eidem rectæ lineæ* EF *ſunt parallelæ ſed non in eodem cum illa plano, hæ quoque ſunt inter ſe parallelæ.*

In plano parallelarum AB, EF duc HG perpendicularem ad EF item in plano parallelarum EF, CD duc IG perpendicularem ad EF. *a* ergo EG recta eſt plano per HG, GI, eidemque plano *b* rectæ ſunt AH, & CI. *c* ergo AH, & CI parallelæ ſunt. Q. E. D.

a 4. 11.
b 8. 11.
c 6. 11.

PROP. X.

Si duæ rectæ lineæ AB, AC *ſe mutuo tangentes ad duas rectas* ED, DF *ſe mutuo tangentes ſint parallelæ, non autem in eodem plano, illæ angulos æquales* (BAC, EDF) *comprehendent.*

Sint AB, AC, DE, DF æquales inter ſe, & ducantur AD, BC, EF, BE, CF. Cum AB, DE *a* ſint parallelæ & æquales, *b* etiam BE, AD parallelæ ſunt, & æquales. Eodem modo CF, AD parallelæ ſunt, & æquales *c* ergo etiam BE, FC ſunt parallelæ & æquales. Æquantur ergo BC, EF. Cum igitur triangula BAC, EDF ſibi mutuo *d* æquilatera ſint, anguli BAC, EDF *e* æquales erunt. Q. E. D.

a *hyp.* & *conſtr.*
b 33. 1.
c 2. *ax.* 1. & 9. 11.
d 33. 1.
e 8. 1.

PROP. XI.

A dato puncto A *in ſublimi ad ſubjectum planum* BC *perpendicularem rectam lineam* AI *ducere.*

In plano BC duc quamvis DE, ad quam ex A *a* duc perpendicularem AF. ad eandem per F in plano BC *b* duc normalem FH. tum ad FH *a* demitte perpendicularem AI. erit AI recta plano BC.

a 12. 1.
b 12. 1.

Nam

Nam per I *c* duc KI L parall. DE. Quia DE *d* recta eſt ad AF, & FH, *e* erit DE recta plano IFA; adeoque & K L eidem plano *f* recta eſt. *g* ergo ang. K I A rectus eſt. atqui ang. A I F etiam *h* rectus eſt. *l* ergo AI plano BC recta eſt. Q. E. D.

c 31. 1.
d *conſtr.*
e 4. 11.
f 8. 11.
g 3. *def.* 11.
h *conſtr.*
l 4. 11.

PROP. XII.

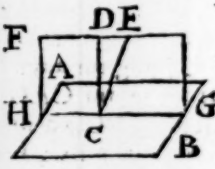
Dato plano BC *à puncto* A, *quod in illo datum eſt, ad rectos angulos rectam lineam* AF *excitare.*

A quovis extra planum puncto D *a* duc DE rectam plano BC; & juncta EA *b* duc AF parall. DE. *c* perſpicuum eſt AF plano BC rectam eſſe. Q. E. F.

a 11. 11.
b 31. 1.
c 8. 11.

Practice perficiuntur hoc, & præcedens problema, ſi duæ normæ ad datum punctum applicentur, ut patet ex 4. 11.

PROP. XIII.



Dato plano AB, *à puncto* D, *quod in illo datum eſt, duæ rectæ lineæ* C D, CE *ad rectos angulos non excitabuntur ab eadem parte.*

Nam utraque CD, CE plano AB recta eſſet, eædemq; *a* adeo parallelæ forent, quod parallelarum definitioni repugnat.

a 6. 11.

PROP. XIV.

Valet hæc converſa.

Ad quæ plana CD, FE, *eadem recta linea* AB *recta eſt; illa ſunt parallela.*

Si nega, plana CD, FE concurrant, ita ut communis ſectio ſit recta GH; ſume in hac quodvis punctum I, ad quod in propoſitis planis ducantur rectæ IA, IB. unde in triangulo IAB, duo anguli IAB, IBA *a* recti ſunt. *b* Q. E. A.

a *hyp.* & 3. *def.* 11.
b 17. 1.

PROP. XV.

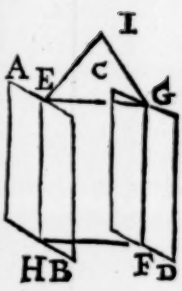
Si duæ rectæ lineæ AB, AC *ſe mutuo tangentes, ad duas rectas* DE, DF *ſe mutuo tangentes ſint parallelæ, non in eodem conſiſtentes plano; parallela ſunt, quæ per illa dicuntur, plana* BAC, EDF.

Ex A *a* duc AG rectam plano EF. *b* Sintque GH, GI parallelæ ad DE, DF. *c* erunt hæ parallelæ etiam ad AB, AC. Cum igitur anguli IGA, HGA *d* ſint recti, *e* erunt etiam CAG, BAG recti. *f* ergo GA recta eſt plano BC; *g* atqui eadem recta eſt plano EF. *h* ergo plana BC, EF ſunt parallela. Q. E. D.

a 11. 11.
b 31. 1.
c 9. 11.
d 3. *def.* 11.
e 29. 1.
f 4. 11.
g *conſtr.*
h 14. 11.

PROP.

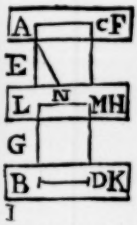
PROP. XVI.



Si duo plana parallela AB, CD, *plano quopiam* HEIGF *secentur, communes illorum sectiones* EH, GF *sunt parallelæ.*

Nam si dicantur non esse parallelæ, cum sint in eodem plano secanti, convenient alicubi, puta in I. quare cum totæ HEI, FGI *a* sint in planis AB, CD productis, etiam hæc convenient, contra hypoth. a 1. 11.

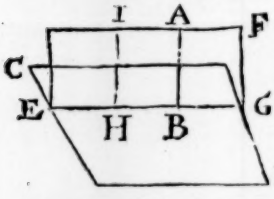
PROP. XVII.



Si duæ rectæ lineæ ALB, CMD *parallelis planis* EF, GH, IK *secentur, in easdem rationes secabuntur* (AL. LB :: CM. MD.)

Ducantur in planis EF, IK rectæ AC, BD. item AD occurrens plano GH in N; junganturque NL, NM. Plana triangulorum ADC, ADB faciunt sectiones BD, LN; & AC, NM *a* parallelas. ergo AL. LB *b* :: AN. ND *b* :: CM. MD. Q. E. D. a 16. 11. b 2. 6.

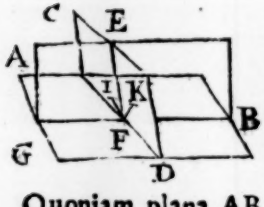
PROP. XVIII.



Si recta linea AB *plano cuipiam* CD *ad rectos sit angulos; & omnia, quæ per ipsam* AB *plana* (EF, &c.) *eidem plano* CD *ad rectos angulos erunt.*

Ductum sit per AB planum aliquod EF, faciens cum plano CD sectionem EG; è cujus
a 31. 1. aliquo puncto H, in plano EF *a* ducatur HI paral-
b 8. 11. lel. AB. *b* erit HI recta plano CD; pariterque
c 4. *def.* 11. aliæ quævis ad EG perpendiculares. *c* ergo planum EF plano CD rectum est; eademque ratione quævis alia plana per AB ducta plano EF recta erunt. Q. E. D.

PROP. XIX.



Si duo plana A B, CD, *se mutuo secantia, plano cuidam* G H *ad rectos sint angulos, communis etiam illorum sectio* EF *ad rectos eidem plano* (GH) *angulos erit.*

Quoniam plana AB, CD ponuntur recta plano GH, patet ex 4. def. 11. quod ex puncto F in utroque plano A B, C D duci possit per-
a 13. 11. pendicularis plano GH; quæ *a* unica erit, & propterea eorundem planorum communis sectio. Q. E. D.

PROP.

PROP. XX.

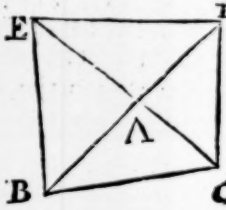
Si ſolidus angulus ABCD *tribus angulis planis* BAD, DAC, BAC *contineatur; ex his duo quilibet, utut aſſumpti, tertio ſunt majores.*

Si tres anguli ſunt æquales, patet aſſertio; ſi inæquales, maximus eſto BAC. ex quo *a* aufer BAE=BAD; & fac AD=AE; ducanturque BEC, BD, DC.

Quoniam latus BA commune eſt, & AD *b*= AE; & ang. BAE *b* = BAD; *c* erit BE=BD. ſed BD + DC *d* ⊏BC. *e* ergo DC⊏EC. cum igitur AD *b* =AE, & latus AC commune eſt, ac DC⊏EC *f*, erit ang. CAD⊏EAC. *g* ergo ang. BAD+CAD⊏BAC. Q. E. D.

a 23. 1.
b *conſtr.*
c 4. 1.
d 20. 1.
e 5. *ax.* 1.
f 25. 1.
g 4. *ax.* 1.

PROP. XXI.



Omnis ſolidus angulus A *ſub minoribus quam quatuor rectis angulis planis continetur.*

Latera enim ſolidi anguli A ſecans planum utcunque faciat figuram multilateram BCDE, & totidem triangula ABC, ACD, ADE, AEB. Omnes angulos polygoni voco X; & ſummam angulorum ad trigonorum baſes voco Y. quare X + 4 Rect. *a* = Y + A. Quia vero (ex angulis ad B) *b* eſt ang. ABE + ABC⊏CBE; idemque verum ſit de angulis ad C, ad D, ad E. *c* liquet fore Y ⊏X. proinde erit A⌐4 Rect. Q. E. D.

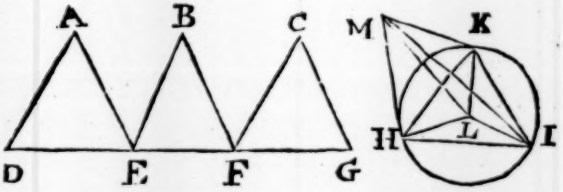
a 32. 1. & *ſch.* 32. 1.
b 20. 11.
c 5. *ax.* 1.

PROP. XXII.

Si fuerint tres anguli plani A, B, HCI, *quorum duo utlibet assumpti reliquo sint majores; comprehendant autem ipsos rectæ lineæ æquales* AD, AE, FB, &c. *fieri potest, ut ex rectis lineis* DE, FG, HI, *æquales illas rectas connectentibus triangulum constituatur.*

a 22. I. Ex ils *a* constitui potest triangulum, si duæ quælibet reliqua majores existant; sed ita se res habet. Nam *b* fac ang. HCK=B, & CK=CH, b 23. I. ducanturque HK, IK. *c* ergo KH=FG. & quia c 4. I. ang. KCI *d*⊏A; erit KI⊏DE. sed KI *f*⊐ d *hyp.* HI+KH (FG;) ergo DE⊐HI+FG. Simili e 24. I. argumento quævis duæ reliqua majores f 20. I. ostendentur; & proinde ex iis triangulum *a* constitui potest. Q. E. D.

PROP. XXIII.



Ex tribus angulis planis A, B, C, *quorum duo quomodocunque assumpti reliquo sunt majores, solidum angulum* MHIK *constituere.* **Oportet autem illos tres angulos quatuor rectis minores esse.*

* 21. 11.

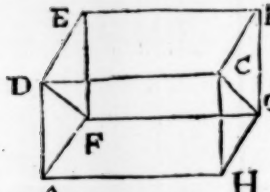
Fac

Fac AD, AE, BE, BF, CF, CG æquales inter ſe. Ex ſubtenſis DE, EF, FG (hoc eſt, ex æqualibus HI, IK, KH) *a* fac triang. HKI. circa quod *b* deſcribatur circulus LHKI. * Quoniam vero AD ⊏ HL; *c* ſit A Dq = H Lq + LMq. *d* ſitque LM recta plano circuli HKI; & ducantur HM, KM, IM. Quoniam igitur ang. HLM *e* rectus eſt, *f* erit MHq = HLq + LMq *g* = ADq. ergo MH = AD. ſimili argumento MK, MI, AD (id eſt, AE, EB, &c.) æquantur; ergo cum HM = AD, & MI = AE; & DE *b* = HI, *k* erit ang. A = HMI; *k* ſimiliter ang. IMK = B. *k* & ang. HMK = C. Factus eſt igitur angulus ſolidus ad M ex tribus planis datis. Q. E. F. Aſſumptum eſt fore A D ⊏ H L. Hoc autem conſtat. Nam ſi AD = vel ⊐ HL, erit ang. A *a* =, *b* vel ⊏ HLI. Eodem modo erit B =, vel ⊏ HLK, & C =, vel ⊏ KLI. quare A + B + C *quatuor rectos aut exæquabunt, aut excedent, contra hypoth. quin potius ſit AD ⊏ HL. Q. E. D.

a 22. 11. & 22. 1.
b 5. 4.
* *Vid. Clavium.*
c *ſcb.* 47. 1.
d 12. 11.
e 3. *def.* 11.
f 47. 1.
g *conſtr.*
h *conſtr.*
k 8. 1.
a *conſtr.* & 8. 1.
b 21. 1.
* 4. *cor.* 13. 1.

PROP.

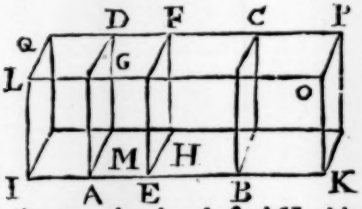
PROP. XXIV.



Si ſolidum AB *parallelis planis contineatur, adverſa illius plana* (AG, DB, &c.) *parallelogramma ſunt ſimilia & æqualia.*

Planum AC ſecans
a 16. 11. plana parallela AG, DB, *a* facit ſectiones AH, DC parallelas. Eadem ratione AD, HC parallelæ ſunt. Ergo ADCH eſt parallelogrammum. Simili argumento reliqua parallelepipedi plana
b 35. *def.* 1. ſunt *b* parallelogramma. Quum igitur AF ad
c 10. 11. HG, & AD ad HC parallelæ ſint, *c* erit ang.
d 34. 1. FAD = CHG; ergo ob AF *d* = HG, & AD *d* =
e 7. 5. HC, ac *e* propterea AF. AD :: HG. HC, trian-
g 6. 6. gula FAD, GAH *g* ſimilia ſunt & *h* æqualia; pro-
h 4. 1. inde & parallelogramma AE, HB ſimilia ſunt &
k 6. *ax.* 1. *k* æqualia. idemque de reliquis oppoſitis planis oſtendetur. ergo, &c.

PROP. XXV.



Si ſolidum parallelepipedum ABCD *plano* EF *ſecetur adverſis planis* AD, BC *parallelo, erit quemadmodum baſis* AH *ad baſim* BH, *ita ſolidum* AHD *ad ſolidum* BHC.

Concipe Ppp. ABCD produci utrinque. accipe AI = AE, & BK = EB; & pone plana IQ, KP planis AD, BC parallela. parallelo-
a 36. 1. & gramma IM, AH, *a* & DL, DG, *b* & IQ, AD,
1. *def.* 6. EF, &c. *a* ſimilia ac æqualia ſunt; *c* quare Ppp.
b 24. 11. AQ = AF; atque eadem ratione Ppp. BP =
c 10. *def.* 11 BF. ergo ſolida IF, EP ſolidorum AF, EC æque-

quemultiplicia ſunt, ac baſes IH, KH baſium AH, BH. Quod ſi baſis IH ⌐, =, ¬ KH, *d* erit ſimiliter ſolidum IF ⌐, =, ¬ EP. *e* proinde AH. BH :: AF. EC. Q. E. D. d 24. 11. & 9. *def.* 11. e 6. *def.* 5.

Hæc eadem omni priſmati accommodari poſſunt; unde

Coroll.

Si priſma quodcunque ſecetur plano oppoſitis planis parallelo, ſectio erit figura æqualis, & ſimilis planis oppoſitis.

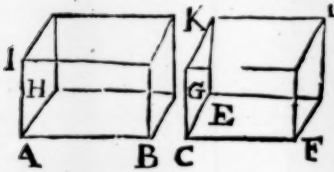
PROP. XXVI.

Ad datam rectam lineam AB, *ejuſque punctum* A, *conſtituere angulum ſolidum* AHIL, *æqualem ſolido angulo dato* CDEF.

A puncto quovis F *a* demitte FG plano DCE rectam; ducanturque rectæ DF, FE, EG, GD, CG. Fac AH = CD, & ang. HAI = DCE. & AI = CE; atque in plano HAI, fac ang. HAK = DCG, & AK = CG. Tum erige KL rectam plano HAI, & ſit KL = GF. ducaturque AL. erit angulus ſolidus AHIL par dato CDEF. Nam hujus conſtructio illius conſtitutionem penitus æmulatur, ut facile patebit examinanti. ergo factum. a 11. 11.

PROP.

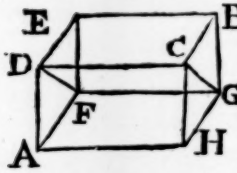
PROP. XXVII.



A data rectâ linea AB, *dato ſolido parallelepipedo* CD *ſimile & ſimiliter poſitum parallelepipedum* AK *deſcribere.*

Ex angulis planis BAH, HAI, BAI, quæ-
a 26. 11. quales ſint ipſis FCE, ECG, FCG, *a* fac angu-
b 12. 6. lum ſolidum A ſolido C parem. item *b* fac FC.
c 22. 5. CE :: BA. AH. *b* ac CE.CG :: AH. AI (*c* unde erit ex æquali FC.CG :: BA. AI;) & perficiatur Ppp. AK. erit hoc ſimile dato.
d 1. *def.* 6. Nam per conſtr. Pgra *d* BH, FE; *d* & HI,
e 24. 11. EG; & *d* BI, FG ſimilia ſunt, & *e* horum ideo oppoſita illorum oppoſitis. ergo ſex plana ſolidi
f 9. *def.* 11. AK ſimilia ſunt ſex planis ſolidi CD. *f* proinde AK, CD ſimilia ſolida exiſtunt. Q. E. F.

PROP. XXVIII.

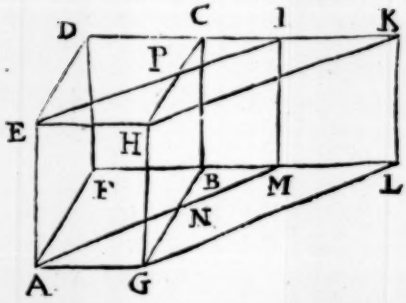


Si ſolidum parallelepipedum AB *plano* FGCD *ſecetur per diagonios* DF, CG *adverſorum planorum* AE, HB, *bifariam ſecabitur ſolidum* AB *ab ipſo plano* FGCD.

a 24. 11. Nam quia DC, FG *a* æquales & parallelæ
b 34. 1. ſunt, *b* planum FGCD eſt pgr. & propter *a* pgra AE, HB æqualia, & ſimilia, *b* etiam triangula AFD, HGC, CGB, DFB æqualia & ſimilia ſunt. Atqui pgra AC, AG ipſis FB, FD *a* etiam æqualia & ſimilia ſunt. ergo priſmatis FGCDAH omnia plana æqualia ſunt, & ſimili-
c 9. *def.* 11. a planis omnibus priſmatis FGCDEB; & *c* proinde hoc priſma illi æquatur. Q. E. D.

PROP.

PROP. XXIX.



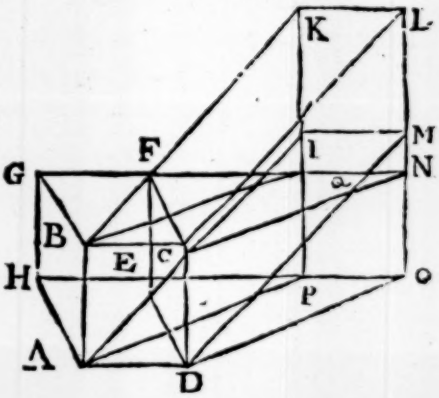
Solida parallelepipeda A G H E F B C D, A G H E M L K I *ſuper eandem baſim* A G H E *conſtituta, & * in eadem altitudine; quorum inſiſtentes lineæ* AF, AM *in iiſdem collocantur rectis lineis* AG, FL, *ſunt inter ſe æqualia.*

* *Id eſt, inter parallela plana*

Nam ſi ex *a* æqualibus priſmatis AFMEDI, A G H E, G B L H C K commune auferatur priſma FLKD, *& fic intellige in ſequent.* N B M P C I, addaturque utrinque ſolidum A G N E H P, *b* erit Ppp. AGHEFBCD = AGHEMLKI. Q. E. D.

a 10. *def.* 11. & 35. 1
b 3. & 2. *ax.* 1.

PROP. XXX.



Solida parallelepipeda A D B C H E F G,

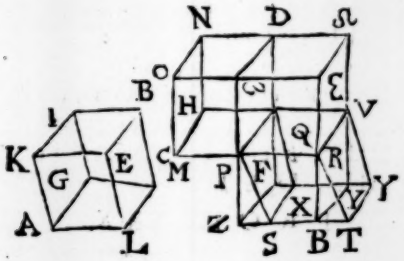
AD-

ADCBIMLK *ſuper eandem baſim* ADCB *conſtituta, & in eadem altitudine, quorum inſiſtentes lineæ* AH, AI *non in iiſdem collocantur rectis lineis, inter ſe ſunt æqualia.*

Nam produc rectas HEO, GFN, & LMO, KIP; & duc AP, DO, BQ, CN. *a* erunt tam DC, AB, HG, EF, PQ, ON; quam AD, HE, GF, BC, KL, IM, QN, PO æquales inter ſeſe & parallelæ. *b* Quare Ppp. ADCBPONQ utrique Pppo. ADCBHEFG, ADCBIMLK æquale eſt; & *c* proinde hæc ipſa inter ſe æqualia ſunt. Q. E. D.

a 34. 1.
b 29. 11.
c 1. *ax.* 1.

PROP. XXXI.



Solida parallelepipeda ALEKGMBI, CPωOHQDN *ſuper æquales baſes* ALEK, *Altitudo, eſt perpendicularis à plano baſis* CPωO *conſtituta, & * in eadem altitudine, æqualia ſunt inter ſe.*

Habeant primo parallelepipeda AB, CD latera ad baſes recta; & ad latus CP productum *ad planum oppoſitum.* fiat pgr. PRTS æq. & ſimile pgro KELA; adeoque Ppp. PRTSQVYX æq. & ſim. Pppo AB. Producantur OωE, NDϛ, ωPZ, DQF, ERB, ϛVγ, TSZ, YXF; & duc Eϛ, Bγ, ZF.

Plana OEϛN, CRVH, STYF *c* parallela ſunt inter ſe; *d* & pgra ALEK, CPωO, PRTS, PRBZ æqualia ſunt. Cum igitur Ppp. CD

a 18. 6.
b 27. 11 & 10. *def.* 11.
c 30 *def.* 11
d *hyp.* & 35. 3.

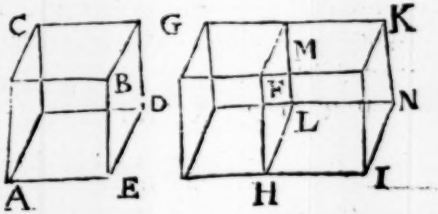
CD. PV ∫ω e :: pgr. Cω (PRBZ.) Rω :: Ppp. PRBZQVγF. PV ∫ω, *f* erit Ppp. CD *f* = PRBZQVγF *g* = PRVQSTYX *b* = AB. Q. E. D.

e 25. 11.
f 9. 5.
g 29. 12.
h *constr.*

Sin Pppa AB, CD latera baſibus obliqua habeant; ſuper eaſdem baſes, & in eadem altitudine, ponantur parallelepipeda, quorum latera baſibus ſint recta. *k* Ea inter ſe, & obliquis æqualia erunt; *m* proinde & obliqua AB, CD æquantur. Q. E. D.

k 29. 11.
m 1. *ax.* 1.

PROP. XXXII.

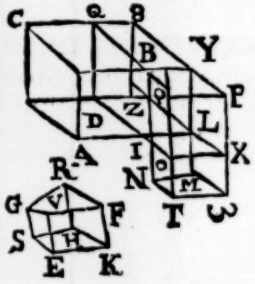


Solida parallelepipeda ABCD, EFGL *ſub eadem altitudine, inter ſe ſunt ut baſes* AB, EF.

Producta EHI, *a* fac pgr. FI = AB, & *b* comple Ppp. FINM. Liquet eſſe Ppp. FINM. (*c* ABCD.) EFGL *d* :: FI. (AB) EF. Q. E. D.

a 45. 1.
b 31. 5.
c 31. 11.
d 25. 12.

PROP. XXXIII.



Similia ſolida parallelepipeda, ABCD, EFGH, *inter ſe ſunt in triplicata ratione homologorum laterum* AI, EK.

Producantur rectæ AIL, DIO, BIN. & *a* fiant IL, IO, IN ipſis EK, KH, KF æquales, *b* adeoque

a 3. 1.
b 17. 11.

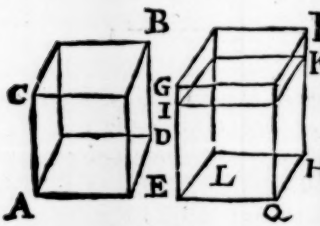
&

c 31. 1. & Ppp. IXMT æq. & ſim. Pppo EFGH.
d *hyp.* *c* Perficiantur Ppp. *a* IXPB, DLYQ. Itaque *d*
e 1. 6. erit AI. IL. (EK) :: DI. IO (HK) :: BI. IN.
f 32. 11. (KF;) hoc eſt Pgr. AD DL :: DL. IX :: BO. IT; *f* id eſt Ppp. ABCD DLQY ::
g *conſtr.* DLQY. IXBP :: IXBP. IXMT. (*g* EFGH.)
h 10. *def.* 5. *b* ergo ratio ABCD ad EFGH triplicata eſt ra-
k 1. 6. tionis ABCD ad DLQY, *k* vel AI ad EK. Q. E. D.

Coroll.

Hinc, ſi fuerint quatuor lineæ rectæ continuæ proportionales, ut eſt prima ad quartam, ita eſt parallelepipedum ſuper primam deſcriptum ad parallelepipedum ſimile ſimiliterque deſcriptum ſuper ſecundam.

PROP. XXXIV.



Æqualium ſolidorum parallelepipedorum ADCB, EHGF *baſes & altitudines reciprocantur* (AD. EH :: EG. AC) *Et quorum ſolidorum parallelepipedorum* ADCB, EHGF *baſes & altitudines reciprocantur, illa ſunt æqualia.*

Sint primo latera CA, GE ad baſes recta; ſi jam ſolidorum altitudines ſint pares, etiam
a 3. 1. baſes æquales erun. & res clara eſt. Sin altitu-
b 31. 1. dines inæquales ſint, à majori EG *a* detrahe EI
c 32. 11. = AC. & per I *b* duc planum IK parallelum
d 17. 5. baſi EH. itaque
e 1. 6. 1. *Hyp.* AD. EH *c* :: Ppp. ADCB. EHIK *d* :
f *conſtr.* Ppp. EHGF. EHIK *c* :: GL. IL *e* :: GE. IE
g 11. 5. & (ſ AC;) *g* Iquet igitur eſſe AD. EH :: GE. AC
32. 11. Q. E. D.

2. *Hyp.*

2. *Hyp.* ADCB. EHIK *b* :: AD. EH *k* :: EG. EI *l* :: GL. IL *m* :: Ppp. EHGF. EHIK, *n* quare Ppp. ADCB = EHGF. Q. E. D.

Sint deinde latera ad baſes obliqua. Eriguntur ſuper iiſdem baſibus, in altitudine eadem, parallelepipeda recta. Erunt obliqua parallelepipeda his æqualia. Quare cum hæc per 1. partem reciprocent baſes & altitudines, etiam illa reciprocabunt. Q. E. D.

h 32. 11.
k *hyp.*
l 1. 6.
m 32. 11.
n 9. 5.

Coroll.

Quæ de parallelepipedis demonſtrata ſunt Prop. 29, 30, 31, 32, 33, 34. *etiam conveniunt priſmatis triangularibus, quæ ſunt dimidia parallelepipedi, ut patet ex* Pr. 28. *Igitur,*

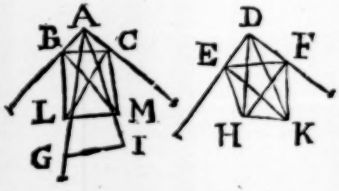
1. Priſmata triangularia æque alta ſunt ut baſes.

2. Si eandem vel æquales habeant baſes, & eandem altitudinem, æqualia ſunt.

3. Si ſimilia fuerint, eorum proportio triplicata eſt proportionis homologorum laterum.

4. Si æqualia ſunt, reciprocant baſes & altitudines, & ſi reciprocant baſes & altitudines, æqualia erunt.

PROP. XXXV.



Si fuerint duo plani anguli BAC, EDF *æquales, quorum verticibus* A, D, *ſublimes rectæ lineæ* AG, DH *inſiſtant, quæ cum lineis primo poſitis angulos contineant æquales, utrumq; utriq;* (ang. GAB = HDE; & GAC = HDF.) *in ſublimibus autem lineis* AG, DH *quælibet ſumpta fuerint puncta* G, H;

& ab his ad plana BAC, EDF, *in quibus consistunt anguli primum positi* BAC, EDF, *ductæ fuerint perpendiculares* GI, HK; *à punctis vero* I, K *quæ in planis à perpendicularibus fiunt, ad angulos primum positos adjunctæ fuerint rectæ lineæ* AI, DK; *hæ cum sublimibus* AG, DH *æquales angulos* GAM, HDK *comprehendent.*

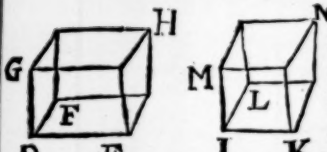
Fiant DH, AL æquales, & GI, LM parallelæ; & MC ad AC, MB ad AB, KF ad DF, KE ad DE perpendiculares, ducanturque rectæ
a 8. 11. BC, LB, LC, atque EF, HF, HE; *a* estque LM
b 3. *def.* 11 recta plano BAC; *b* quare anguli LMC, LMA, LMB; eademque ratione anguli HKF, HKD,
c 47. 1. HKE recti sunt. Ergo ALq *c* = LMq + AMq *c* = LMq + CMq + ACq *c* = LCq + ACq;
d 48. 1. *d* ergo ang. ACL rectus est. Rursus ALq *e* =
e 47. 1. LMq + MAq *e* = LMq + BMq + BAq *e* = BLq + BAq. *d* ergo ang. ABL etiam rectus est. Simili discursu anguli DFH, DEH recti
f 26. 1. sunt, *f* ergo AB = DE; *f* & BL = EH; *f* & AC = DF; & CL = FH. *g* quare etiam BC
g 4. 1. = EF, *g* & ang. ABC = DEF *g* & ang. ACB
h 3. *ax.* 1. = DFE. unde reliqui è rectis anguli CBM,
k 26. 1. BCM reliquis FEK, EFK æquantur. *k* ergo
l 47. 1. CM = FK, *l* ideoque & AM = DK. ergo si
m *constr.* ex LAq *m* = HDq. auferatur AMq = DKq,
n 47. 1. & *n* remanet LMq = HKq. quare trigona LAM,
3. *ax.* HDK sibi mutuo æquilatera sunt. *o* ergo ang.
o 8. 1. LAM = HDK. Q. E. D.

Coroll.

Itaque si fuerint duo anguli plani æquales, quorum verticibus sublimes rectæ lineæ æquales insistant, quæ cum lineis primo positis angulos contineant æquales, utrumque utrique; erunt à punctis extremis linearum sublimium ad plana angulorum primo positorum demissæ perpendiculares inter se æquales; nempe LM = HK.

PROP.

PROP. XXXVI.

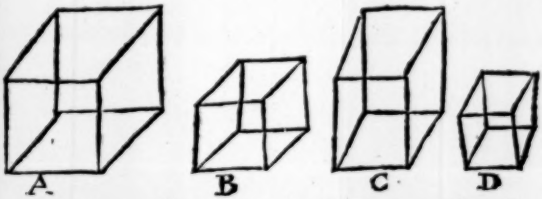


Si tres rectæ lineæ DE, DG, DF *proportionales fuerint; quod ex his tribus fit ſolidum parallelepipedum* DH, *æquale eſt deſcripto à media linea* DG (IL) *ſolido parallelepipedo* IN, *quod æquilaterum quidem ſit, æquiangulum vero prædicto* DH.

Quoniam DE. IK *a* :: IL. DF, *b* erit pgr. LK = FE. & propter angulorum planorum ad D & I, ac linearum GD, IM æqualitatem, etiam altitudines parallelepipedorum æquales ſunt, ex coroll. præced. *c* ergo ipſa inter ſe æqualia ſunt, Q. E. D.

a hyp.
b 14. 6.
c 31. 11.

PROP. XXXVII.

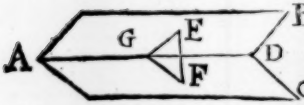


Si quatuor rectæ lineæ A, B, C, D *proportionales fuerint, & ſolida parallelepipeda* A, B, C, D *quæ ab ipſis & ſimilia, & ſimiliter deſcribuntur, proportionalia erunt. Et ſi ſolida parallelepipeda, quæ & ſimilia, & ſimiliter deſcribuntur, fuerint proportionalia* (A. B :: C. D.) *& ipſæ rectæ lineæ* A, B, C, D *proportionales erunt.*

Nam rationes parallelepipedorum *a* triplicatæ ſunt rationum, quas habent lineæ. ergo ſi A. B :: C D. *b* erit Ppp. A. Ppp. B :: Ppp. C. Ppp. D. & vice verſa.

a 33. 11.
b ſch. 23. 5.

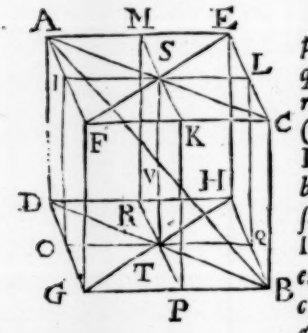
PROP. XXXVIII.



Si planum AB *ad planum* AC *rectum fuerit, & ab aliquo puncto* E *eorum, quæ sunt in uno planorum* (AB) *ad alterum planum* AC *perpendicularis* EF *ducta fuerit, in planorum communem sectionem* AD *cadet ducta perpendicularis* EF.

Si fieri potest, cadat F extra intersectionem
a 12. 1. AD. In plano AC *a* ducatur FG perpendicula-
b 4, & 3. ris ad AD, jungaturque EG. Angulus FGE *b* re-
def. 11. ctus est; & EFG rectus ponitur. ergo in triangu-
c 17. 1. lo EFG sunt duo anguli recti. Q. E. D.

PROP. XXXIX.



Si solidi parallelepipedi AB, *eorum quæ ex adverso planorum* AC, DB *latera* (AE, FC, AF, EC, & DH, GB, DG, HB) *bifariam secta sint; per sectiones autem planæ* ILQO, PKMR *sint extensa; planorum communis sectio* ST, *& solidi parallelepipedi diameter* AB, *bifariam se mutuo secabunt.*

a 34. 1. Ducantur rectæ SA, SC, TD, TB. Propter
b 29. 1. *a* latera DO, OT lateribus BQ. QT, *b* angu-
c 4. 1. losque alternos TOD, TQB æquales, *c* etiam
d *sch.* 15. 1. bases DT, TB, & anguli DTO, BTQ æquan-
e 34. 1. tur. *d* ergo DTB est recta linea. eodem modo
f 9. 11. & AS C recta est linea. Porro *e* tam AD ad FG,
1. *ax.* *c* quam FG ad CB; *f* ideoque AD ad CB, *g* ac
g 33. 1. proinde AC ad DB parallelæ & æquales sunt.

b quare

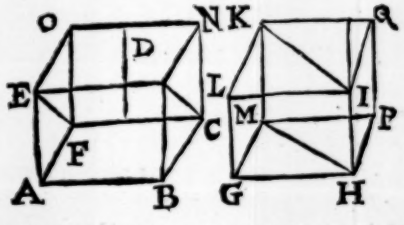
b quare AB, & ST in eodem plano ABCD exſiſtunt. Itaque cum anguli AVS, BVT ad verticem, & alterni ASV, BTV æquentur; *k* & AS = BT; erit AV = BV, *l* & SV = VT. Q. E. D.

h 7. 11.
k 7. *ax.* 1.
l 26. 1.

Coroll.

Hinc, in omni parallelepipedo diametri omnes ſe mutuo biſecant in uno puncto, V.

PROP. XL.



Si fuerint duo priſmata ABCFED, GHMLIK *æqualis altitudinis, quorum hoc quidem habeat baſim* ABCF *parallelogrammum, illud vero* GHM *triangulum; duplum autem fuerit parallelogrammum* ABCF *trianguli* GHM; *æqualia erunt ipſa priſmata* ABCFED, GHMLIK.

Nam ſi perficiantur parallelepipeda AN, GQ, *a* erunt hæc æqualia ob *b* baſium AC, GP, & *c* altitudinum æqualitatem. *d* ergo etiam priſmata, *e* horum dimidia, æqualia erunt. Q. E. D.

a 31. 11.
b 34. 1. & 7. *ax.*
c *hyp.*
d 28. 11.
e 7. *ax.* 1. *And. Tar.*

Schol.

Ex hactenus demonſtratis habetur dimenſio priſmatum triangularium, & quadrangularium, ſeu parallelepipedorum, ſi nimirum altitudo ducatur in baſim.

Ut ſi altitudo ſit 10 pedum, baſis vero pedum quadratorum 100 (menſurabitur autem baſis per Sch. 35. 1. vel per 41. 1.) multiplica 100 per 10;

proveniunt 1000 pedes cubici pro soliditate prismatis dati.

Vide schol. 35. 1. Nam quemadmodum rectangulum, ita & parallelepipedum rectum producitur ex altitudine ducta in basim. Ergo quodvis parallelepipedum producitur ex altitudine in basim ducta, ut patet ex 31. hujus.

Deinde cum totum parallelepipedum producatur ex altitudine in totam basim, semissis ejus (hoc est prisma triangulare) producetur ex altitudine ducta in dimidiam basim, nempe triangulum.

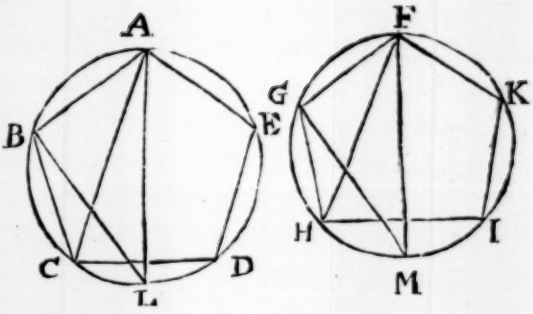
Monitum.

Nota, litterarum quæ designant angulum solidum primam esse semper ad punctum, in quo est angulus; litterarum vero quæ denotant pyramidem, ultimam esse ad verticem pyramidis.

Ex. gr. Angulus solidus ABCD est ad punctum A; pyramidis quoque B C D A vertex est ad punctum A, & basis triangulum BCD.

LIB. XII.

PROP. I.



Quæ sunt in circulis ABD, FGI *polygona similia* ABCDE, FGHIK, *inter se sunt, ut quadrata à diametris* AL, FM.

Ducantur AC, BL, FH, GM. Quoniam *a* ang. ABC = FGH, *a* atque AB. BC :: FG. GH, *b* erit ang. ACB (*c* ALB) = FHG (*c* FMG.) anguli autem ABL, FGM *d* recti, ac proinde æquales sunt. *e* ergo triangula ABL, FGM æquiangula sunt. *f* quare AB. FG :: AL. FM. *g* ergo ABCDE. FGHIK :: ALq. FMq.

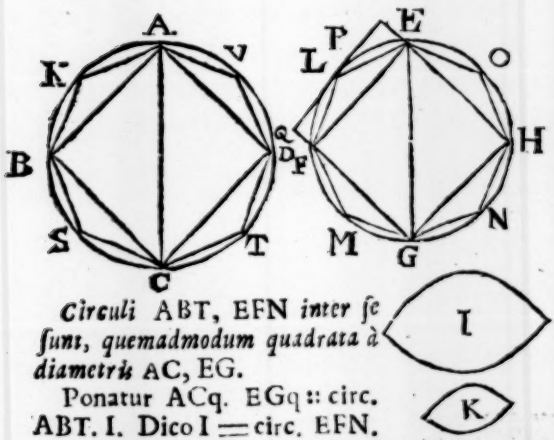
a 1. *def.* 6.
b 6. 6.
c 21. 3.
d 31. 3.
e 32. 3.
f *cor.* 4. 6.
g 22. 6.

Coroll.

Hinc (quia AB. FG :: AL. FM :: BC. GH, &c.) polygonorum similium circulo inscriptorum *b* ambitus sunt ut diametri.

h 1. 12. & 12. 5.

PROP. II.



Circuli ABT, EFN *inter se sunt, quemadmodum quadrata à diametris* AC, EG.

Ponatur ACq. EGq :: circ. ABT. I. Dico I = circ. EFN.

Nam primo, si fieri potest, sit I ⊐ circ. EFN, sitque excessus K. Circulo EFN inscribatur
a *sch.* 7. 4. quadratum EFGH, *a* quod dimidium est circumscripti quadrati, adeoque semicirculo majus.
b 30. 3. *b* Bisecta arcus EF, FG, GH, HE, & ad puncta bisectionum junge rectas EL, LF, &c. per L
c *sch.* 27. 3. duc tangentem PQ (*c* quæ ad EF parallela est,) & produc HEP, GFQ; estque triangulum
d 41. 1. ELF *d* dimidium parallelogrammi EPQF, adeoque majus dimidio segmenti ELF; pariterque reliqua triangula ejusmodi reliquorum segmentorum dimidia superant. Et si iterum bisecentur arcus EL, LF, FM, &c. rectæque adjungantur, eodem modo triangula segmentorum semisses excedent. Quare si quadratum EFGH è circulo EFN, & e reliquis segmentis triangula
e 1. 10. detrahantur, & hoc fiat continuo, tandem *e* restabit magnitudo aliqua minor quam K. Eousque perventum sit, nempe ad segmenta EL, LF, FM, &c. minora quam K, simul sumpta,

pta. ergo I (*f* circ. EFN − K) ⊐ polyg. ELFMGNHO (circ. EFN − segm. EL + LF &c.) Circulo ABT inſcriptum *g* puta ſimile polygonum AKBSCTDV. itaque quum AKBSCTDV. ELFMGNHO *h* :: ACq. EGq *k* :: circ. ABT. I. ac polyg. AKBSCTDV *l* ⊐ circ. ABT. *m* erit polyg. ELFMGNHO ⊐ I. ſed prius erat I ⊐ ELFMGNHO. quæ repugnant.

f *hyp.* & 1. *ax.*
g 30. 3. & 1. *poſt.* 1.
h 1. 12.
k *hyp.*
l 9. *ax.* 1.
m 14. 5.

Rurſus, ſi fieri poteſt, ſit I ⊏ circ. EFN. Quoniam igitur ACq. EGq *n* :: circ. ABT. I; inverſeque I. circ. ABT :: EGq. ACq. pone I. circ. ABT :: circ. EFN. K. *o* ergo circ. ABT ⊏ K, *p* atque EGq. ACq :: circ. EFN. K. Quæ repugnare modo oſtenſum eſt.

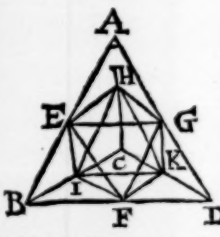
n *hyp.*
o 14. 5.
p 11. 5.

Ergo concludendum eſt, quod I = circ. EFN, Q. E. D.

Coroll.

Hinc, ut circulus eſt ad circulum, ita polygonum in illo deſcriptum ad ſimile polygonum in hoc deſcriptum.

PROP. III.



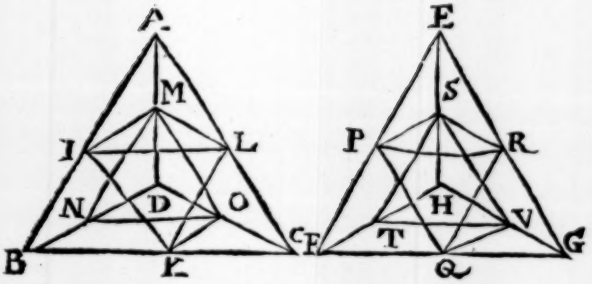
Omnis pyramis ABDC *triangularem habens baſim, dividitur in duas pyramides* AEGH, HIKC *æquales & ſimiles inter ſe, triangulares habentes baſes, & ſimiles toti* ABDC; *& in duo priſmata æqualia* BFGEIH, FGDIHK; *quæ duo priſmata majora ſunt dimidio totius pyramidis* ABDC.

Latera pyramidis biſecentur in punctis E, F, G, H, I, K; junganturque rectæ EF, FG, GE, EI, IF, FK, KG, GH, HE. Quoniam latera pyra-

a 2. 6. pyramidis proportionaliter ſecta ſunt, *a* erunt HI, AB; & GF, AB; & IF, DC; atque HG, DC, &c. parallelæ; proinde & HI, FG, & GH, FI parallelæ ſunt. liquet igitur triangula ABD,
b 29. 1. AEG, EBF, FDG, HIK *b* æquiangula eſſe; &
c 26. 1. quatuor ultima *c* æquari. eodem modo triangula ACB, AHE, EIB, HIC, FGK æquiangula ſunt; & quatuor poſtrema inter ſe æqualia. Similiter triangula BFI, FDK, IKC, EGH; & denuo triangula AHG, GDK, HKC, EFI, ſimilia ſunt & æqualia. Quinetiam triang. HIK ad ADB, & EGH ad BDC, & EFI ad ADC, & FGK ad
d 15. 11. ABC *d* parallela ſunt. Ex quibus perſpicue ſequitur primo, pyramides AEGH, HIKC æquales
e 10. *def.* 11 eſſe; totique ABDC, & inter ſe *e* ſimiles. deinde ſolida BFGEIH, FGDIHK priſmata eſſe, & quidem æque alta, nempe ſita inter parallela plana ABD, HIK. verum baſis BFGE baſis FDG
f 2. *ax.* 1. *f* duplex eſt. *g* quare dicta priſmata æqualia ſunt.
g 40. 11. quorum alterum BFGEIH pyramide BEFI, hoc eſt, AEGH majus eſt, totum ſua parte; proinde duo priſmata majora ſunt duabus pyramidibus, totiuſque adeo pyramidis ABDC dimidium excedunt. Q. E. D.

PROP.

PROP. IV.



Si fuerint duæ pyramides ABCD, EFGH *ejuſdem altitudinis, triangulares habentes baſes* ABC, EFG; *ſit autem illarum utraque diviſa & in duas pyramides* (AILM, MNOD; & EPRS, STVH) *æquales inter ſe, & ſimiles toti; & in duo priſmata æqualia* (IBKLMN, KLCNMO; & PFQRST, QRGTSV;) *ac eodem modo diviſa ſit utraque pyramidum, quæ ex ſuperiore diviſione natæ ſunt, idque ſemper fiat; erit ut unius pyramidis baſis ad alterius pyramidis baſim, ita & omnia, quæ in una pyramide, priſmata ad omnia, quæ in altera pyramide priſmata, multitudine æqualia.*

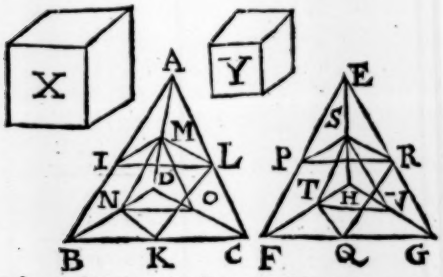
Nam (adhibendo conſtructionem præcedentis) BC. KC *a* :: FG. QG. *b* ergo triang. ABC eſt ad ſimile triang. LKC, ut EFG ad *c* ſimile RQG. ergo permutando ABC. EFG *d* :: LKC. RQG *e* :: Priſm. KLCNMO. QRGTSV (nam hæc æque alta ſunt) *f* :: IBKLMN. PFQRST. *g* quare triang. ABC. EFG :: Priſm. KLCNMO + IBKLMN. Priſm. QRGTSV + PFQRST. Q. E. D.

a 15. 5.
b 22. 6.
c 2. 6. &c.
d 16. 5.
e *ſch.* 34. 11
f 7. 5.
g 12. 5.

Sin ulterius ſimili pacto dividantur pyramides MNOD, AILM; & EPRS, STVH, erunt quatuor nova priſmata hic effecta ad quatuor iſthic

iſtbic producta, ut baſes MNO & AIL ad baſes STV & EPR, hoc eſt ut LKC ad RQG, vel ut
h 12. 5. ABC ad EFG. *h* quare omnia priſmata pyramidis ABCD ad omnia ipſius EFGH ita ſe habent, ut baſis ABC ad baſim EFG. Q.E.D.

PROP. V.



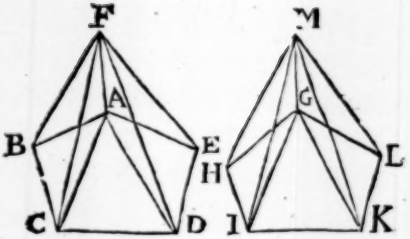
Sub eadem altitudine exiſtentes pyram ABCD, EFGH, *triangulares habentes baſes* ABC, EFG, *inter ſe ſunt ut baſes* ABC, EFG.

Sit triang. ABC. EFG :: ABCD. X. Dico X = pyr. EFGH. Nam, ſi poſſibile eſt, ſit X ⊐ EFGH; ſitque Y exceſſus. Dividatur pyramis EFGH in priſmata & pyramides, & reliquæ pyramides ſimiliter, *a* donec relictæ pyramides EPRS, STVH minores evadant ſolido Y.
a 1. 10. Quum igitur pyr. EFGH = X + Y; liquet reliqua priſmata PFQRST, QRGTSV ſolido X majora eſſe. Pyramidem ABCD ſimiliter diviſam concipe;
b 4. 12. *b* eritque priſm. IBKLMN + KLCNMO. PFQRST + QRGTSV ::
c *hyp.* ABC. EFG. *c* :: pyr. ABCD. X. *d* ergo X ⊏
d 14. 5. priſm. PFQRST + QRGTSV; quod repugnat prius affirmatis.

Rurſus, dic X ⊏ pyr. EFGH. pone pyr.
e *hyp. &* EFGH. Y :: X. pyr. ABCD *e* :: EFG. ABC.
cor. 4. 5. quia EFGH *f* ⊐ X, *g* erit Y ⊐ pyr. ABCD,
f *ſuppoſ.* quod fieri nequit, ex jam dictis. Concludo igitur,
g 14. 5. quod X = pyr. EFGH. Q.E.D.

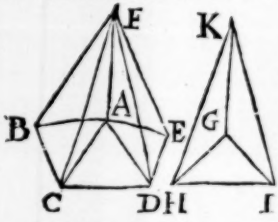
PROP.

PROP. VI.



Sub eadem altitudine exiſtentes pyramides ABCDEF, GHIKLM, *& polygonas habentes baſes* ABCDE, GHIKL, *inter ſe ſunt ut baſes* ABCDE, GHIKL.

Duc rectas AC, AD, GI, GK. Eſt baſ. ABC. ACD *a* :: pyr. ABCF. ACDF. *b* ergo compoſite a 5. 12.
ABCD. ACD :: pyr. ABCDF. ACDF. *a* atquib 18. 5.
etiam ACD. ADE :: pyr. ACDF. ADEF. *c* ergo ex æquali ABCD. ADE :: ABCDF. ADEF. *b* ergo componendo ABCDE. ADE :: pyr. c 22. 5.
ABCDEF. ADEF. porro ADE. GKL *d* :: pyr. d 5. 12.
ADEF. GKLM; ac, ut prius, atque inverſe GKL. GHIKL :: pyr. GKLM. GHIKLM. *c* ergo iterum ex æqualibus, ABCDE. GHIKL :: pyr. ABCDEF. GHIKLM. Q. E. D.



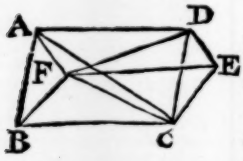
Si baſes non habent latera æque multa, demonſtratio ſic procedet. Baſ. ABC. GHI *e* :: pyr. ABCF. GHIK. *e* atque e 5. 12.
ACD. GHI :: pyr. f 24. 5.
ACDF. GHIK. *f* ergo baſ. ABCD. GHI :: pyr. ABCDF. GHIK.
iam baſ. ADE. GHI :: pyr. ADEF.
ergo baſ. ABCDE. GHI :: pyr.
GHIK. *f*. GHIK.

e Quinet

ABCDEF

PROP.

PROP. VII.

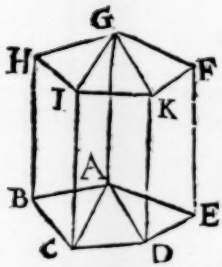


Omne prisma ABCDFE *triangularem habens basim, dividitur in tres pyramides* ACBF, ACDF, CDFE *æquales inter se, triangulares bases habentes.*

Ducantur parallelogrammorum diametri AC, CF, FD. Triang. ACB *a* = ACD *b* ergo æque altæ pyramides ACBF, ACDF æquantur. eodem modo pyr. DFAC = pyr. DFEC. atqui ACDF, & DFAC una eademque sunt pyramis. *c* ergo tres pyramides ACBF, ACDF, DFEC, in quas divisum est prisma, inter se æquales sunt. Q. E. D.

a 34. 1.
b 5. 12.
c 1. *ax.* 1.

Coroll.



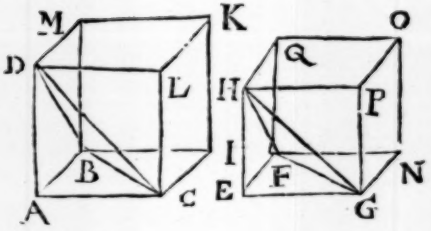
Hinc, quælibet pyramis tertia est pars prismatis eandem cum illa habentis & basim & altitudinem: sive, prisma quodlibet triplum est pyramidis eandem cum ipso habentis basim & altitudinem.

Nam resolve prisma polygonum ABCDEGHIKF in trigona prismata, & pyramidem ABCDEH in trigonas pyramides. *a* Erunt singulæ partes prismatis triplæ singularum partium pyramidis. *b* proinde totum prisma ABCDEGHIKF totius pyramidis ABCDEH triplum est. Q.E.D.

a 7. 12.
b 1. 5.

PROP.

PROP. VIII.



Similes pyramides ABCD, EFGH, *quæ triangulares habent baſes* ABC, EFG, *in triplicata ſunt ratione homologorum laterum* AC, EG.

a Perficiantur parallelepipeda ABICDMKL, EFNGHQOP; quæ *b* ſimilia ſunt & pyramidum ABCD, EFGH *c* ſextupla; *d* ideoque in eadem cum ipſis ratione ad ſe invicem, *e* hoc eſt in triplicata homologorum laterum. Q. E. D.

a 27. 11.
b 9. *def.* 11.
c 28. 11. &
7. 12.
d 15. 5.
c 33. 11.

Coroll.

Hinc, etiam ſimiles polygonæ pyramides rationem habent laterum homologorum triplicatam; ut facile probabitur reſolvendo has in trigonas pyramides.

PROP. IX.

Vide Schema præced.

Æqualium pyramidum ABCD, EFGH, *& triangulares baſes* ABC, EFG *habentium, reciprocantur baſes & altitudines: & quarum pyramidum triangulares baſes habentium reciprocantur baſes & altitudines, illæ ſunt æquales.*

1. *Hyp.* Perfecta parallelepipeda ABICDMKL, EFNGHQOP æqualium pyramidum ABCD, EFGH (utrumque utriuſque) *a* ſextupla ſunt, ac æqualia ideo inter ſe. ergo alt. (H) alt.

a 28. 11. & 7. 12.

b 34. 11. alt. (D) *b* :: ABIC. EFNG *c* :: ABC. EFG.
c 15. 5. Q. E. D.

d *hyp.* 2. *Hyp.* Alt. (H.) alt. (D) *d* :: ABC. EFG *e* ::
e 15. 5. ABIC. EFNG. *f* ergo parallelepipeda ABIC-
f 34. 11. DMKL, EFNGHQOP æquantur; *g* proinde
g 6. *ax.* 1. & pyramides ABCD, EFGH, horum ſubſextuplæ, pares ſunt. Q. E. D.

Eadem polygonis pyramidibus conveniunt: nam hæ ad trigonas reduci poſſunt.

Coroll.

Quæ de pyramidibus demonſtrata ſunt Prop. 6, 8, 9. *etiam conveniunt quibuſcunque priſmatis, cum hæc tripla ſint pyramidum eandem baſim & altitudinem habentium. itaque* 1. Priſmatum æque altorum eadem eſt proportio, quæ baſium.

2. Similium priſmatum proportio triplicata eſt proportionis laterum homologorum.

3. Æqualia priſmata reciprocant baſes & altitudines; & quæ reciprocant, ſunt æquales.

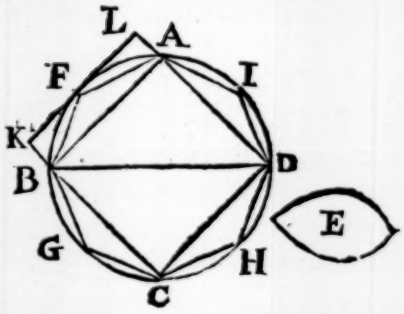
Schol.

Ex hactenus demonſtratis elicitur dimenſio quorumcunque priſmatum & pyramidum.

a cor. 1. *hujus; & ſch.* 40. 12. b 7. 12. *a* Priſmatis ſoliditas producitur ex altitudine in baſim ducta; *b* itaque & pyramidis ex tertia altitudinis parte ducta in baſin.

PROP.

PROP. X.



Omnis conus tertia pars est cylindri habentis eandem cum ipso basin ABCD, *& altitudinem æqualem.*

Si negas, primo Cylindrus triplum coni superet excessu E. Prisma super quadratum circulo ABCD inscriptum *a* subduplum est prismatis super quadratum eidem circulo circumscriptum sibi & cylindro æque alti. ergo prisma super quadratum ABCD superat cylindri semissem. eodem modo prisma super basim AFB cylindro æque altum segmenti cylindrici AFB *b* dimidio majus est. Continuetur bisectio arcuum, & detrahantur prismata, donec segmenta cylindri relicta, nempe ad AF, FB, &c. minora evadant solido E. Itaque cylind. — segment. AF, FB, &c. (prisma ad basim AFBGCHDI) *c* majus est quam cylind. — E (*d* triplum coni.) ergo pyramis dicti prismatis *e* pars tertia (ad eandem basim sita, ejusdemque altitudinis) cono æque alto ad basim ABCD circulum major est, pars toto. Q. E. A.

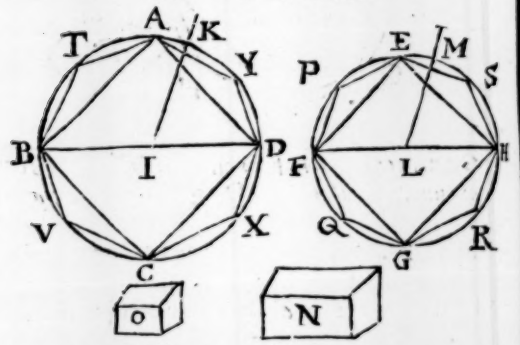
Vide fig. 2. hujus.
a *sch. 7. 4. & cor. 9. 12.*
b *sch. 27. 3. & cor. 9. 12.*
c *5. ax. 1.*
d *hyp.*
e *cor. 7. 12.*

Sin conus tertia parte cylindri major dicatur, sit iidem excessus E. Ex cono detrahe pyramides, ut in priori parte prismata ex cylindro, donec reliqua coni segmenta aliqua, puta ad AF,

FB, BG, &c. minora ſolido E. ergo con.—E (*f* ⅓ cylindr.) ⊐ pyr. AFBGCHDI (con.—ſegment. AF, FB, &c.) ergo priſma pyramidis triplum (æque altum ſcilicet atque ad eandem baſim) cylindro ad baſim ABCD majus eſt, pars toto. Q.E.A. Quare fatendum eſt, quod cylindrus triplo cono æquatur. Q.E.D.

f *hyp.*

PROP. XI.



Sub eadem altitudine exiſtentes cylindri, & coni ABCDK, EFGHM, *inter ſe ſunt ut baſes* ABCD, EFGH.

Sit circ. ABCD. circ. EFGH :: con. ABCDK. N. Dico N = con. EFGHM.

Nam ſi fieri poteſt, ſit N ⊐ con. EFGHM, ſitque exceſſus O. Suppoſita præparatione, & argumentatione præcedentis; erit O majus ſegmentis conicis EP, PF, FQ, &c. ideoque ſolidum N ⊐ pyr. EPFQGRHSM. *a* Fiat in circulo ABCD ſimile polygonum ATBVCXDY. Quia pyr. ABVYK. pyr. EFQSM *b* :: polyg. ATBVY. polyg. EPFQS *c* :: circ. ABCD. circ. EFGH *d* :: con. ABCDK. N. *e* erit pyram. EPFQGRHSM ⊐ N. contra modo dicta.

a 30. 3. & 1. *poſt.*
b 6. 12.
c *cor.* 2. 12.
d *hyp.*
e 14. 5.

Rurſus dic N ⊏ con. EFGHM. pone con. EFGHM. O :: N. con. ABCDK *f* :: circ. EFGH, ABCD. *g* ergo O ⊐ con. ABCDK, quod

quod abſurdum eſt, ex oſtenſis in priori parte. f *hyp. &*
Itaque potius dic, A B C D. EFGH :: con. *invertendo.*
ABCDK. EFGHM. Q. E. D. g 14. 5.

Idem demonſtrabitur de cylindris, ſi conorum & pyramidum loco concipiantur cylindri & priſmata. ergo, &c.

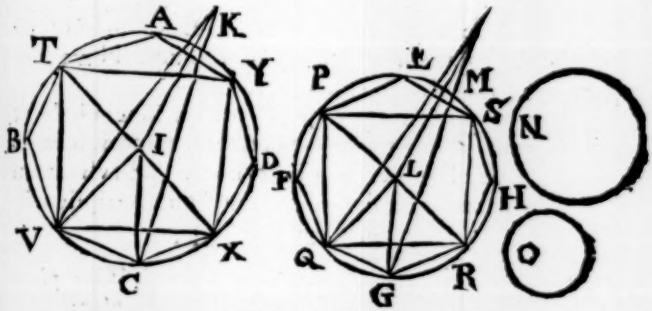
S C H O L.

Ex his habetur dimenſio cylindrorum & conorum quorumcunque. Cylindri rectæ ſoliditas producitur ex baſe circulari (*a* pro cujus dimenſione conſulendus eſt Archimedes) ducta in altitudinem. *b* igitur & cujuſcunque cylindri.

c Itaque coni ſoliditas producitur ex tertia parte altitudinis ducta in baſim.

a 1. *Prop. de dimenſ. circ.*
b 11. 12.
c 10. 12.

PROP. XII.



Similes coni & cylindri ABCDK, EFGHM *in triplicata ratione ſunt diametrorum* TX, PR, *quæ in baſibus* ABCD, EFGH.

Habeat conus A ad aliquod N rationem triplicatam TX ad PR. dico N = con. EFGHM; Nam ſi fieri poteſt, ſit N ⊐ EFGHM; ſitque exceſſus O. ergo ut in Prioribus, N ⊐ pyr. EPFQGRHSM. Sint axes conorum IK LM, adducanturque rectæ VK, CK, VI, CI; & QM, GM, QL, GL. Quoniam coni ſimiles ſunt, *a* eſt VI. IK :: QL. LM. anguli vero VIK, QLM *b* recti ſunt. *c* ergo trigona VIK, QLM,

a 24. *def.* 11
b 18 *def.* 11
c 6. 6.

d 4. 6. QLM æquiangula ſunt; *d* unde VC, VI :: QG. QL. item VI. VK :: QL. QM. ergo ex æ-
e 7. 5. quali VC. VK :: QG. QM. *e* quinetiam VK. CK :: QM. MG. ergo rurſus ex æquo VC.
f 5. 6. CK :: QG. GM. *f* ergo triangula VKC, QMG ſimilia ſunt; ſimilique argumento reliqua
g 9. *def.* 11. hujus pyramidis triangula reliquis illius. *g* quare
h *cor.* 8. 12. pyramides ipſæ ſimiles ſunt. *h* ſunt vero hæ in
k 4. 6. triplicata ratione VC ad QG, *k* hoc eſt VI ad
l 15. 5. QL, *l* vel TX ad PR. *m* ergo pyr. ATBVC.
m *byp.* & 11. 5. XDYK. pyr. EPFQGRHSM :: con. ABCDK. N. *n* unde pyr. EPFQGRHSM ⊐ N; quod
n 14. 5. repugnat prius dictis.

Rurſus, dic N. ⊏ con. EFGHM. ſit con.
o *Prius & inverſe.* EFGHM. O :: N. con. ABCDK *o* :: pyr. EPRM. ATCK *p* :: GQ. VC ter :: *q* PR.
p *cor.* 8. 12. TX ter. ergo O *r* ⊐ ABCDK. quod modo
q 4. 6. repugnare oſtenſum eſt. Proinde N = con.
r 14. 5. EFGHM. Q. E. D.

Quoniam vero quam proportionem habent coni, eandem quoque obtinent cylindri, eorum tripli, habebit quoque cylindrus ad cylindrum proportionem diametrorũ in baſibus triplicatã.

PROP. XIII.

Si cylindrus ABCD *plano* EF *ſecetur adverſis planis* BC, AD *parallelo*; *erit ut cylindrus* AEFD *ad cylindrum* EBCF, *ita axis* GI *ad axem* IH.

a 3. 1. Producto axe, *a* ſume GK = GI, & HL = IH = LM. & concipe per puncta K, L, M, plana duci circulis AD, BC parallela. *b* ergo cylind. ED = cyl. AN. & cylin. EC *b* = BO *b* = OP. itaque cylin-
b 11. 12.

drus

drus EN cylindri ED æque multiplex eſt, ac axis IK axis IG. pariterque cylindrus FP æque multiplex eſt cylindri BF, ac axis IM axis IH. prout vero IK =, ⊏, ⊐ IM, *c* ſic cylindr. EN =, ⊏, ⊐ EP. *d* ergo cyl. AEFD. cyl. EBCF :: GI. IH. Q. E. D.

c 11. 12.
d 6. *def.* 5.

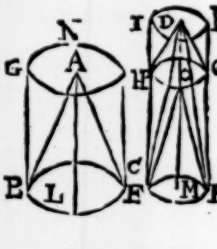
PROP. XIV.

Super æqualibus baſibus AB, CD *exiſtentes coni* AEB, CFD, *& cylindri* AH, CK, *inter ſe ſunt ut altitudines* ME, NF.

Productis cylindro HA & axe EM, ſume ML = FN; & per punctum L ducatur planum baſi AB parallelum. *a* erit cyl. AP = CK. *b* atqui cylind. AH. AP. (CK) :: ME. ML. (NF.) Q. E. D. Idem de conis cylindrorum ſubtriplis dictum puta. * imo de priſmatis & pyramidibus.

a 11. 12.
b 13. 12.
* *Adhibe* 9 & 7. 12.

PROP. XV.



Æqualium conorum BAC, EDF, *& cylindrorum* BH, EK, *reciprocantur baſes & altitudines* (BC. EF :: MD. LA:) *& quorum conorum, & cylindrorum reciprocantur baſes & altitudines, illi ſunt æquales.*

Si altitudines pares ſint, etiam baſes pares erunt; & res clara eſt. Sin altitudines ſint impares, aufer MO = LA.

a 14. 12.
b *conſtr.*

1. *Hyp.* Eſtque MD. MO (*a* LA) *b* :: cyl. EK (*c* BH.) EQ *d* :: circ. BC. EF. Q. E. D.

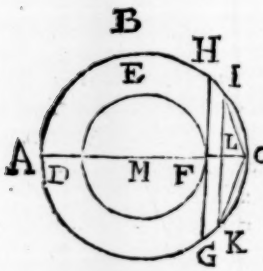
c *hyp.*
d 11. 12.

e *hyp.* f 14. 12. g 11. 5. h 11. 12. k 9. 5.

2. *Hyp.* BC. EF *e* :: DM. OM (LA) *f* :: Cyl. EK. EQ *g* :: BC. EF *b* :: BH. EQ. *k* Ergo cylind. EK = BH. Q. E. D.

Simili argumento utere de conis.

PROP. XVI.



Duobus circulis ABCG, DEF *circa idem centrum* M *existentibus, in majori circulo* ABCG *polygonum æquilaterum, & parium laterum inscribere, quod non tangat minorem circulum* DEF.

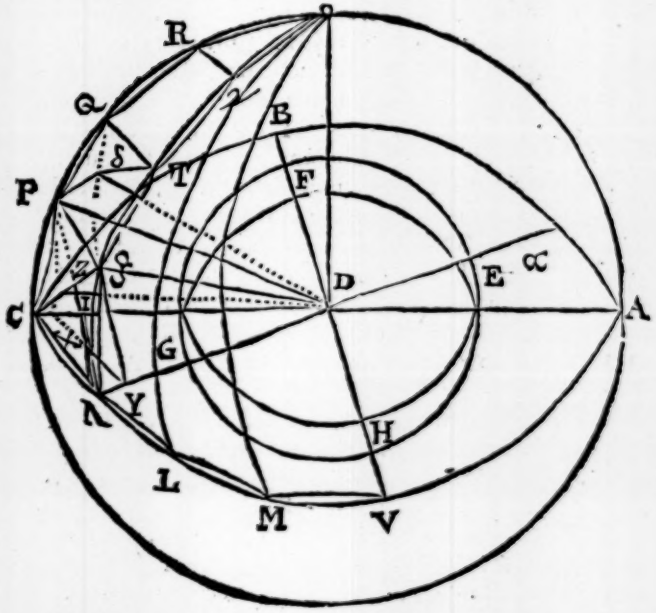
Per centrum M extendatur recta AC secans circulum DEF in F. ex quo erige perpendicularem FH. *a* Biseca semicirculum ABC, ejusque semissem BC, atque ita continuo, *b* donec arcus IC minor evadat arcu HC. ab I demitte perpendicularem IL. Liquet arcum IC totum circulum metiri, numerumque arcuum esse parem, adeoque subtensam IC latus esse *c* polygoni inscriptibilis, quod circulum DEF minime continget. Nam HG *d* tangit circulum DEF; *e* cui parallela est IK, extraque sita, *f* quare IK circulum non tangit, multoque magis CI, CK; & reliqua polygoni latera, longius à centro distantia, circulum DEF non tangunt. Q. E. F.

a 30. 3. b 1. 10. c *seb.* 16. 4. d *cor.* 16. 3. e 28. 1. f 34. *def.* 1.

Coroll. Nota, quod IK non tangit circulum DEF.

PROP.

PROP. XVII.



Duabus ſphæris ABCV, EFGH *circa idem centrum* D *exiſtentibus, in majori ſphæra* ABCV *ſolidum polyedrum inſcribere, quod non tangat ſuperficiem minoris ſphæræ* EFGH.

Secentur ambæ ſphæræ plano per centrum faciente circulos EFGH, ABCV. ducanturque diametri AC, BV ſecantes perpendiculariter. Circulo ABCV *a* inſcribatur polygonum æquilaterum VMLNC, &c. circulum EFGH minime tangens. ducta diametro N*α*, erectaque DO recta ad planum ABC. per DO, perque diametros AC, N*α* erigi concipiantur plana DOC, DON, quæ ad circulum ABCV *b* recta erunt, ideoque in ſuperficie ſphæræ *c* quadrantes

a 16. 11.

b 18. 11.

c *cor.* 33. 6.

d 4. 1. efficient DOC, DON. in quibus *d* aptentur rectæ CP, PQ, QR, RO, NS, ST, Tγ, γO ipſis CN, NL, &c pares, & æque multæ. In reliquis quadrantibus OL, OM, &c. inque tota ſphæra eadem conſtructio fiat. Dico factum.

A punctis P, S ad planum A B C V demitte
e 38. 11. perpendiculares PX, SY, *e* quæ in ſectiones AC,
f 12. *ax.* Nα cadent. Quoniam igitur tam *f* anguli recti
g 27. 3. PXC, SYN, *g* quam PCX, SNY, *h* æqualibus
peripheriis inſiſtentes, *f* pares ſunt, triangula
h 32. 1. PCX, SNY *h* æquiangula ſunt. Cum igitur PC
k *conſtr.* *k* = SN, *l* etiam PX = SY, *l* & XC = YN;
l 26. 1. *m* quare DX = DY. *n* ergo DX. XC :: DY.
m 3. *ax.* 1. YN. *o* ergo parallelæ ſunt YX, NC. quia vero
n 7. 5. PX, SY pares, & cum eidem plano ABCV re-
o 2. 6. ctæ, etiam *p* parallelæ ſunt, *q* erunt YX,
p 6. 11. SP etiam pares & parallelæ. *r* ergo SP, NC
q 33. 1. inter ſe parallelæ ſunt. ergo *ſ* quadrilaterum
r 9. 11. NCPS, eademque ratione SPQT, TQRG,
ſ 7. 11. ſed & *t* triangulum γRO totidem ſunt plana.
t 2. 11. Eodem modo tota ſphæra ejuſmodi quadrilate-
ris & triangulis repleta oſtendetur. quare inſcri-
ptum eſt polyedrum.

u 11. 11. A centro D *u* duc DZ rectum plano NCPS; & junge ZN, ZC, ZS, ZP. Quoniam DN.
x 4. 6. NC *x* :: DY. YX; eſt NC *y* ⊏ YX (SP;) pa-
y 14. 5. riterque SP ⊏ TQ, & TQ ⊏ γR. Et quia
z 3. *def.* 11. anguli DZC, DZN, DZS, DZP, *z* recti ſunt,
a 15. *def.* 1. latera vero DC, DN, DS, DP *a* æqualia, &
b 47 1. DZ commune, *b* erunt ZC, ZN, ZS, ZP æ-
c 15. *def.* 1. quales inter ſe; proinde circa quadrilaterum
d *conſtr.* NCPS *c* deſcribi poteſt circulus, in quo (*b*
e 28. 3. NS, NC, CP *d* æquales, & NC ⊏ SP) NC
f 33. 6. *e* pluſquam quadrantem ſubtendit. *f* ergo ang.
g 12. 2. NZC ad centrum obtuſus eſt. *g* ergo NCq ⊏
h 32. 1. 2 ZCq (ZCq + ZNq.) Sit NI ad AC nor-
k 9. *ax.* 1. malis. ergo cum ang. ADN (*h* DNC +
l 5. 1. DCN) ſit *k* obtuſus, *l* erit ſemiſſis ejus DCN

recti

recti ſemiſſe major; propterea que eo minor eſt reliquus è recto ang. GNI. *n* unde IN ⊏ IC. n 19. 1. ergo NCq (NIq + ICq) *o* ⊐ 2 INq. itaque o 47. 1. IN ⊏ ZC. & conſequenter DZ *p* ⊏ DI. atqui p 47. 1. punctum I eſt *q* extra ſphæram EFGH. ergo q *cor.* 16. punctum Z potiori jure eſt extra ipſam. adeoque 12. planum NCPS (cujus *r* proximum centro punctum eſt Z) ſphæram EFGH non contingit. Et ſi ad planum SPQT demittatur perpendicularis D♌, punctum ♌, adeoque & planum SPQT adhuc ulterius à centro elongatur; idemque eſt de reliquis polyedri planis. ergo polyedrum ORQPCN, &c. majori ſphæræ inſcriptum, minorem non contingit. Q. E. F. r 47. 1.

Coroll.

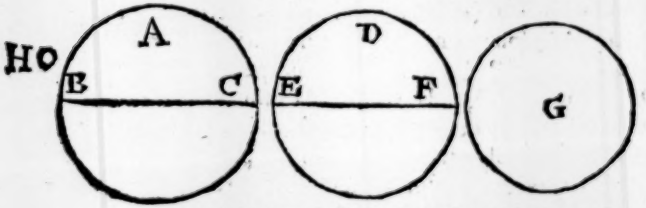
Hinc ſequitur, *Si in quavis alia ſphæra deſcribatur ſolidum polyedrum, ſimile prædicto ſolido polyedro, proportionem polyedri in una ſphæra ad polyedrum in altera eſſe triplicatam ejus quam habent ſphærarum diametri.*

Nam ſi ex centris ſphærarum ad omnes angulos baſium dictorum polyedrorum rectæ lineæ ducantur, diſtribuentur polyedra in pyramides numero æquales & ſimiles, quarum homologa latera ſunt ſemidiametri ſphærarum; ut conſtat, ſi intelligatur harum ſphærarum minor intra majorem circa idem centrum deſcripta, congruent enim ſibi mutuo lineæ rectæ ductæ à centro ſphæræ ad baſium angulos, ob ſimilitudinem baſium, ac propterea pyramides efficientur ſimiles. Quare cum ſingulæ pyramides in una ſphæra, ad ſingulas pyramides illis ſimiles in altera ſphæra *a* habeant proportionem triplicatam laterũ homologorum, hoc eſt, ſemidiametrorum ſphærarum; ſint autem *b* ut una pyramis ad unam pyramidem, ita omnes pyramides, hoc eſt, ſolidum polyedrum ex his compoſitum, ad omnes pyramides, a *cor.* 8. 12. b 12. 5.

mides, id est, ad solidum polyedrum ex illis constitutum; habebit quoque polyedrum unius sphæræ ad polyedrum alterius sphæræ proportionem triplicatam semidiametrorum, *c* atque adeo diametrorum.

c 15. 5.

PROP. XVIII.



Sphæræ BAC, EDF *sunt in triplicata ratione suarum diametrorum* BC EF.

Sit sphæra BAC ad sphæram G in triplicata ratione diametri BC ad diametrum EF. Dico G=EDF. Nam si fieri potest, sit G⌐EDF, & cogita sphæram G concentricam esse ipsi EDF. Sphæræ EDF *a* polyedrum sphæram G non tangens, sphæræque BAC simile polyedrum inscribatur. *b* Hæc polyedra sunt in triplicata ratione diametrorum BC, EF, *c* id est, sphæræ BAC ad G. *d* Proinde sphæra G major est polyedro sphæræ EDF inscripto, pars toto.

a 17. 12.
b cor. 17. 12
c hyp.
d 14. 5.

Rursus, si fieri potest, sit sphæra G⌐EDF. Sitque ut sphæra EDF ad aliam sphæram H, ita G ad BAC, *e* hoc est in triplicata ratione diametri EF ad BC; cum igitur BAC *f* ⌐H, incurrimus absurditatem prioris partis. Quin potius sphæra G=EDF. Q. E. D.

e hyp. inverſ.
f 14. 5.

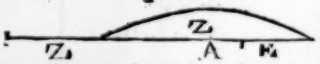
Coroll.

Hinc, ut sphæra ad sphæram, ita est polyedrum in illa descriptum ad polyedrum simile in hac descriptum.

LIB. XIII.

PROP. I.

Si recta linea z *secundum extremam & mediam rationem secetur* (z. a :: a. e;) *majus segmentum* a *assumens dimidium totius* z, *quintuplum potest ejus, quod à dimidia totius* z *describitur, quadrati.*



Dico Q. a + ½ z = 5 Q. ½ z. *a* hoc est aa + ¼ zz + za = zz + ¼ zz. *b* vel aa + za = zz. Nam ze + za *c* = zz. & ze *d* = aa. *e* ergo aa + za = zz. Q. E. D.

a 4. 2.
b 3. ax. 1.
c 2. 2.
d *hyp.* & 16. 6.
e 2. ax. & 1. ax.

PROP. II.

Si recta linea ½ z + a *sui ipsius segmenti* ½ z *quintuplum possit, duplæ prædicti segmenti* (z) *extrema ac media ratione sectæ majus segmentum est* a, *reliqua pars ejus quæ à principio rectæ* ½ z + a.

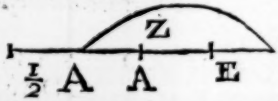
Dico z. a :: a. e. Nam quia per hyp. * aa + ¼ zz + za = zz + ¼ zz; vel aa + za = zz *a* = ze + za, *b* erit aa = ze. *c* quare z. a :: a. e. Q. E. D.

Vide fig. præced.

* 4. 2.
a 2. 2.
b 3. ax. 1.
c 17. 6.

PROP. III.

Si recta linea z *secundum extremam ac mediam rationem secetur* (z. a :: a. e;) *minus segmentum* e *assumens dimidium majoris segmenti* a, *quintuplum potest ejus, quod à dimidia majoris segmenti* a *describitur, quadrati.*



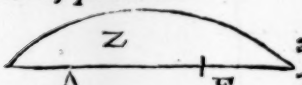
Dico Q. e + ½ a = 5 Q. ½ a. *a* hoc est ee + ¼ aa + ea = aa + ¼ aa. *b* vel ee + ea = aa. Nam ee + ea *c* = ze *d* = aa. Q. E. D.

a 4. 2.
b 3. ax. 1.
c 3. 2.
d *hyp.* & 17. 6.

PROP.

PROP. IV.

Si recta linea z *secundum extremam ac mediam rationem secetur* (z. a :: a. e;) *quod à tota* z, *quodque à minori segmento* e, *utraque simul quadrata, tripla sunt ejus, quod à majori segmento* a *describitur, quadrati.*



Dico zz + ee = 3 aa. *a* vel aa + ee + 2 ae + ee = 3 aa. Nam ae + ee *b* = ze *c* = aa.

a 4. 1.
b 3. 2.
c 17. 6.
d 2. *ax.*

d ergo aa + 2 ae + 2 ee = 3 aa. Q. E. D.

PROP. V.

D A C B *Si recta linea* AB *secundum extremam & mediam rationem secetur in* C, *apponaturque ei* AD *æquali majori segmento* AC; *tota recta linea* DB *secundum extremam ac mediam rationem secatur, & majus segmentum est quæ à principio recta linea* AB.

a *hyp.* Nam quia AB. AD *a* :: AC. CB, invertendoque AD. AB :: CB. AC; erit componendo DB. AB :: AB. AC. (AD.) Q. E. D.

Schol.

Quod si fuerit BD. BA :: BA. AD. erit BA. AD :: AD. BA—AD. Nam dividendo est BD—BA (AD) BA :: BA—AD. AD. ergo inverse, BA. AD :: AD. BA—AD. Q. E. D.

PROP. VI.

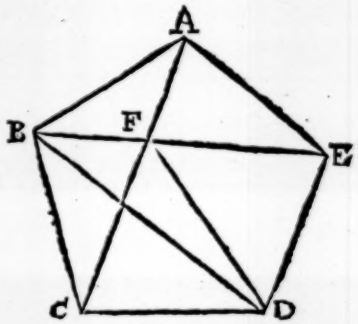
D A C B *Si recta linea rationalis* AB *extrema ac media ratione secetur in* C; *utrumque segmentorum* (AC, CB) *irrationalis est linea, quæ vocatur apotome.*

a 3. 1. Majori segmento AC *a* adde AD = ½ AB;
b 1. 13. *b* ergo DCq = 5 DAq. *c* ergo DCq ⊏ DAq.
c 6. 10. proinde cum AB, *c* ideoque ejus semissis D A
d *hyp.* sint ῥ, etiam DC est ῥ. Quia vero 5. 1 :: non
e *sch.* 12. 10

Q. Q. f est DC ⊓ DA. *g* ergo DC—AD, id est AC est apotome. Insuper quia AC q *b*=AB × BC, & AB est *β*, *k* etiam BC est apotome. Q. E. D.

f 9. 10.
g 74. 10.
h 17. 6.
k 98. 10.

PROP. VII.



Si pentagoni æquilateri ABCDE *tres anguli, sive qui deinceps* EAB, ABC, BCD, *sive* EAB, BCD, CDE *qui non deinceps sint, æquales fuerint, æquiangulum erit ipsum pentagonum* ABCDE.

Paribus deinceps angulis subtendantur rectæ BE, AC, BD.

Quoniam latera EA, AB, BC, CD, angulique inclusi *a* æquantur, *b* erunt bases BE, AC, BD, *c* angulique AEB, ABE, BAC, BCA pares. *d* quare BF=FA, & *e* proinde FC=FE. ergo triangula FCD, FED sibi mutuo æquilatera sunt; *f* unde ang. FCD=FED, *g* proinde ang. AED =BCD. Eodem pacto ang. CDE reliquis æquatur. quare pentagonum æquiangulum est. Q. E. D.

a *hyp.*
b 4. 1.
c 4. & 5. 1.
d 6. 1.
e 3. *ax.* 1.
f 8. 1.
g 2. *ax.* 1.

Sin anguli EAB, BCD, CDE, qui non deinceps, statuantur pares, *h* erit ang. AEB=BDC, & BE=BD, *k* ideoque ang. BED=BDE; *l* totus proinde ang. AED=CDE. ergo propter angulos A, E, D deinceps æquales, ut prius, pentagonum æquiangulum erit. Q. E. D.

h 4. 1.
k 5. 1.
l 2. *ax.*

PROP.

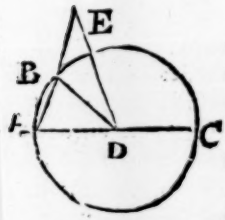
PROP. VIII.

Si pentagoni æquilateri & æquianguli A B C D E *duos angulos* BCD, CDE, *qui deinceps sint, subtendant rectæ lineæ* B D, CE; *hæ extrema ac media ratione se mutuo secant, & majora ipsarum segmenta* B F, *vel* EF *æqualia sunt pentagoni lateri* BC.

Circa pentagonum *a* describe circulum ABD.
b Arcus BD = BC. *c* ergo ang. FCD = FDC.
d ergo ang. BFC = 2 FCD (FCD + FDC.)
Atqui arcus BAE *b* = 2 ED, proinde ang. BCF *e* = 2 FCD = BFC. *f* quare BF = BC. Q. E. D. Porro quia triangula BCD, FCD *g* æquiangula sunt, *h* erit BD. DC (BF) :: CD (BF.) FD. pariterque E C. E F :: E F, F C. Q. E. D.

a 14. 4.
b 28. 3.
c 27. 3.
d 32. 1.
e 33. 6.
f 6. 1.
g 27. 3.
h 4. 6.

PROP. IX.



Si hexagoni latus BE, *& decagoni* AB, *in eodem circulo* A B C *descriptorum componantur, tota recta linea* AE *extrema ac media ratione secatur,* (AE. BE :: BE. AB) *& majus ejus segmentum est hexagoni latus* BE.

Duc diametrum ABC, & junge rectas DB, DE. Quoniam ang. BDC *a* = 4 BDA, estque ang. BDC *b* = 2 DBA (DAB + DBA,) erit DBA (*b* BDE + BED) *c* = 2 BDA *d* = 2 BDE: proinde ang. DBA, vel DAB *e* = ADE. Itaque trigona ADE, ADB æquiangula sunt, *f* quare AE. AD. (*g* BE) :: AD. (BE.) AB. Q. E. D.

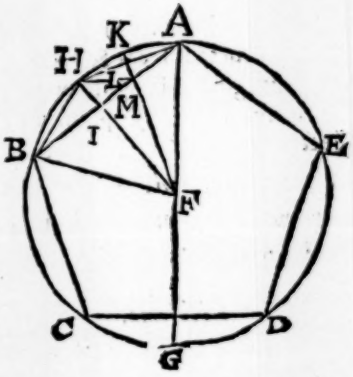
a *hyp.* & 27. 3.
b 32. 1.
c 7. *ax.* 1.
d 5. 1.
e 1. *ax.* 1.
f 4. 6.
g *cor.* 15. 4.

Coroll.

Coroll.

Hinc, ſi latus hexagoni alicujus circuli ſecetur extrema ac media ratione; majus illius ſegmentum erit latus decagoni ejuſdem circuli. *ſch. 5. 13.*

PROP. X.



Si in circulo ABCE *pentagonum æquilaterum* ABCDE *deſcribatur; pentagoni latus* AB *poteſt & hexagoni latus* FB, *& decagoni latus* AH, *in eodem circulo deſcriptorum.* a 28. 3. & 3. *ax.*

Duc diametrum AG. Biſeca arcum AH in K. Et duc FK, FH, FB, BH, HM. b *hyp.* & 7. *ax.*

Semicirc. AG — arc. AC *a* = AG — AD. hoc eſt, arc. CG = GD *b* = AH = HB. ergo arc. BCG = 2 BHK; *c* adeoque ang. BFG = 2 BFK. *d* ſed ang. BFG = 2 BAG. *e* ergo ang. BFK = BAG. Trigona igitur BFM, FAB *f* æquiangula ſunt. *g* quare AB. BF :: BF. BM. *b* ergo AB × BM = BFq. Rurſus ang. AFK *k* = HFK; & FA = FH; *m* quare AL = LH, *m* & anguli FLA, FLH pares, ac proinde recti ſunt. ergo ang. LHM *m* = LAM *n* = HBA. Trigona igitur AHB, AMH *o* æquiangula ſunt. *p* qua-

c 33. 6. d 20. 3. e 1. *ax.* 1. f 32. 1. g 4. 6. h 17. 6. k 27. 3. m 4. 1. n 17. 3. o 32. 1. p 4. 6.

re

q 17. 6. r 2. 2. s 2. ax. re AB. AH :: AH. AM. q ergo AB × AM = AHq. Quum igitur ABq r = AB × BM + AB × AM, s erit ABq = BFq + AHq. Q. E. D.

Coroll.

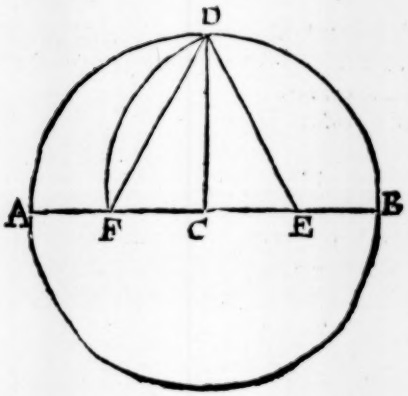
1. Hinc, linea recta (FK) quæ ex centro (F) arcum quempiam (HA) biſecat, etiam rectam (HA) illi arcui ſubtenſam biſecat ad angulos rectos.

2. Diameter circuli (AG) ex angulo quovis (A) pentagoni ducta biſecat & arcum (CD,) quem latus pentagoni illi angulo oppoſitum ſubtendit, & latus ipſum (CD) oppoſitum, idque ad angulos rectos.

Schol.

Hic, ut promiſimus, praxim trademus expeditam problematis 11. 4.

Problema.



Invenire latus pentagoni circulo A D B *inſcribendi.*

Duc diametrum A B. cui perpendicularem

CD

CD ex centro C erige. Biſeca CB in E. Fac EF = ED. erit DF pentagoni latus.

Nam BF × FC + ECq *a* = EFq *b* = EDq *c* = DCq + ECq. *d* ergo BF × FC = DCq, vel BCq. *e* quare BF. BC :: BC. FC. ergo quum BC ſit latus hexagoni, *f* erit FC latus decagoni, proinde DF *b* = √ DCq + FCq *g* eſt latus pentagoni. Q. E. F.

a 6. 2.
b *conſtr.*
c 47. 1.
d 3. *ax.*
e 17. 6.
f 9. 13.
g 10. 13.
h 47. 1.

PROP. XI.

Si in circulo ABCD *rationalem habente diametrum* AG, *pentagonum æquilaterum* ABCDE *deſcribatur; pentagoni latus* AB *irrationalis eſt linea, quæ vocatur minor.*

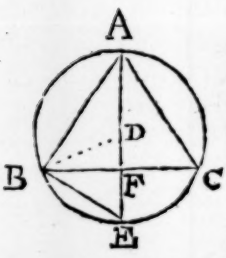
Duc diametrum BFH, rectaſque AC, AH; & * fac FL = $\frac{1}{4}$ radii FH, & CM = $\frac{1}{4}$ CA.

* 10. 6.

Ob angulos AKF, AIC *a* rectos, & communem CAI, trigona AKF, AIC *b* æquiangula ſunt; *c* ergo CI. FK *c* :: CA. FA (FB) *d* :: CM. FL. ergo permutando FK, FL :: CI. CM *d* :: CD. CK (2 CM.) *e* componendo igitur CD + CK. CK :: KL. FL. *f* proinde Q: CD + CK (*g* 5 CKq.) CKq :: KLq. FLq. ergo KLq = 5 FLq. Itaque ſi BH (*p*) ponatur 8, erit FH 4; FL 1. & FLq. 1. BL 5. & BLq 25. KLq 5. è quibus liquet BL, & KL eſſe *p b* ⌐. *k* ideoque BK eſſe Apotomen, cujus congruens KL. cum vero BLq − KLq = 20, *l* erit BL ⌐ √ BLq − KLq. * unde BK erit apotome quarta. Quoniam igitur ABq *m* = HB × BK, *n* erit AB minor. Q. E. D.

a *cor.* 10. 13.
b 32. 1.
c 4. 6.
d 15. 5.
e 18. 5.
f 22. 6.
g 1. 13.
h 9. 10.
k 74. 10.
l 9. 10.
*4 *def.* 85. 10.
m *cor.* 8. 6. & 17. 6.
n 95. 10.

PROP. XII.



Si in circulo ABEC *triangulum æquilaterum* ABC *describatur, trianguli latus* AB *potentia triplum est ejus lineæ* AD, *quæ ex* D *centro circuli ducitur.*

Protracta diametro AE, duc BE. Quoniam arcus BE *a* = EC, arcus BE sextans est pars circumferentiæ. *b* ergo BE = DE. hinc AEq *c* = 4 DEq (4 BEq) *d* = ABq + BEq (= ADq.) *e* proinde ABq = 3 ADq. Q. E. D.

a *cor.* 10. 13.
b *cor.* 15. 4.
c 4. 2.
d 47. 1.
e 3. *ax.* 1.

Coroll.

1. AEq. ABq :: 4. 3.

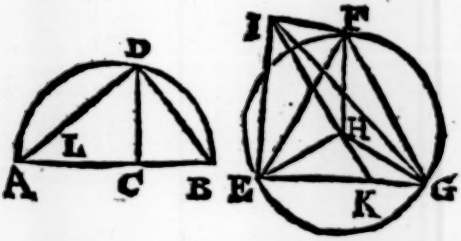
2. ABq. AFq :: 4. 3. *f* Nam ABq. AFq :: AEq. ABq.

3. DF = FE. Nam triang. EBD *g* æquilaterum est; *h* & BF ad ED perpendicularis, *h* ergo EF = FD.

f *cor.* 8. 6. & 21. 6.
g *cor.* 15. 4.
h *cor.* 3. 3.

4. Hinc AF = DE + DF = 3 DF.

PROP. XIII.



Pyramidem EGFI *constituere, & data sphæræ complecti; & demonstrare quod sphæræ diameter* AB

AB *potentia ſit ſeſquialtera lateris* EF *ipſius pyramidis* EGFI.

Circa AB deſcribe ſemicirculum ADB. *a* ſitque AC = 2 CB. ex puncto C erige perpendicularem CD; & junge AD, DB. Tum radio HB = CD deſcribe circulum HEFG; cui *b* inſcribe triangulum æquilaterum EFG. ex H *c* erige IH = CA rectum plano EFG, produc IH ad K; *d* ita ut IK = AB. rectaſque adjunge IE, IF, IG. erit EFGI pyramis expetita.

Nam quia anguli ACD, IHE, IHF, IHG *e* recti ſunt; & CD, HE, HF, HG *e* pares, *e* atque IH = AC; *f* erunt AD, IE, IF, IG æquales inter ſe. Quia vero AC (2 CB.) CB *g* :: ACq. CDq. erit ACq = 2 CDq. itaque ADq *f* = ACq + CDq *h* = 3 CDq = 3 HEq *k* = EFq. *l* ergo AD, EF, IE, IF, IG pares ſunt, adeoque pyramis EFGI eſt æquilatera. Quod ſi punctum C ſuper H collocetur, & AC ſuper HI, rectæ AB, IK *m* congruent, utpote æquales. quare ſemicirculus ADB axi AB vel IK circumductus *n* tranſibit per puncta, E, F, G, * adeoque pyramis EFGI ſphæræ inſcripta erit. Q. E. F. liquet vero eſſe BAq. ADq *o* :: BA. AC *p* :: 3. 2. Q. E. D.

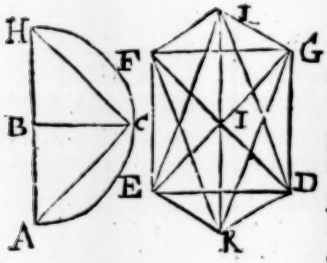
a 10. 6.
b *cor.* 15. 4.
c 12. 11.
d 3. 1.
e *conſtr.*
f 47. 1.
g 20. 6.
h 2. *ax.*
k 12. 13.
l 1. *ax.* 1.
m 8. *ax.* 1.
n 15. *def.* 11.
* 31. *def.* 11.
o *cor.* 8. 6.
p *conſtr.*

Corollaria.

1. ABq. HEq :: 9. 2. Nam ſi ABq ponatur 9, erit ADq (8Fq) 6. *q* proinde HEq erit 2.
2. Si L centrum fuerit, erit AB. LC :: 6. 1. Nam ſi AB ponatur 6, erit AL, 3; *r* ideoque AC 4; quare LC erit 1. Hinc
3. AB. HI :: 6. 4 :: 3. 2. unde
4. ABq. HIq :: 9. 4.

q 12. 13.
r *conſtr.*

PROP. XIV.



Octaedrum KEF. GDL *conſtituere, & data ſphæra complecti, qua & pyramidem; & demonſtrare, quad ſphæræ diameter* A H *potentia ſit dupla lateris* A C *ipſius Octaedri.*

Circa AH deſcribe ſemicirculum ACH. ex centro B erige perpendicularem BC. duc AC, HC. Super ED = AC *a* fac quadratum EFGD, cujus diametri DF, EG ſecantes in centro I. ex I duc IL = AB *b* rectam plano EFGD. produc IL, *c* donec IK = IL. Connexis KE, KF, KG, KD, LE, LF, LG, LD; erit KEFGDL octaedrum quæſitum.

a 46. 1.
b 12. 11.
c 3. 1.

Nam AB, BH, FI, IE, &c. æqualium quadratorum ſemidiametri æquales ſunt inter ſe. *d* quare triangulorum rectangulorum LIE, LIF, FIE, &c. bales LF, LE, FE, &c. æquantur. proinde octo triangula LFE, LFG, LGD, LDE, KEF, KFG, KGD, KDE æquilatera ſunt, *e* atque octaedrum conſtituunt, quod ſphæræ cujus centrum I, radius IL, vel AB, inſcribi poteſt. (quoniam AB, IL, IF, IK, &c. *f* æquales ſunt.) Q. E. F. porro liquet AHq (LKq) *g* = 2 ACq (2 LDq.) Q. E. D.

d 4. 1.
e 27. *def.* 11
f *conſtr.*
g 47. 1.

Corollaria.

1. Hinc manifeſtum eſt, in Octaedro tres diametros EG, FD, LK ſe mutuo ad angulos rectos ſecare in centro ſphæræ.

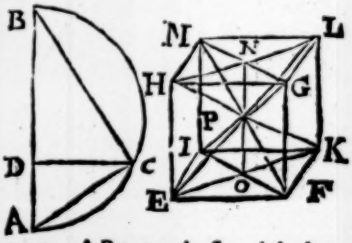
2. Item, tria plana EFGD, LEKG, LEKD eſſe quadrata, ſe mutuo ad angulos rectos ſecantia.

3. Octa-

3. Octaedrum dividitur in duas pyramides ſimiles & æquales EFGDL, & EFGDK, quarum baſis communis eſt quadratum EFGD.

4. Denique, baſes octaedri oppoſitæ, inter ſe parallelæ ſunt. 15. 11.

PROP. XV.



Cubum EFGHIKLM *conſtituere, & ſphæra complecti, quæ & priores figuras; & demonſtrare, quod ſphæræ diameter* AB *potentia ſit tripla lateris* EF *ipſius cubi.*

Super AB deſcribe ſemicirculum ACB; & fac AB = 3 DA. ex D erige perpendicularem DC, & junge BC ac AC. Tum ſuper EF = AC conſtrue quadratum EFGH, cujus plano rectæ inſiſtant EI, FK, HM, GL ipſi EF pares, quas connecte rectis IK, KL, LM, IM. Soljdum EFGHIKLM cubus eſt, ut ſatis conſtat ex conſtructione.

In quadratis oppoſitis EFKI, HGLM duc diametros EK, FI, HL, MG, per quas ducta plana EKLH, FIMG ſe interſecent in recta NO. Hæc diametros cubi EL, FM, GI, HK *c* biſecabit in P, centro cubi. *d* ergo P centrum erit ſphæræ per puncta cubi angularia tranſeuntis. Porro EL*q* *e* = EK*q* + KL*q* *e* = 3 KL*q*, *f* vel 3 AC*q*. atqui AB*q*. AC*q* *g* :: BA. DA *f* :: 3. 1. ergo AB = EL. Quare cubum fecimus, &c. Q. E. F.

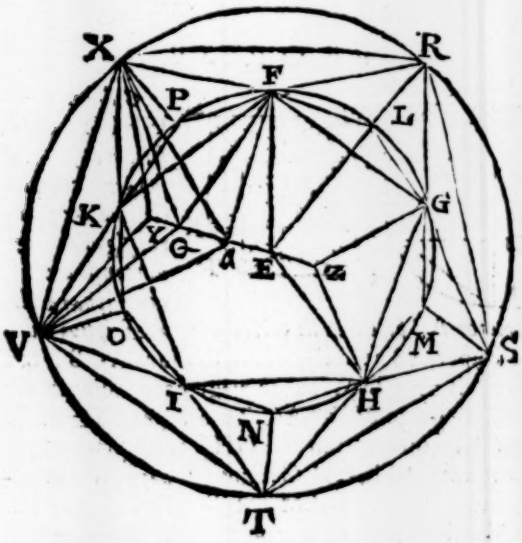
a 10. 6.
b 46. 1.
c *cor.* 39. 11.
d 15. *def.* 1. & 14. *def.* 11.
e 47. 1.
f *conſtr.*
g *cor.* 8. 6.
h 14. 5.

Coroll.

1. Hinc, omnes diametri cubi inter ſe æquales ſunt, ſeſeque mutuo in centro ſphæræ biſecant. Eademque ratione rectæ quæ quadratorum oppoſitorum centra conjungunt, biſecantur in eodem centro.

k 47. 1. l 13. 13. m 15. 13. 2. Diameter ſphæræ poteſt latus tetraedri, & cubi. nempe ABq k = l BCq + m ACq.

PROP. XVI.



Icoſaedrum ZGHIKFYVXRST *conſtituere, & ſphæra complecti, qua & antedictas figuras; & demonſtrare, quod icoſaedri latus* FG *irrationalis eſt linea, quæ vocatur minor.*

Super AB diametrum ſphæræ deſcribe ſemicirculum ADB; & a fac AB = 5 BC. ex C erige normalem CD, & duc AD ac BD. Ad intervallum EF = BD deſcribe circulum EFKNG;

a 10. 6.

cui

b cui inſcribe pentagonum æquilaterum FKIHG. Biſeca arcus FG, GH, &c. ac connecte rectas FL, LG, &c. latera nempe decagoni. Tunc *c* erige EQ, LR, MS, NT, OV, PX ipſi EF æquales, rectaſque plano FKNG. & connecte RS, ST, TV, VX, XR; item FX, FR, GR, GS, HS, HT, IT, IV, KV, KX. Denique producta EQ, ſume QY = FL; & EZ = FL; rectaſque duci concipe ZG, ZH, ZI, ZK, ZF; ac YV, YX, YR, YS, YT. Dico factum.

b 11. 4.
c 12. 11.

Nam ob EQ, LR, MS, NT, OV, PX *d* æquales *e* & parallelas, etiam quæ illas jungunt, EL, QR, EM, QS, EN, QT, EO, QV, EP, QX *f* pares & parallelæ ſunt. Item ideo LM (vel FG,) RS, MN, ST, &c. æquales ſunt inter ſe. *g* ergo planum per EL, EM, &c. plano per QR, QS, &c. æquidiſtans, *h* & circulus QXRSTV è centro Q, circulo EPLMNO æqualis eſt; atque RSTVX eſt pentagonum æquilaterum. Duci vero intellectis EF, EG, EH, &c. ac QX, QR, QS, &c. quia FRq *k* = FLq + LRq, *l* vel FRq *m* = FGq, *n* erunt FR, FG, adeoque omnes RS, FG, FR, RG, GS, GH, &c. æquales inter ſe. Proinde 10 triangula RFX, RFG, RGS, &c. æquilatera ſunt & æqualia. Rurſus ob ang. XQY *o* rectum, erit XYq *p* = QXq + QYq *q* = VXq vel FGq. quare XY, VX, hiſque ſimiliter YV, YT, YS, YR, ZG, ZH, &c. æquantur: Ergo alia decem trigona conſtituta ſunt æquilatera, & æqualia, tam ſibi mutuo, quam decem prioribus; ac proinde factum eſt Icoſaedrum.

d *conſtr.*
e 6. 11.
f 33. 1.
g 15. 11.
h 1. *def.* 3.
k 47. 1.
l *conſtr.*
m 10. 13.
n *ſch.* 48. 1.
& 1. *ax.*
o *cor.* 14. 11
p 47. 2.
q 10. 13.

Porro, biſecta EQ in α, duc rectas αF, αX, αV; & propter QX *r* = QV, & commune latus αQ, angulique EQX, EQV rectos; *ſ* erit αX = αV. ſimilique argumento omnes, αX, αR, αS, αT, αV, αF, αG, αH, αI, αK æquantur.

r 15. *def.* 1.
ſ 4. 1.

t 9. 13. Quoniam autem Z Q. Q E t :: QE. ZE, erit
u 3. 13. $Z\alpha q\, u = 5\, E\alpha q\, x = EQq\,(EFq)\; E\alpha q\, y = \alpha Fq$.
x 4. 2. ergo $Z\alpha = \alpha F$. pari pacto $\alpha F = Y\alpha$. ergo
y 47. 1. ſphæra, cujus centrum α, radius αF, per 12 puncta icoſaedri angularia tranſibit.
z 15. 5. Denique, z quia $Z\alpha . \alpha E$ :: ZY. QE; a ideoque
a 22. 6. $Z\alpha q . \alpha Eq$ :: ZYq. QEq b erit $ZYq = 5\, QEq$,
b 14. 5. vel 5 BDq : atqui ABq. BDq c :: AB. BC :: 5.
c cor. 8. 6. 1. d ergo ZY = AB. Q. E. F.
d 1. ax. 1. Itaque ſi AB ponatur $\dot{p}$, e erit $FF = \sqrt{}$ AB x
e ſch. 12. 10 BC. etiam $\dot{p}$; proinde FG pentagoni, idemque
f 11. 13. Icoſaedri 5 latus, f eſt minor. Q. E. D.

Coroll.

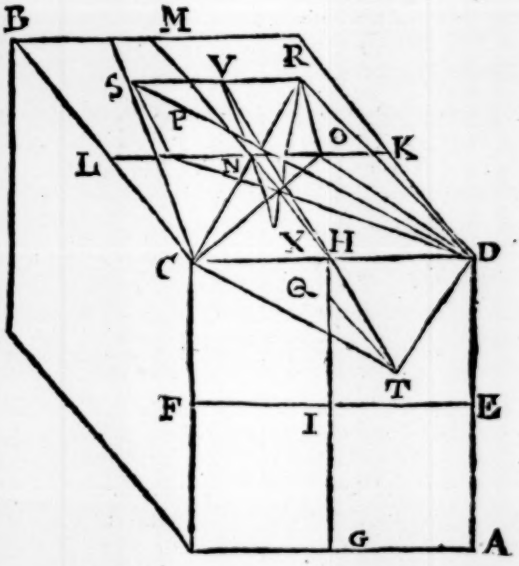
1. Ex dictis infertur, ſphæræ diametrum eſſe potentia quintuplum ſemidiametri circuli quinque latera icoſaedri ambientis.

2. Item manifeſtum eſt, ſphæræ diametrum eſſe compoſitam ex latere hexagoni, hoc eſt, ex ſemidiametro, & duobus lateribus decagoni circuli ambientis quinque latera icoſaedri.

3. Conſtat denique latera icoſaedri oppoſita,
a 33. 1. qualia ſunt RX, HI, eſſe parallela. Nam RX a
b ſch. 26. 3. parall. LP. b parall. HI.

PROP.

PROP. XVII.



Dodecaedrum conſtituere, & ſphæra complecti, qua & prædictas figuras; & demonſtrare, quod dodecaedri latus RS *irrationalis eſt linea, quæ vocatur apotome.*

Sit AB cubus datæ ſphæræ inſcriptus, cujus latera omnia biſecentur in punctis E, H, F, G, K, L, &c. rectæque adjungantur KL, MH, HG, EF. *a* Fac HI. IQ :: IQ. QH; & ſume NO, NP pares ipſi IQ. Erige OR, PS rectas plano DB, & QT plano AC. ſintque OR, PS, QT ipſis IQ, NO, NP æquales. Connexis DR, RS, SC, CT, DT, erit DRSCT pentagonum Dodecaedri expetiti. Nam duc NV parall. OR, & protracta NV ad occurſum cum cubi centro X, connecte rectas DS, DO, DP, CR, CP, HV, HT, RX. Quia DOq *a* = DKq (*b* KNq) + KOq *c* = 3 ONq (3 ORq) *d* erit DRq = 4

a 30. 6.

a 47. 1. *b* 7. ax. 1. *c* 4. 13. *d* 47. 1.

e 4. 2. = 4 ORq e = OPq, vel RSq. ergo DR = RS. Simili argumento DR, RS, SC, CT, TP pa-
f *constr.* 9. res ſunt. Quia vero OR f = g & parall. PS,
6. 11. g erunt RS, OP, & h conſequenter RS, DC eti-
g 33. 1. am parallelæ; h ergo hæ cum ſuis conjungenti-
h 9. 1. bus DR, CS, VH in uno ſunt plano. quinetiam
k 7. 11. quia HI. IQ k :: IQ (TQ.) QH k :: HN.
k *constr.* NV; & tam TQ, HN, quam QH, NV k rectæ
l 6. 11. eidem plano, l adeoque & parallelæ exiſtunt,
m 32. 6. m erit THV recta linea. n ergo Trapezium
n 1. & 2. 11 DRSC, & triang. DTS in uno ſunt plano per rectas DC, TV extenſo. ergo DTCSR eſt pentagonum, & quidem æquilaterum, ex antedictis. Porro, o quia PK. KN :: KN. NP; &
o 5. 13. DSq p = DPq + PSq (PNQ) = p DKq + PKq
p 47. 1. + NPq, q erit DSq = DKq + 3 KNq = 4 DKq
q 1. *ax.* 2. (4DHq) r = DCq. ergo DS = DC; unde tri-
& 4. 13. gona DRS, DCT ſibi mutuo æquilatera ſunt.
r 4. 2. $ſ$ ergo ang. DRS = DTC; & eodem pacto ang.
ſ 8. 1. CSR = DTC. ergo * pentagonum DTCSR
* 7. 13. etiam æquiangum eſt. Ad hæc, quia AX, DX,
t 15. 13. CX, &c. ſunt cubi ſemidiametri, s erit XN =
u 1. *ax.* 1. IH, vel KN, u adeoque XV = KP. unde ob angulum x rectum RVX, z erit RXq = XVq + RVq
x 29. 1. (NPq) = KPq + NPq a = 3 KNq b =
z 47. 1. AXq, vel DXq, &c. ergo RX, AX, DX, & ea-
a 4. 13. dem ratione XS, XT, AX æquales ſunt inter ſe.
b 15. 13. Et ſi eadem methodo, qua conſtructum eſt pentagonum DTCSR, fabricentur 12 ſimilia pentagona tangentia duodecim cubi latera, ea Dodecaedrum conſtituent; ac per eorum puncta angularia tranſiens ſphæra, cujus radius AX, vel RX, Dodecaedrum complectetur. Q. E. F.

c *conſtr.* Denique, quia KN. NO c :: NO. OK, d
d 15. 5. erit KL. OP :: OP. OK + PL. Itaque ſi
e 15. 13. ſphæræ diameter AB ponatur ρ, erit KL e = $\sqrt{\frac{1}{3}}$
f *ſch.* 12. 13 ABq f etiam ρ. g unde OP, vel RS latus dode-
g 6. 13. caedri apotome erit. Q. E. D.

Coroll.

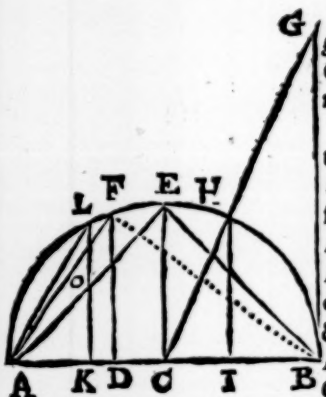
Coroll.

1. Hinc, ſi latus cubi ſecetur extrema ac media ratione, majus ſegmentum erit latus dodecaedri in eadem ſphæra deſcripti.

2. Si rectæ lineæ ſectæ extrema ac media ratione, minus ſegmentum ſit latus dodecaedri, majus ſegmentũ erit latus cubi ejuſdem ſphæræ.

3. Liquet etiam latus cubi æquale eſſe lineæ rectæ ſubtendenti angulum pentagoni dodecaedri eadem ſphæra comprehenſi.

PROP. XVIII.



Latera quinq; figurarum exponere, & inter ſe comparare.

Sit AB diameter ſphæræ, ac AEB ſemicirculus. ſitque AC $a = \frac{1}{2}$ AB, & AD $b = \frac{1}{3}$ AB. Erige perpendiculares CE, DF, & BG = AB. junge AF, AE, BE, BF, CG, ex H demitte perpendicularem HI, & ſumpta CK = GI, ex K erige perpendicularem KL, & connecte AL. Denique *c* fac AF. AO :: AO. OF. a 10 1. b 10. 6. c 30. 6.

Itaque 3. 2 *d* :: AB. BD *e* :: ABq. BFq, latus Tetraedri. & 2. 1 :: *a* AB. AC :: ABq. BEq, *f* latus Octaedri. d *conſtr.* e *cor.* 8. 6. f 14. 13.

Item 3. 1 *d* :: AB. AD *e* :: ABq. AFq, *g* latus Hexaedri. g 15. 13. h *conſtr.*

Porro, quia AF. AO *b* :: AO. OF. *k* erit AO k *cor.* 17. 13.

l 4. 6. AO latus Dodecaedri. denique BG (2 BC.)
m 24 5. BC *l* :: HI. IC. *m* ergo HI = 2 CI *n* = KI. ergo
n *constr.* HIq *o* = 4 CIq. proinde CHq = *p* 5 CIq. *q* ergo
o 4. 2. ABq = 5 KIq. *r* itaque KI, vel HI, eſt radius circuli circumſcribentis pentagonum icoſaedri; &
p 47. 1. AK, vel IB, *r* eſt latus decagoni eidem circulo in-
q 15. 5. ſcripti. unde AL *ſ* erit latus pentagoni, *t* idemque
r *cor.* 16. 13 Icoſaedri latus. Ex quibus liquet BF, BE, AF
ſ 10. 13. eſſe ῥ ☐. & AL, AO eſſe ῥ ☐; atque BF
t 16. 13. ⊏ BE; & BE ⊏ AF; ac AF ⊏ AO. Quia
u 1. 6. vero 3 AFq = ABq *u* = 5 KLq. ac AF × AO
x 4. *ax.* 1. ⊏ AF × OF, *x* ideoque AF × AO + AF × OF
y 1. 2. ⊏ 2 AF × OF, *y* hoc eſt AFq ⊏ ϛ 2 AOq. *a* e-
z 17. 6. rit 3 AFq (5 KLq) ⊏ 6 AOq. proinde KL
a 47. 1. ⊏ AO; & fortius, AL ⊏ AO.

Jam vero ut hæc latera numeris exprimamus, ſi AB ponatur $\sqrt{60}$, erit ex jam dictis ad calculum exactis, BF = $\sqrt{40}$. & BE = $\sqrt{30}$. & AF = $\sqrt{20}$. item AL = $\sqrt{:30-\sqrt{180}}$ (nam AK = $\sqrt{15}-\sqrt{3}$. & KL (HI) = $\sqrt{12}$.) denique AO = $\sqrt{:30-\sqrt{500}}$ ($\sqrt{25}-\sqrt{5}$.)

SCHOL.

SCHOL.

Præter jam dictas figuras nullam dari posse figuram solidam regularem (nempe quæ figuris planis ordinatis & æqualibus contineatur) admodum perspicuum est. Nam ad anguli solidi constitutionem requiruntur ad minimum tres anguli plani; *a* hique omnes simul 4 rectis minores esse debent. *b* Atqui 6 anguli trigoni æquilateri, 4 quadratici, & 3 hexagonici, figillatim 4 rectos exæquant; quatuor vero pentagonici, 3 heptagonici, 3 octagonici, &c. 4 rectos excedunt. ergo solummodo ex 3, 4, vel 5 triangulis æquilateris, ex 3 quadratis, vel 3 pentagonis, effici potest angulus solidus. Proinde, præter quinque prædicta, nulla existere possunt corpora regularia.

a 21. 11.

b *Vid. schol.* 32. 1.

Ex P. Herigonio.

Proportiones sphæræ, & 5 figurarum regularium eidem inscriptarum.

Sit diameter sphæræ 2. Erunt

Peripheria circuli majoris, 6 | 28318.

Superficies circuli majoris, 3 | 14159.

Superficies sphæræ, 12 | 56637.

Soliditas sphæræ, 4 | 18859.

Latus tetraedri, 1 | 62299.

Latus

Superficies tetraedri, 4 | 6188.

Soliditas tetraedri, 0 | 15132.

Latus hexaedri, 1 | 1547.

Superficies hexaedri, 8.

Soliditas hexaedri, 1 | 5396.

Latus octaedri, 1 | 41421.

Superficies octaedri, 6 | 9282.

Soliditas octaedri, 1 | 33333.

Latus dodecaedri, 0 | 71364.

Superficies dodecaedri, 10 | 51462.

Soliditas dodecaedri, 2 | 78516.

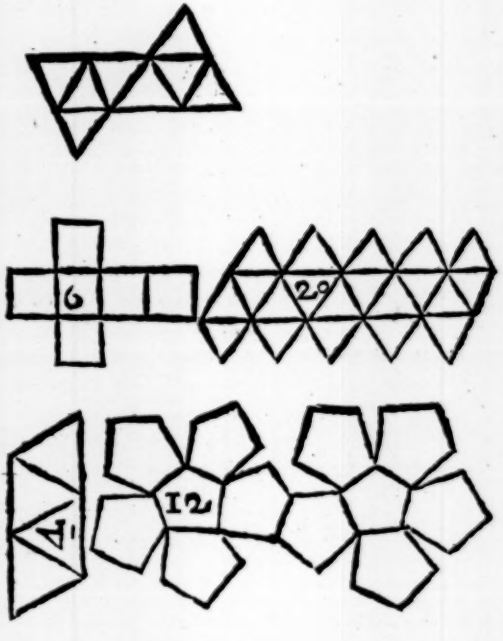
Latus Icosaedri, 1 | 05146.

Superficies Icosaedri, 9 | 57454.

Soliditas Icosaedri, 2 | 53615.

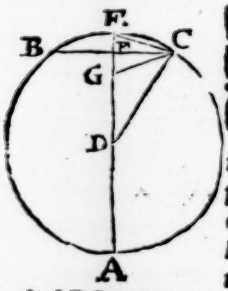
Quod

Quod ſi ex charta conficiantur quinque figuræ æquilateræ & æquiangulæ ſimiles his quæ ſunt in ſubjecta figura, componentur quinque figuræ ſolidæ, ſi rite complicentur.



LIB. XIV.

PROP. I.



Quæ ex D *centro circuli cujuspiam* ABC *in pentagoni eidem circulo inscripti latus* BC *ducitur perpendicularis* DF, *dimidia est utriusque lineæ simul, & lateris hexagoni* DE, *& lateris decagoni* EC *eidem circulo* ABC *inscripti.*

a 4. 1. Sume FG = FE, & duc CG. *a* Estque CE
b 5. 1. = CG. ergo ang. CGE *b* = CEG *b* = ECD.
c 32. 1. ergo ang. ECG *c* = EDC *d* = $\frac{1}{4}$ ADC *e* =
d *hyp.* & $\frac{1}{2}$ CED ($\frac{1}{2}$ ECD.) proinde ang. GCD =
33. 6. ECG = EDC. *g* quare DG = GC (CE.) er-
e 20. 3. go DF = CE (DG) + EF = $\frac{DE + CE}{2}$.
f 7. *ax.* Q. E. D.
g 8. 1.

PROP. II.

A G B C
D H E F

Si binæ rectæ lineæ AB DE *extrema ac media ratione secentur* (AB. AG :: AG. GB. *&* DE. DH :: DH. HE;) *ipsæ similiter secabuntur, in easdem scilicet proportiones.* (AG. GB :: DH. HE.)

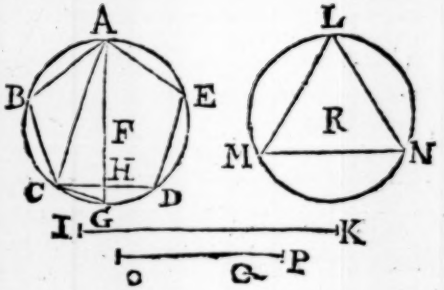
a 17. 6. Accipe BC = BG & EF = EH. Estque
b 8. 2. AB × BG *a* = AGq. quare ACq *b* = 4 ABG
c 1. *ax.* 1. + A Gq *c* = 5 A Gq. Similiter erit D Fq =
d 22. 5. & 5 DHq. *d* ergo AC. AG :: DF. DH. compo-
32. 6. nendo igitur AC + AG, AG :: DF + DH.

DH,

DH. hoc eſt 2 AB. AG :: 2 DE. DH. *e* proinde AB. AG :: DE. DH. unde *f* dividendo AG. GB :: DH. HE. Q. E. D.

e 22. 5.
f 17. 5.

PROP. III.

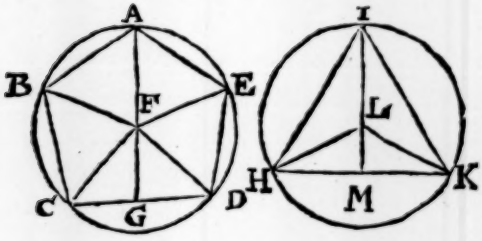


Idem circulus ABD *comprehendit & Dodecaedri pentagonum* ABCDE, *& Icoſaedri trianguum* LMN, *eidem ſphæræ inſcriptorum.*

Duc diametrum AG, rectaſque AC, CG. ſitque IK diameter ſphæræ, *a* & IKq = 5 OPq. fiatque OP. OQ :: OQ. QP. Quia ACq + CGq *c* = AGq *d* = 4 FGq ; & ABq *e* = FGq + CGq. *f* erit ACq + ABq = 5 FGq. porro, quia CA. AB *g* :: AB. CA − AB ; ac OP. OQ :: OQ. QP. *h* ideoque CA. OP :: AB. OQ. *k* erit 3 ACq (*l* IKq.) 5 OPq (*m* IKq) :: 3 ABq. 5 OQq. ergo 3 ABq = 5 OQq. Verum ob ML *n* latus pentagoni circuo inſcripti, cujus radius OP, erunt 15 RMq = 5 MLq *p* = 5 OPq + 5 OQq = * 3 ACq + 3 ABq *q* = 15 FGq. *r* ergo RM = FG. *ſ* proinde circ. ABD = circ. LMN. Q. E. D.

a *ſch.* 47. 1.
b 30. 6.
c 47. 1.
d 4. 2.
e 10. 13.
f 2. & 3. *ax.*
g 8. 13.
h 2. 13. & 16. 5.
k 22. 6. & 4. 5.
l 15. 13.
m *conſtr.*
n *cor.* 16. 13
o 12. 13.
p 10. 13.
p 15. 5, & ſupra.
* *Prius.*
r 1. *ax.* 1. & *ſch.* 48. 1
ſ 1. *def.* 3.

PROP. IV.



Si ex F *centro circuli pentagonum dodecaedri* ABCDE *circumſcribentis ducatur perpendicularis* FG *ad pentagoni unum latus* CD; *erit quod ſub dicto latere* CD, *& perpendiculari* FG *comprehenditur rectangulum trigeſies ſumptum, icoſaedri ſuperficiei æquale.* item,

Si ex centro L *circuli triangulum icoſaedri* HIK *circumſcribentis, perpendicularis* LM *ducatur ad trianguli unum latus* HK; *erit quod ſub dicto latere* HK, *& perpendiculari* LM *comprehenditur rectangulum trigeſies ſumptum, icoſaedri ſuperficiei æquale.*

Duc FA, FB, FC, FD, FE. *a* Erunt trianguli CFD, DFE, EFA. AFB, BFC æqualia. atqui CD×FG *b* = 2 triang. CFD. ergo 30 CD×GF *c* = 60 CFD *d* = 12 pentag. ABCDE *e* = ſuperf. dodecaedri. Q. E. D.

Duc LI, LH, IK. eſtque HK×LM *f* = 2 triang. LHK. ergo 30 HK×LM *g* = 60 HLK = 20 HIK *b* = ſuperfic. icoſaedri. Q. E. D.

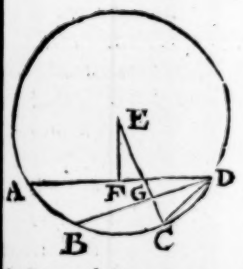
a 8. 1.
b 41. 1.
c 15. 5.
d 6. *ax.*
e 17. 3.
f 41. 1.
g 15. 5.
h 16. 13.
k 15. 5.

Coroll.

CD×FG. HK×LM *k* :: ſuperfic. dodecaed. ad ſuperf. icoſaedri.

PROP

PROP. V.



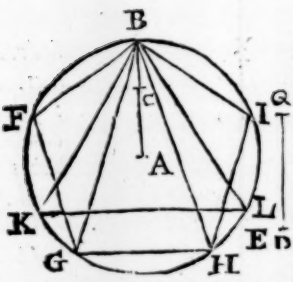
X *Superficies dodecaedri ad ſuperficiem icoſaedri in eadem ſphæra deſcripti eandem proportionem habet, quum* H *latus cubi ad* AD *latus icoſaedri.*

H Circulus A B C D *a* circumſcribat tam dodecaedri pentagonum, quam icoſaedri triangulum; quorum latera BD, AD; ad quæ demittantur ex E centro perpendiculares EF, EGC. & connectatur CD. [a 3. 14.]

Quoniam EC+CD. EC *b* :: EC. CD. erit EG (*c* ½ EC+½ CD.) EF (*d* ½ EC) *e* :: EF. EG—EF (½ CD.) atqui H. BD *f* :: BD. H—BD. *g* ergo H. BD :: EG. EF. proinde H × EF =BD × EG. quum igitur H. AD *b* :: H × EF. AD × EF. erit H. AD :: BD × EG. AD × EF :: *l* ſuperfic. dodecaedri ad ſuperfic. icoſaedri. Q. E. D.

[b 9. 13.]
[c 1. 14.]
[d *cor.* 13.]
[e 15. 5.]
[f *cor.* 17. 13.]
[g 2. 14.]
[h 1. 6.]
[k 7. 5.]
[l *cor.* 4. 14.]

PROP. VII.



Si recta linea AB *secetur extrema ac media ratione; erit ut recta* BF *potens id, quod à tota* AB, *& id quod à majori segmento* AC, *ad rectam* E, *potentem id quod à tota* AB, *& id quod à minori segmento* BC; *ita latus cubi* BG *ad latus icosaedri* BK *eidem sphæræ cum cubo inscripti.*

Circulo, cujus semidiameter AB, inscribantur dodecaedri pentagonum BFGHI, & icosaedri triangulum BKL. *a* quare BG latus cubi erit eidem sphæræ inscripti. igitur BKq *b* = 3 ABq; & Eq *c* = 3 ACq. ergo BKq. Eq *d* :: ABq. ACq *e* :: BGq. BFq. permutando igitur BGq, BKq :: BFq. Eq. *f* unde BG. BK :: BF. E. Q.E.D.

a *cor.* 17. 13.
b 12. 13.
c 4. 13.
d 15. 5.
e 2. 14.
f 22. 6

PROP. VII.

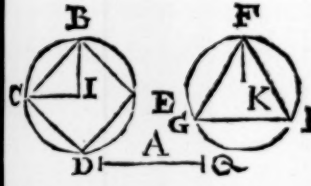
Dodecaedrum est ad Icosaedrum, ut cubi latus ad latus Icosaedri, in una eademque sphæra inscripti.

Quoniam *a* idem circulus comprehendit & dodecaedri pentagonum & icosaedri triangulum, *b* erunt perpendiculares à centro sphæræ ad plana pentagoni & trianguli ductæ inter se æquales. itaque si dodecaedrum & icosaedrum intelligantur esse divisa in pyramides, ductis rectis à centro sphæræ ad omnes angulos, omnium pyramidum altitudines erunt inter se æquales. Cum igitur pyramides æque altæ *c* sint ut bases, & superficies dodecaedri sit æqualis 12 pentagonis, superficies vero icosaedri 20 triangulis; erit

a 3. 14.
b 47. 1.
c 5, & 6 12.

erit dodecaedrum ad icosaedrum, ut superficies dodecaedri ad superficiem icosaedri, *d* hoc est, ut latus cubi ad latus icosaedri. d 5. 14.

PROP. VIII.



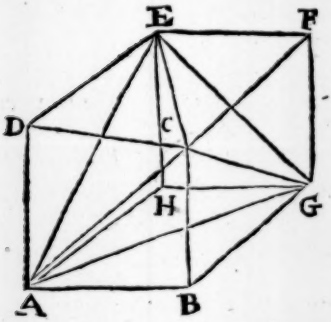
Idem circulus BCDE *comprehendit & cubi quadratũ* BCDE *& octacdri triangulum* FGH, *ejusdem sphæræ.*

Sit A diameter sphæræ. Quoniam Aq *a* = 3 BCq *b* = 6 BIq; itemque Aq *c* = 2 GFq = 6 KFq; erit BI = KF. *e* ergo circulus CBED = GFH. Q. E. D.

a 15. 13.
b 47. 1.
c 14. 13.
d 12. 13.
e 2. *def.* 1.

LIB. XV.

PROP. I.



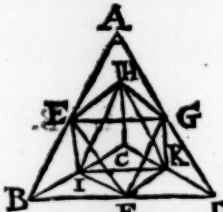
IN *dato cubo* ABGHDCFE *pyramidem* AGEC *describere.*

Ab angulo C duc diametros CA, CG, CE; Easque connecte diametris AG, GE, EA. Hæ omnes inter se *a* æquales sunt, utpote æqualium quadratorum diametri. ergo triangula CAG, CGE, CEA, EAG æquilatera sunt, ac æqualia: proinde AGEC est pyramis, quæ cubi angulis insistit, eique idcirco *b* inscribitur. Q. E. F.

a 47. 1.

b 31. *def.* 11

PROP. II.



In data pyramide ABDC *octaedrum* EGKIFH *describere.*

a Biseca latera pyramidis in punctis E, I, F, K, G, H; quæ connecte 12 rectis EF, FG, GE, &c. Hæ omnes *b* æquales sunt inter se. proinde 8 triangula EHI, IHK, &c. æquilatera sunt & æqualia, adeoq; constituunt *c* octaedrum *d* in data pyramide descriptum. Q. E. F.

a 10. 1.

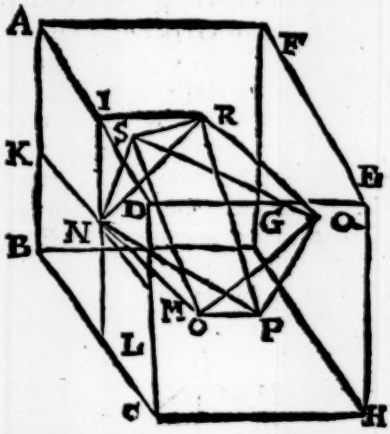
b 4. 1.

c 27. *def.* 11

d 31. *def.* 11

PROP.

PROP. III.

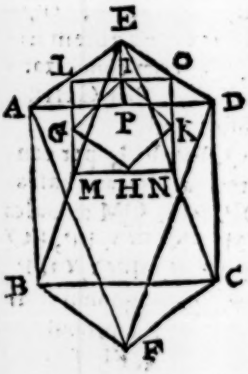


In dato cubo CHGBDEFA *octaedrum* NPQSOR *describere.*

Connecte quadratorum * centra N, P, Q, S, O, R, 12 rectis NP, PQ, QS, &c. quæ *a* æqualia sunt inter se, ideoque 8 triangula efficiunt æquilatera & æqualia. proinde *b* inscriptum est cubo Octaedrum NPQSOR. Q. E. F.

* 8. 4.
a 4. 1.
b 31. & 27. *def.* 11.

PROP. IV.



In dato octaedro ABCDEF *cubum inscribere.*

Latera pyramidis EABCD, cujus basis quadratum ABCD, bisecentur rectis LM, MN, NO, OL; quæ *a* æquales sunt & *b* parallelæ lateribus quadrati ABCD. *c* ergo quadrilaterum LMNO est quadratum.

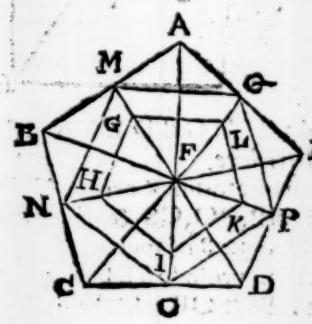
a 4. 1.
b 2. 6.
c 29. *def.* 1.

Eodem modo, si latera quadrati LMNO bisecentur

centur in punctis G, H, K, I, & connectantur GH, HK, KI, IG, erit GHKI quadratum. Quod si eadem arte in reliquis 5 pyramidibus octaedri centra triangulorum rectis conjungantur, describentur quadrata similia & æqualia quadrato GHKI. quare sex hujusmodi quadrata cubum constituent, qui quidem intra octaedrum descriptus erit, *d* cum octo ejus anguli tangant octo octaedri bases in earum centris. Q. E. F.

d 31. def. 11

PROP. V.



In dato Icosaedro Dodecaedrum inscribere.

Sit ABCDEF pyramis Icosaedri, cujus basis pentagonum ABCDE; centra autem triangulorum G, H, I, K, L; quæ connectantur rectis GH, HI, IK, KL, LG. Erit GHIKL pentagonum dodecaedri inscribendi.

* 5. 4.

Nam rectæ FM, FN, FO, FP, FQ, per centra triangulorum transeuntes, *a* bisecant bases. *b* ergo rectæ MN, NO, OP, PQ, QM æquales sunt inter se. quinetiam FM, FN, FO, FP, FQ *c* pares sunt. *d* ergo anguli MFN, NFO, OFP, PFQ, QFM æquantur. pentagonum igitur GHIKL æquiangulum est; *c* proinde & æquilaterum, cum FG, FH, FI, FK, FL *f* pares sint. Quod si eadem arte in reliquis undecim pyra-

a cor. 3. 3.
b 4. 1.
c 4. 1.
d 8. 1.
e 4. 1.
f 12. 13.

pyramidibus icoſaedri, centra triangulorum rectis lineis connectantur, deſcribentur pentagona æqualia & ſimilia pentagono GHIKL. quamobrem 12 hujuſmodi pentagona dodecaedrum conſtituent; quod quidem in icoſaedro erit deſcriptum, cum viginti anguli dodecaedri in centris viginti baſium icoſaedri conſiſtant. Quapropter in dato icoſaedro dodecaedrum deſcripſimus. Q. E. F.

FINIS.

EUCLIDIS
DATA

succincte demonstrata;

Una cum Emendationibus quibusdam & Additionibus ad ELEMENTA

EUCLIDIS

nuper opera.

Opera
Mri. IS. BARROW, *Cantabrigiensis*,
Coll. *Trin.* Soc.

LONDINI,
Excudebat *J. Redmayne*, 1678.

Ornatiſſimo viro

D. JACOBO STOCK,

Amico ſuo & patrono ſingulari.

NEc publica, nec tui nominis luce dignum cenſeo hunc pauſorum dierum partum puſillum & præmaturum. Qui quidem quod ſe mundo, quodque Tibi, ſpectandum obtulerit, duplici nomine arrogantiæ ſpeciem incurrit. Sed utrinque parata eſt excuſatio qualiſcunque. Nam amico obtemperatum oportuit jubenti mitterem hunc libellum Euclidæis (quæ cognatione proxima attingit) Elementis ſubjungendum. In eum quicquid eſt in publicum aut peccati aut meriti protinus rejicio, facti cujus author fuit, rationem redditurum. Inte autem delictum quod maxime aggravat, idem potenter extenuat, Tibi tantum debere. Nam cum iis, qui Diis ipſis ſacrificia, ac modica magnis Regibus donaria offerre non dubitarunt, ſatius eſſe credo, etiam pro immenſis beneficiis parum, quam nihil rependere. Sufficiat igitur regeſſiſſe, me Tibi multis magniſque nominibus obſtrictum fore; vices, quas potuero maximas, referre debere; ultra vota & grates nihil poſſe; illa privatim, has publice perſolutas præcellere; quibus

bus agendis, quam jamdiu ſpe & ſtudio aucupor, occaſionem nondum comparere; præſtare hanc oblatam prehendere, quamvis exilem, quam elapſam nequicquam pœnitentia proſequi. Eſto igitur hæc oblatio pignus quoddam & præludium futuræ amplioris, in qua meritorum in me Tuorum hiſtoria uberior ac diſtinctior commemoranda occurret. Quæ ſimpliciter agnoſcere, non aut fuſe deſcribere, aut digne prædicare, præſentis eſt inſtituti. Ac revera jam brevis ſum ἐκὼν ἀέκουτί γε θυμῷ, neceſſitate potius coactus, quam inductus conſilio. Nam me vela ventis turgentia alio avocant; ac vereor ne hæc pene currenti calamo exequentem, quæ hæc ad te perferet, amica manus, importuna patientia præſtoletur. Quid ſupereſt igitur, niſi ut te domi ſtudiis ac rebus honeſtis animum intendentem ſalutari præſentia tutetur, eum exorem venerandi ac ἀῤῥήτου nominis; quem tantæ beneficentiæ benignum remuneratorem jugibus votis exopto; idemque me extemplo ſuper Tyrrhenos, Ionios, Ægeoſque fluctus longinquam profectionem ſuſceptur͠u comitetur. Obteſtor autem, ne tenuis opellæ patrociniu͠m reſpuas, quod ultro impertire dignatus es

Tibi devinctiſſimus
& obſequentiſſimo,
I. B.

E U-

EUCLIDIS Data.

Definitiones.

I. DAta magnitudine dicuntur ſpatia, lineæ, anguli, quibus æqualia poſſumus invenire.

II. Ratio dari dicitur, cui poſſumus eandem invenire.

III. Rectilineæ figuræ ſpecie dari dicuntur, quarum & ſinguli anguli dati ſunt, & laterum rationes ad invicem datæ ſunt.

Hinc, datæ ſunt ſpecie figuræ, quibus ſimiles inveniri poſſunt.

IV. Poſitione dari dicuntur puncta, lineæ, angulique, quæ eundem ſitum ſemper obtinent.

V. Circulus magnitudine dari dicitur, cujus ea quæ ex centro datur magnitudine.

VI. Poſitione & magnitudine dari dicitur circulus, cujus datur centrum poſitione, & ea quæ ex centro magnitudine.

VII. Circuli ſegmenta magnitudine dari dicuntur, in quibus dati ſunt magnitudine anguli & ſegmentorum baſes.

VIII. Poſitione & magnitudine dari dicuntur circuli ſegmenta, in quibus anguli magnitudine dati ſunt, & ſegmentorum baſes poſitione & magnitudine.

IX. Magnitudo magnitudine major eſt data, quando ablata data, reliqua eidem æqualis eſt.

X. Magnitudo magnitudine minor eſt data, quando adjuncta data, tota eidem æqualis eſt.

Ut ſi A data ſit, erit A + B ⊏ B data. At B ⊐ A + B data.

XI. Magnitudo magnitudine major eſt data quam in ratione, quando ablata data, reliqua ad eandem habet rationem datam.

XII.

XII. Magnitudo magnitudine minor est data quam in ratione, quando adjuncta data tota ad eandem rationem habet datam.

Ut si A data sit, & $\frac{B}{C}$ detur, erit A+B⊏C; data q. in r. sin $\frac{A+B}{C}$ detur, erit B⊐C data q. in r.

PROP. I.

A. B. *Datarum magnitudinum* A, B,
a. b. *ad invicem datur ratio.*

Nam quia A*datur, *a* inveniri potest aliqua a=A. Eodem jure sume b=B. *b* estque a. b :: A. B. *c* quare ratio $\frac{A}{B}$ data est. Q. E. D.

*hyp.
a 1. *def.*
b *sch.* 7. §
c 2. *def.*

PROP. II.

A. B. *Si data magnitudo* A *ad aliam*
a. b. *aliquam* B *habeat rationem datam, datur etiam hæc alia magnitudo.*

Nam ob A*datam, *a* sume a=A; ac ob $\frac{A}{B}$ *datam, *b* sit a=A. *c* ergo b=B. *a* quare B datur. Q. E. D.

*hyp.
a 1. *def. d.*
b 2. *def. d.*
c 9. 5.

PROP. III.

A. B. *Si quotlibet datæ magnitudines*
a. b. *A, B componantur, etiam ea* A+B *quæ ex his componitur, data erit.*

Nam *a* cape a=A, & b=B; *b* estque a+b =A+B. *a* quare A+B datur. Q. E. D.

a 1. *def.*
b 2. *ax.* 1.

PROP. IV.

A. B. *Si à data magnitudine* A *auferatur data magnitudo* B, *etiam reliqua* A−B *dabitur.*
a. b.

a Sint enim a = A, & b=B, ergo A−B= a−b. *a* proinde A−B datur. Q. E. D.

a 1. *def. d.*
b 3. *ax.* 1.

PROP.

PROP. V.

A. B. C. D. *Si magnitudo* A *ad sui-ipsius ali-quam partem* B *habeat rationem datam, etiam ad reliquam* A—B *habebit rationem datam.*

a *hyp.* b 2. *def. d.* c *cor.* 9. 5. Nam, quia $\frac{A}{B}$ *a* data est, *b* sit A. B :: C. D. ergo A. A — B :: C. C — D. *b* proinde $\frac{A}{A-B}$ datur. Q. E. D.

PROP. VI.

A. B. C. D. *Si componantur duæ magnitudines* A, B, *habentes ad invicem rationem datam, etiam quæ ex his componitur magnitudo* A+B, *habebit ad utramque* A *&* B *rationem datam.*

a 2. *def. d.* b 18. 5. c 2. *def. d.* Nam *a* sit A. B :: C. D. *b* ergo A+B. B :: C+D. D. *c* quare A+B datur, Similiter B+A datur. Q. E. D. $\frac{A+B}{B}$

PROP. VII.

A. B. *Si data magnitudo* A+B *data ratione secetur, utrumque segmentorum* A, *&* B *datum est.*

* *hyp.* a 6. *dat.* b 2. *dat.* Nam ob $\frac{A}{B}$ *datam, *a* erit $\frac{A+B}{A}$ data. *b* ergo A datur. Eodem modo B datur. Q. E. D.

PROP. VIII.

A. C. B. D. E. F. *Quæ* A, B *ad idem* C *rationem habent datam, habebunt ad invicem rationem datam.*

a 1. *def. d.* Nam *a* sit A. C :: D. E. *a* & C. B :: E. F. quare ex æquali A. B :: D. F. *a* ergo $\frac{A}{B}$ datur. Q. E. D.

Coroll.

Rationes ex datis rationibus compositæ, datæ sunt. Ut $\frac{A}{B}$ fit ex $\frac{A}{C}$, & $\frac{C}{B}$ datis.

PROP.

PROP. IX.

A. B. C. D. E. F. *Si duæ, pluresve magnitudines* A,B,C *ad invicem habeant rationem datam, habeant autem illæ magnitudines* A,B,C *ad alias quasdam* D,E,F *rationes datas, etsi non easdem; illæ aliæ magnitudines* D, E, F *etiam ad invicem habent rationes datas.*

Nam ratio $\frac{D}{E}$ *a* fit ex *b* datis $\frac{D}{A}, \frac{A}{B}, \frac{B}{E}$, *c* ergo $\frac{D}{E}$ datur. Eadem de causa datur $\frac{E}{F}$. Q.E.D.

a 20. *def.* 5.
b *hyp.*
c *cor.* 8. *dat.*

PROP. X.

A. B. C. *Si magnitudo magnitudine major fuerit data, quam in ratione; & simul utraque illa eadem major erit data quam in ratione. Sin autem simul utraq; magnitudo eadem magnitudine major fuerit data, quam in ratione; & reliqua illa eadem major erit data quam in ratione; aut reliqua data est cum consequente, ad quam habet altera magnitudo rationem datam.*

1. Sint A, & B datæ. *a* erit $\overline{B+C}$ data. *b* ergo $\overline{A+B+C}\sqsubset C$ data q. in r. Q. E. D.

a 6. *dat.*
b 11. *def. d.*

2. Sint A, & $\overline{B+C}$ datæ: *c* ergo B datur, proinde $\overline{A+B}\sqsubset C$ data q. in r. Q. E. D.

c 17. 5.

3. Sint $\overline{A+B}$, & C datæ. *d* Liquet $\overline{B+C}$ dari. Q. E. D.

d 5. *dat.*

PROP. XI.

A. B. C. *Si magnitudo magnitudine major fi data quam in ratione, eadem simul utraque major erit data quam in ratione. Et si eadem simul utraque major sit data quam in ratione, eadem reliqua magnitudine major erit data quam in ratione.*

a 6. dat. b 11. def d. c 5. dat.

1. A, & $\frac{B}{C}$ dantur. *a* ergo $\frac{B}{B+C}$ datur. proinde *b* A+B⊏B+C data q. in r. Q. E. D.

2. A, & $\frac{B}{B+C}$ dantur. *c* ergo $\frac{B}{C}$ datur. proinde *b* A+B⊏C data q. in r. Q. E. D.

PROP. XII.

A. B. C.

Si fuerint tres magnitudines A, B, C, *& prima cum secunda* (A+B) *data fit, secunda quoque cum tertia* (B+C) *data fit; aut prima* A *tertiæ* C *æqualis est, aut altera altera major data.*

a 4. ax. 1. b 4. dat.

Nam si A+B, & B+C pares sint, *b* liquet A & C æquari; sin istæ impares fuerint, *b* liquet excessum A—C, vel C—A dari. Q. E. D.

PROP. XIII.

D, A+B, C. E

Si fuerint tres magnitudines D, A+B, C, *& earum prima* D *ad secundam* A+B *habeat rationem datam; secunda autem* A+B *tertia* C *major fit data quam in ratione; prima quoque* D *major erit tertia* C *data quam in ratione.*

a 2. def. d. b 19. 5. c 2. dat. d 2 def. d. c 8. dat. f 11. def. d.

Sint A, & $\frac{B}{C}$ ac $\frac{D}{A+B}$ datæ; *a* fitque A+B. D :: A. E *b* :: B. D — E. ergo *c* E, *d* & $\frac{B}{D-E}$ & (ob $\frac{B}{C}$ datam, *e* $\frac{C}{D-E}$ dantur. *f* quare D(E+: D—E)⊏C data q. in r. Q. E. D.

PROP. XIV.

A. B. E. C. D.

Si duæ magnitudines A *&* C *ad invicem habeant rationem datam, utrique autem illarum adjiciatur data magnitudo* B *&* D; *totæ* A+B, C+D, *aut habent rationem datam, aut altera* A+B *altera* C+D *major erit data quam in ratione.*

Nam

Nam ſi A. C :: B. D *a* :: A+B. C+D ob $\frac{A}{C}$ *b* datam, *c* liquet $\frac{A+B}{C+D}$ dari.

a 13. 5.
b *hyp.*
c 2. *def. d.*

Saltem *d* ſit A. C :: E. D. *a* :: A+E. C+D. Ergo *c* $\frac{A+E}{C+D}$, ac *e* E, *f* ideoque B—E dantur. *g* proinde A+B (A+E :+B—E.) ⊏ C+D data q. in r. Q. E D.

d 2. *def. d.*
e 2. *dat.*
f 4. *dat.*
g 11. *def. d.*

PROP. XV.

A. C. B. D. E.

Si duæ magnitudines A *&* C *habeant ad invicem rationem datam, & ab utraque harum auferatur data magnitudo* B *&* D; *reliquæ magnitudines* A—B, C—D *ad invicem habebunt aut rationem datam, aut altera* A—B, *altera* C—D *major erit data quam in ratione.*

b Nam ſi A. C :: B. D *a* :: A—B. C—D. ob $\frac{A}{C}$ datam, *c* liquet $\frac{A-B}{A-C}$ dari.

a 19. 5.
b *hyp.*
c 2. *def. d.*

Saltem *d* ſit A. C :: E. D *a* :: A—E. C—D. Ergo *c* $\frac{A-E}{C-D}$, & *e* E, ac *f* ideo E—B dantur. *g* proinde A—B (A—E :+E—B) ⊏ C—D data q. in r. Q. E. D.

d 2. *def. 2.*
e 2. *dat.*
f 4. *dat.*
g 11. *def. d.*

PROP. XVI.

B. C. A. D. E.

Si duæ magnitudines B, C *habeant rationem datam, & ab una quidem illarum* C *auferatur data magnitudo* D, *alteri autem* B *adjiciatur data magnitudo* A; *tota* A+B *reſidua* C—D *major erit data quam in ratione.*

Sit enim C. B *a* :: D. E *b* :: C—D. B—E. ergo *c* $\frac{C-D}{B-E}$, & *d* E, ac *e* ideo E+A dantur. *f* proinde B+A (E+A :+B—E) ⊏ C—D data q. in r. Q. E. D.

a 2. *def. d.*
b 19. 5.
c 2. *def. d.*
d 2. *dat.*
e 3. *dat.*
f 11. *def. d.*

PROP. XVII.

A+B. D+E. C. *Si fuerint tres magnitudines* A+B, C, D+E; *& prima quidem* A+B *secunda* C *major sit data quam in ratione, tertia quoque* D+E *eadem secunda* C *major sit data quam in ratione; prima* A+B *ad tertiam* D+E *aut rationem habebit datam, aut altera altera major erit data quam in ratione.*

Nam ob A, D, & $\frac{B}{C}$, $\frac{E}{C}$ *a* datas, *b* erit $\frac{B}{E}$ data. ergo per 14. hujus.

a *hyp.*
b 8. *dat.*

PROP. XVIII.

A+C. E. G. B+D. F. H. *Si fuerint tres magnitudines, atque ex his una utraque reliquarum major sit data quam in ratione; reliquæ duæ aut datam rationem habebunt ad invicem, aut altera altera major erit data quam in ratione.*

Datæ sint A, B, $\frac{C}{E}$, $\frac{D}{F}$; ac sit A+C=B+D.

Sitque C. C *a* :: A G *b* :: C+A. E+G. itemque D. F *a* :: B. H *b* :: D+B. F+H. *c* ergo C+A *d* hoc est B+D, *c* & B+D, ac *e* idcirco $\overline{E+G}$, $\overline{E+G}$, $\overline{F+H}$ E+G quin & G ac H *f* dantur. ergo per 15. $\overline{F+H}$; (hujus.

a 2. *def. d.*
b 12. 5.
c 2. *def. d.*
d 7. 5.
e 8. 5.
f 2 *dat.*

PROP. XIX.

A+B. E. C+D. F. *Si fuerint tres magnitudines, & prima quidem magnitudo secunda magnitudine major sit data quam in ratione, sit quoque secunda major tertia data quam in ratione; prima magnitudo tertia magnitudine major erit data quam in ratione.*

Sint A, C, & $\overline{C+D}$, $\frac{D}{E}$ datæ; dico A+B $\overline{\ }$E data q. in r.

Nam

Nam ſit C+D. B *a* :: C. F *b* :: D. B—F. ergo *c* $\frac{C}{F}$ & *d* F, ac *e* ideo F+A, & *c* $\frac{D}{B-F}$ *f* ideoque E dantur. *g* proinde A+B (F+A : +B—F) $\frac{}{B-F}$ ⊏E data q. in r. Q. E. D.

a 2. *def. d.*
b 19. 5.
c 2. *def. d.*
d 2. *dat.*
e 3. *dat.*
f 8. *dat.*
g 11. *def. d.*

PROP. XX.

A. C. E. B. D. *Si datæ fuerint duæ magnitudines* A, C; *& auferantur ab ipſis magnitudines* B, D *habentes ad invicem rationem datam; reſiduæ magnitudines* A—B, C—D *aut habebunt ad invicem rationem datam, aut altera* A—B *altera* C—D *major erit data quam in ratione.*

Nam ſi A. C :: B. D *a* :: A—B. C—D ; *b* liquet A—B dari.

a 19. 5.
b 2. *def. d.*

$\frac{C-D}{}$

Saltem ſit D. B *b* :: C. E *a* :: C—D. E—B. ergo *b* $\frac{C}{E}$ & *c* E, ac *d* propterea A—E, *b* itemque C—D datæ ſunt. *e* ergo A—B (A—E : +E $\frac{E-B}{}$ —B) ⊏ C—D data q. in r. Q. E. D.

c 2. *dat.*
d 4. *dat.*
e 11. *def. d.*

PROP. XXI.

A. C. E. B. D. *Si datæ fuerint duæ magnitudines* A, C; *& adjiciantur ipſis aliæ magnitudines* B, D *habentes ad invicem rationem datam, totæ* A+B, C+D *aut habebunt ad invicem rationem datam, aut altera* A+B *altera* C+D *major erit data quam in ratione.*

Nam ſi B. D :: AC *a* :: A+B. C+D, *b* liquet A+B dari.

a 12. 5.
b 2. *def. d.*

$\frac{C-D}{}$

Saltem ſit B. D *b* :: E. C *a* :: B+E. D+C. ergo *c* E, *d* ideoque A—E, & *b* B+E dantur. $\frac{D+C}{}$

c 2. *dat.*
d 4. *dat.*

e 11. def. e ergo A + B (B + E :: + A − E) ⊏ C + D data q. in r. Q. E. D.

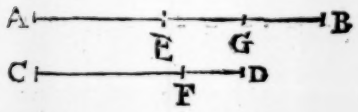
PROP. XXII.

A. B. C. *Si duæ magnitudines* A, B *ad aliam aliquam magnitudinem* C *habeant rationem datam, & simul utraque* A + B *ad eandem* C *habebit rationem datam.*

Nam ob $\frac{A}{C}$, $\frac{B}{C}$ datas, *b* erit $\frac{A}{B}$ data. *c* quare $\frac{A+B}{B}$, *b* ideoque $\frac{A+B}{C}$ data est. Q. E. D.

a *hyp.* b 8. *d.* c 6. *d.*

PROP. XXIII.



Si totum AB *ad totum* CD *habeat rationem datam, habeant autem & partes* AE, EB *ad partes* CF, FD *rationes datas (etsi non easdem ;) habebunt omnia ad omnia rationes datas.*

Nam sit AE. CF *a* :: AG. CD *b* :: GE. FD. *a* ergo $\frac{GE}{FD}$ datur. quare (ob $\frac{EB}{FD}$ *c* datam) *d* erit $\frac{GE}{EB}$ ac *e* ideo $\frac{EB}{GB}$ data. ergo quum *c* $\frac{AB}{CD}$, & *a* $\frac{AG}{CD}$, *d* ideoque $\frac{AB}{AG}$, ac proinde *e* $\frac{AB}{GB}$, dentur, *d* erit $\frac{EB}{AB}$, data. Quare *e* $\frac{AB}{AE}$, & *d* $\frac{AE}{EB}$, & *e* $\frac{EB}{CF}$, dantur. Q. E. D.

a *def. d.* b 19. 5. c *hyp.* d 8. *dat.* e 5. *dat.*

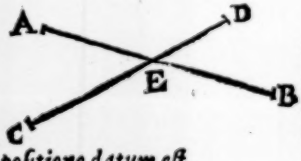
PROP. XXIV.

A B C *Si tres rectæ lineæ,* A, B, C, *proportionales fuerint ; prima autem* A *ad tertiam* C *habeat rationem datam ; & ad secundam* B *habebit rationem datam.*

Nam

Nam A. C *a*:: Aq. Bq. *b* ergo $\frac{Aq}{Bq}$ data eſt. a *cor.* 20 6.
b 2. *def. d.*
proinde $\frac{A}{B}$ *c* datur. Q. E. D c 1. *d.*

PROP. XXV.



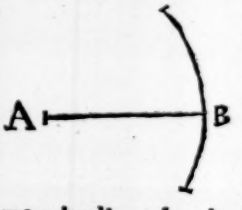
Si duæ rectæ lineæ, A B, C D *poſitione datæ ſe mutuo ſecuerint, punctum* E, *in quo ſe invicem ſecant, poſitione datum eſt.*

a Nam hæ lineæ alibi quam in E, neutrius ſitu mutuo, ſeſe interſecare nequeunt. a 4 *def. d.*

Schol.

a Idem patet de quibuſcunque lineis poſitione datis, ſeque in unico puncto interſecantibus: ut de circuli arcu, & recta, &c.

PROP. XXVI.

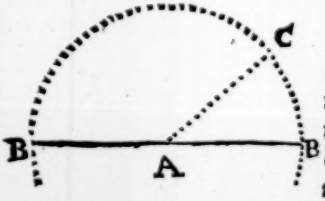


Si rectæ lineæ A B *extremitates* A, B, *poſitione datæ ſint, recta* A B *poſitione & magnitudine data eſt.*

Poſitione quidem, *a* quia inter eoſdem terminos unica recta duci poteſt: & a 14 *ex.*

magnitudine, *b* quia ſi centro A per B ducatur circulus, hujus omnes radii ipſi AB æquantur. b 1. *def. d.*

PROP. XXVII.



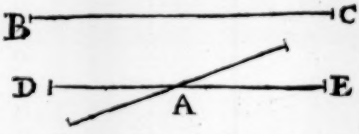
Si rectæ lineæ A B *poſitione & magnitudine datæ, data fuerit una extremitas* A; *& altera extremitas* B *data erit.*

a 1. *def. d.* b 3. *poſt.* c 2. *poſt.* d *cor.* 25. Nam ſi centro A, ſpatio AC *a* = AB *b* ducatur circulus, qui data recta *c* occurrat in B, *d* erit extremitas B data.

Schol.

Vides partes puncti B determinandas eſſe.

PROP. XXVIII.

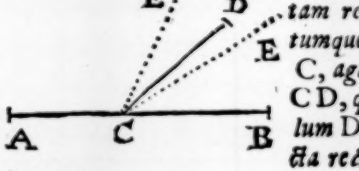


Si per datum punctum A *contra datam poſitione rectam* BC *agatur recta linea* DE, *acta recta* DE *poſitione data eſt.*

a 4. *def. d.* b 30. 1. c 34. *def.* 1. Nam *a* dic alteram per A ad BC fore parallelam. Hæc idcirco ad DE *b* parallela erit. *c* Quod repugnat.

Nota, Vocabulum *contra* in hoc libro paralleliſmum ſignificare.

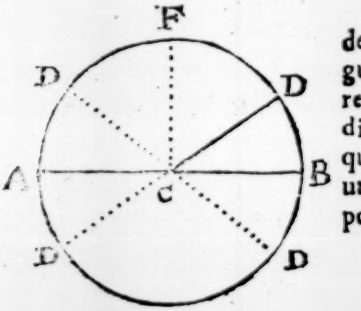
PROP. XXIX.



Si ad poſitione datam rectam AB, *datumque in ea punctum* C, *agatur recta linea* CD, *quæ faciat angulum* DCB *datum*; *acta recta* CD *poſitione data erit.*

a 4. *def. d.* b 9. *ax.* 1. *a* Nam quævis alia CE angulum *b* efficiet majorem, vel minorem dato BCD.

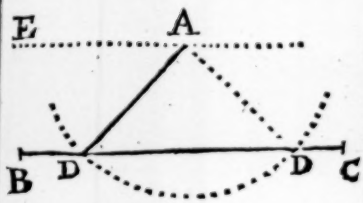
Schol.



Determinari debet ſitus anguli dati tam reſpectu perpendicularis CF, quam ipſius AB, ut cernis in apoſita figura.

PROP.

PROP. XXX.



Si à dato puncto A *in datam positione rectam* BC *agatur recta linea* AD, *quæ faciat angulum* ADC *datum, acta linea* AD *positione data est.*

Nam per A duc AE ad BC parallelam. *a* Hæc positione datur. Item ang. DAE par dato alterno ADC *b* datus est. *c* ergo recta AD positione data est. Q. E. D.

a 28. *dat.*
b 1. *def. d.*
c 29 *dat.*

Schol.

Hinc praxim discimus à dato puncto ducendi rectam, quæ cum data positione recta datum angulum efficit.

PROP. XXXI.

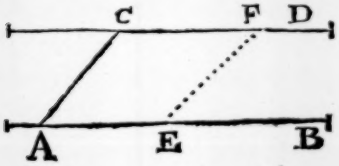
Si à dato puncto A *in datam positione rectam* BC *data magnitudine recta* AD *ducatur, positione quoque data erit.*

Nam puncta D, per quæ transit circulus centro A, *a* spatio AD descriptus, *b* data sunt. *c* ergo AD positione data est. Q. E. D.

a 1. *def. d.*
b *sch.* 25. *d.*
c 26. *d.*

PROP.

PROP XXXII.

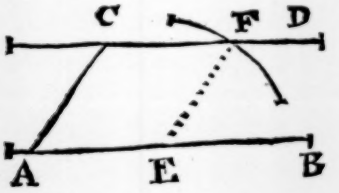


Si in datas positione parallelas rectas AB, CD *agatur recta linea* AC, *quæ faciat angulos datos* BAC, ACD, *acta recta* AC *magnitudine data est.*

Nam ad E (quodvis punctum in AB) fac ang. BEF = *a* BAC. liquet rectas EF, AC *b* parallelas, & *c* pares fore. *d* quare AC data est. Q. E. D.

a 1. *def. d.*
b 29. 1.
c 34. 1.
d 2. *def. d.*

PROP. XXXIII.



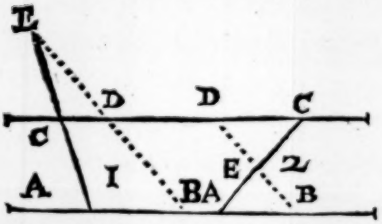
Si in datas positione parallelas rectas AB, CD *agatur magnitudine data recta* AC, *faciet angulos* BAC, ACD *datos.*

Nam ex quovis puncto E in AB, spatio EF *a* = AC describe circulum occurrentem rectæ CD in F. *b* Liquet EF, & AC parallelas esse posse. *c* ergo.

a 1. *def. d.*
b 34. 1.
c 29. 1.

PROP.

PROP. XXXIV.



Si in datas positione parallelas rectas A B, C D *à dato puncto* E *agatur recta linea* E C A, *secabitur data ratione.*

Nam ab E duc rectam E B utcunque parallelis occurrentem in D, & B. *a* liquet esse EC. CA :: ED. DB. *b* quare $\frac{EC}{CA}$ datur. Q. E. D.

a 2. 6.
b 2. def. d.

PROP. XXXV.

Si à dato puncto E *in datam positione rectam* AB *agatur recta linea* EA, *secetur que data ratione; agatur autem per punctum sectionis* C *contra datam positione rectam* A B *recta linea* C D ; *acta linea* CD *positione data est.*

Recta enim E B ducta ab E utcunque in A B, *a* secetur sic ut ED. DB :: EC. CA. ob punctum D datum, *b* erit CD positione data. Q. E. D.

a 10. 6.
b 28 dat.

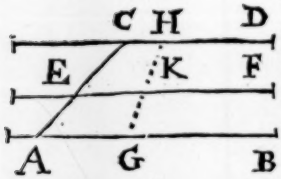
PROP. XXXVI.

Si à dato puncto E *in datam positione rectam lineam* AB *agatur recta linea* E A; *adjiciatur autem ipsi aliqua recta* EC, *quæ ad illam* (E A) *habeat rationem datam; per extremitatem autem* C *adjectæ lineæ* E C *agatur contra datam positione rectam* AB *recta linea* CD ; *acta linea* CD *positione data est.*

Demonstratio parum differt à præcedenti. *Vide fig.* 2.

PROP.

PROP. XXXVII.

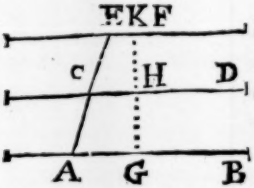


Si in datas positione parallelas rectas AB, CD, *agatur recta linea* AC, *& secetur ratione data; agatur autem per sectionis punctum* E *contra datas positione rectas* AB, CD *linea recta* EF; *acta recta* EF *positione data est.*

a 2.def.d
b 28.dat. & sch.2.6.

Nam duc rectam GH utcunque occurrentem parallelis. Hæc *a* secta sit in K ita ut GK. KH :: AE. EC. *b* Punctum K parallelæ (EF) situm determinat. Q. E. F.

PROP. XXXVIII.

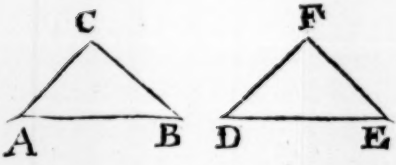


Si in datas positione rectas parallelas AB, CD *agatur recta linea* AC; *adjiciatur autem ipsi quædam recta* CE, *quæ ad actam* AE *habeat rationem datam; per extremitatem autem* E *adjectæ* CE *agatur contra datas positione parallelas* AB, CD *recta linea* EF; *acta recta linea* EF *est data positione.*

Demonstratio perfimilis est præcedenti. Cerne & compara figuras.

PROP.

PROP. XXXIX.



Si trianguli ABC *ſingula latera* AB, BC, AC *magnitudine data ſint, triangulum* ABC *ſpecie datum eſt.*

Nam *a* fac triang. DEF ipſi ABC æquilaterum. Hoc eidem *b* æquiangulum erit. *c* ergo ABC ſpecie datum eſt. Q. E. D.

a 22. 1.
b 5 6.
c 3. *def. d.*

PROP. XL.

Si trianguli ABC *ſinguli anguli,* A, B, C *magnitudine dati ſint, triangulum* ABC *ſpecie datum eſt.*

Nam ad quamvis DE *a* fac triang. DEF ipſi ABC æquiangulum. *b* Hoc eidem ſimile erit. *c* proinde trigonum ABC ſpecie datum eſt. Q. E. D.

a 23. 1.
b 4. 6.
c 3. *def. d.*

PROP. XLI.

Si triangulum ABC *unum angulum* A *datum habeat; circa datum autem angulum* A *duo latera* AB, AC *ad invicem habeant rationem datam; triangulum* ABC *ſpecie datum eſt.*

Nam in uno latere dati anguli ſume quampiam AD; & *a* fit AB. AC :: AD. AE. & duc DE. *b* Liquet trigonum ADE ipſi ABC ſimile fore. *c* Quare ABC ſpecie datum eſt. Q. E. D.

a 1 *def. d.*
b 6. 6.
c 3. *def. d.*

PROP.

PROP. XLII.

Si trianguli ABC *latera ad invicem habeant rationem datam, triangulum* ABC *specie datum est.*

Nam *a* fac A B. BC :: DE. EF. *a* & B C. CA :: EF. FD. *b* Liquet trigonum DEF trigono ABC assimilari. *c* quare A B C specie datum est. Q. E D.

a 12. 6.
b 5. 6.
c 3. *def.* d.

Vide fig. 39.

PROP. XLIII.

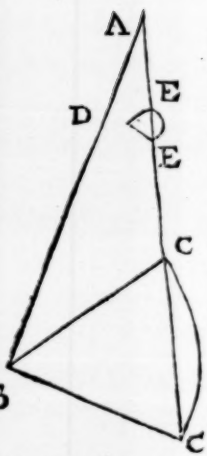
Si trianguli rectanguli ACB *circa unum acutorum angulorum* A *latera* AB, AC *ad invicem rationem habeant datam, triangulum* A C B *specie datum est.*

Nam esto D E F semicirculus utcunque; & *a* fac A B. A C :: D E. D F. inventamque D F *b* adapta in semicirculo; & duc EF. *c* Liquet triang. D F E ipsi A C B assimilari; & *d* proinde ipsum ACB specie dari. Q. E. D.

a 12. 5.
b 1. 4.
c 32. 1. & 4. 6.
d 3. *def.* d.

PROP.

PROP. XLIV.



Si triangulum A B C *habeat unum angulum* A *datum; circa alium autem angulum* A B C *latera* A B, B C *ad invicem habeant rationem datam; triangulum* A B C *ſpecie datum eſt.*

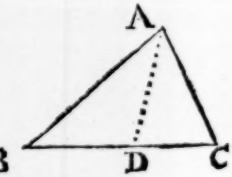
Nam in crure dati anguli ſume quamlibet A D. & *a* fac A B. B C :: A D. D E. centro D ſpatio D E deſcribe circulum, qui ſecet alterum dati anguli latus in E. *b* Eritque triang. A D E ipſi A B C ſimile. *c* quare datur ſpecie triang. A B C. Q. E. D.

a 2. *def. d.*

b 7. 6.

c 3. *def. d.*

PROP. XLV.



Si triangulum B A C *unum angulum* BAC *datum habeat; circa datum autem angulum* BAC *latera ſimul utraque tanquam unum* (BA + AC) *ad reliquum latus* (BC) *rationem habeant datam; triangulum* BAC *ſpecie datum eſt.*

Datum angulum B A C *a* biſecet recta A D. *b* ergo B A. A C :: B D. D C. & componendo BA + AC. AC :: BC. DC. permutando igitur BA + AC. BC :: AC. DC. ergo ob $\frac{BA+AC}{BC}$ *c* datam, *d* erit $\frac{AC}{DC}$ data, item ang. DAC ſub-duplus

a 9. 1.

b 3. 6.

c *hyp.*

d 2 *def d.*

c 2. dat. duplus dati B A C *c* datur. *f* ergo ang. C datur.
f 44. dat. *g* proinde trigonum ABC ſpecie datum eſt.
g 40. dat.

Coroll.

Hinc in triangulo, datis uno latere AB, uno angulo B A C, & ratione aggregati laterum ad baſim (R ad S;) datur triangulum. Nam datum angulum biſeca, & fac R. S :: AB. BD. & centro B ſpatio B D duc circulum occurrentem rectæ biſecanti in D; & produc BDC. habes triangulum.

PROP. XLVI.

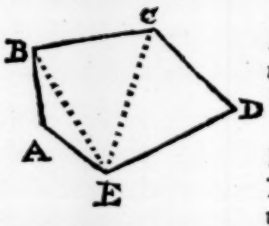
Si triangulum B A C *unum angulum* C *datum habeat: circa alium autem angulum* BAC *latera ſimul utraque tanquam unum* (BA + AC) *habeant ad reliquum* (B C) *rationem datam; triangulum* BAC *ſpecie datum eſt.*

Nam biſecto angulo BAC, erit (ut in præcedenti) $\frac{AC}{DC}$ data. item ang. C *a* datus eſt. ergo ang. D A C, *b* proinde & duplus B A C datur. *c* quare triang BAC ſpecie datur. Q. E. D.

a *hyp.*
b 2. *dat.*
c 40. *dat.*

Deducetur ab hac corollarium ſimile præcedenti.

PROP. XLVII.



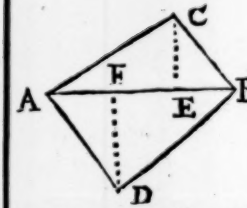
Data ſpecie recti linea ABCDE *in data ſpecie triangula* BAE, CDE BCE *dividuntur.*

Nam ob ang B, & BA *a* dat. *b* erit triang. $\frac{AB}{2}$ BAE ſpecie datum. Simili diſcurſu triang. CDE ſpecie datur. *c* quare ang. DCE datus eſt; Hunc deme ex dato BCD, *d* eſtque reliquus BCE datus. Similiter ang. CBE datur. *e* ergo triang. BCE etiam ſpecie datum eſt. Q. E. D

a *hyp.* & 3. *def.* d
b 41. *dat.*
c 3. *def. d.*
d 4. *dat.*
e 40. *dat.*

PROP.

PROP. XLVIII.

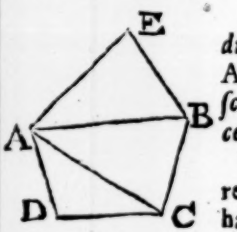


Si ab eadem recta AB *describantur triangula* ACB, ADB *data specie, habebunt ad invicem rationem datam.*

Duc enim perpendiculares CE, DF. Liquet angulos trianguli rectanguli CEB, *a* proinde & $\frac{CE}{CB}$ dari. ergo (quum $\frac{AB}{CB}$ *b* data sit) *c* erit $\frac{CE}{AB}$ data. Simili discursu datur $\frac{DF}{AB}$; *c* quare $\frac{CE}{DF}$, *d* hoc est triang. $\frac{ACB}{ADB}$ datur. Q. E. D.

a 40. *d.*
b *hyp.*
c 8. *d.*
d *sch.* 1. 6.

PROP. XLIX.

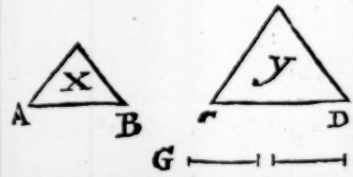


Si ab eadem recta linea AB *duo rectilinea quælibet* BA ABCD, AEB *data specie describantur, habebunt ad invicem rationem datam.*

Nam rectilineum ABCD resolvatur in triangula. *a* hæc specie data sunt. ergo ob communem basim AC, *b* ratio ADC ad ACB & *e* proinde totius ABCD ad ACB datur. *b* item ratio AEB ad ACB. *d* proinde & ABCD ad AEB datur. Q. E. D.

a 47. *d.*
b 48. *d.*
c 6. *d.*
d 8. *d.*

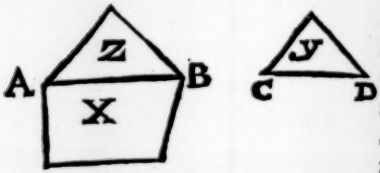
PROP. L.



Si duæ rectæ lineæ AB CD *ad invicem habeant rationem datam: & ab illis similia, similiterque descripta rectilinea* X, Y *habebunt ad invicem rationem datam.*

a 11. 6. Nam ſit AB. CD :: a CD. G. d liquet AB ad
b 8. d. G, e hoc eſt X ad Y dari. Q. E. D.
c cor. 20. 6.

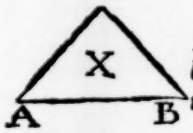
PROP. LI.



Si duæ rectæ lineæ AB, CD *habeant ad invicem rationem datam; & ab illis rectilinea quæcunque* X, Y *ſpecie data deſcribantur; habebunt ad invicem rationem datam.*

a 18. 6. Nam a fac Z ſimile ipſi Y. Ac ob b $\overline{Z}$, c & $\overline{X}$ $\overline{Y}$
b 49. d.
c 50. d. datas, d liquet $\overline{X}$ dari. Q. E. D.
d 8. d. $\overline{Y}$

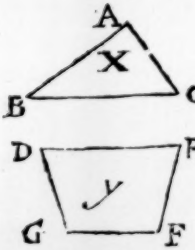
PROP. LII.



Si à data magnitudine recta AB *figura* X *ſpecie data deſcribatur, deſcripta figura* X *magnitudine data eſt.*

a 3. & 1. def. d. Nam ABq a datur ſpecie, &
b 49. d. magnitudine; & b ABq datur. c ergo $\overline{X}$ datur.
c 2. d.

PROP. LIII.



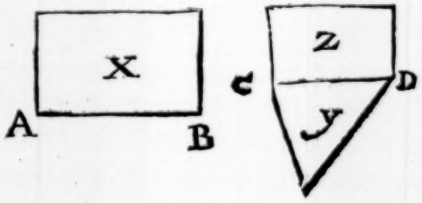
Si duæ figuræ X, Y *ſpecie datæ fuerint; & unum latus unius* BC *ad unum latus alterius* DE *habuerit rationem datam; reliqua quoque latera* AB *ad reliqua* FG *habebunt rationem datam.*

Nam

Nam { a AB, b BC, a DE, EF, a FG } dantur. &c. ergo per 8. dat.

a 3. def. d.
b hyp.

PROP. LIV.



Si duæ figuræ X, Y *ſpecie datæ ad invicem habuerint rationem datam, etiam latera* (AB, CD, &c.) *habebunt ad invicem rationem datam.*

Nam ad CD *a* fiat Z ipſi X ſimilis. *b* Hæc ſpecie datur. *c* ergo Y datur. Proinde ob $\frac{Y}{Z}$ *d* datam, $\frac{Y}{X}$ datur. *e* ergo X. *f* ergo AB datur. ergo per præcedentē. $\frac{X}{Z}$ $\frac{AB}{CD}$

a 18. 6.
b 3. def. d.
c 49. dat.
d hyp.
e 8. dat.
f cor. 20. 6. & 24. dat.

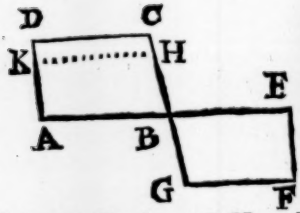
PROP. LV.

Si ſpatium X *magnitudine & ſpecie datum fuerit, ejus latera* (AB, &c.) *magnitudine data erunt.*

Nam ad quamvis CD *a* fac Y ſimile ipſi X. hoc ſpecie & magnitudine datur. *b* ergo Y da- $\frac{Y}{X}$ tur. *c* quare $\frac{CD}{AB}$ datur. *d* ergo AB data eſt. Q. E. D.

a 18. 6.
b 1. dat.
c 54. dat.
d 2. dat.

PROP. LVI.

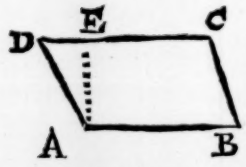


Si duo æquiangula parallelogramma AC, BF *habuerint ad invicem rationem datam, eſt ut primi latus* AB *ad ſecundi latus* BE, *ita reliquum ſecundi latus* BG *ad eam* BH, *ad quam alterum primi latus* BC *habet rationem datam, quam habet parallelogrammum* AC *ad parallelogrammum* BF.

a 1. 6. b 14. 6. & 7. 5.

Nam duc HK parall. AB. Liquet eſſe BC. BH *a* :: AC. AH *b* :: AC. BF. Q. E. D.

PROP. LVII.



Si datum ſpatium AC *ad datam rectam* AB *applicatum fuerit, in angulo* BAD *dato, datur applicationis altitudo* AD.

a 11. 1. b 1. 6. c. 35. 1. d 1. & 2. dat. e 28. & 25. dat.

a Erige perpendicularem AE. eſtque AB. AE *b* :: ABq. AB × AE *c* :: ABq. pgr. AC. *d* ergo AE datur. quare per E duc parallelam DC, *e* hæc abſcindet quæſitam AD. Q. E. F.

PROP. LVIII.

Si datum ad datam rectam applicetur, deficiens data ſpecie figura, latitudines defectus datæ ſunt.

Non differt à vigeſima octava ſextæ.

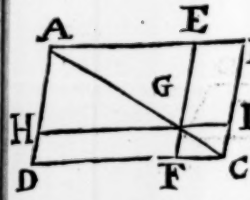
PROP. LIX.

Si datum ad datam rectam applicetur, excedens data ſpecie figura, latitudines exceſſus datæ ſunt.

Eadem eſt cum vigeſima nona ſextæ.

PROP.

PROP. LX.



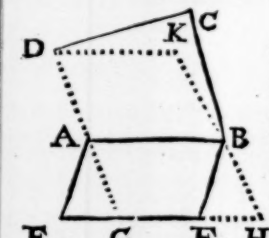
Si datum ſpecie parallelogrammum (H,E, *vel* DB) *dato gnomone* HCE *augeatur, vel minuatur; latitudines gnomonis* HD, EB *datæ ſunt.*

1. *Hyp.* Liquet totum DB tam *a* magnitudine, quam *b* ſpecie dari, *c* proinde & laticudines AB, AD; è quibus auſer *d* datas AE, AH, *e* manent EB, HD datæ. Q.E.D.

2. *Hyp.* Liquet HE *b* ſpecie, & *a* magn. *c* dari, *d* quare & latera AE, AH; hæc deme ex *d* datis AB, AD: *e* remanent EB, HD datæ. Q. E. D.

a 3. *d.* b 24. *6.* c 55. *d.* d *hyp.* e 4. *d.*

PROP. LXI.



Si ad datæ ſpecie figuræ ABCD *unum latus* AB *applicetur parallelogrammum ſpatium* AF *in angulo* BAE *dato; habeat autem data figura* AC *ad parallelogrammum* AF *rationem datam; parallelogrammum* AF *ſpecie datum eſt.*

Ad DAG protractam duc (per B) parallelam, cui occurrant EFH, & DK parall. AB. Ac ob $\frac{AD}{AG}$, & ang. BAD *a* dat. *a* liquet pgr. AK ſpecie dari. *b* ergo $\frac{AK}{AC}$ & *c* proinde $\frac{AK}{AF}$ *d* vel $\frac{AK}{AH}$, *e* hoc eſt $\frac{AD}{AG}$ dantur. *e* ergo $\frac{AB}{AG}$ datur. Item ob angulos E, & GAE *f* notos, *g* datur $\frac{AE}{AG}$; *c* ergo $\frac{AB}{AE}$ datur. *b* unde pgr. AF ſpecie datur. Q. E. D.

a 3. *def.* d. b 49. *d.* c 8. *d.* d 35. 1. e 1. 6. f *hyp.* & 4. *d.* g 40. *d.* h 3. *def.* d.

PROP. LXII.

Si duæ rectæ AB, CD *ad invicem habeant rationem datam; & ab una quidem data ſpecie figura* X *deſcripta ſit, ab altera autem ſpatium parallelogrammum* Y *in angulo dato; habeat autem figura* X *ad parallelogrammum* Y *rationem datam; parallelogrammum* Y *ſpecie datum eſt.*

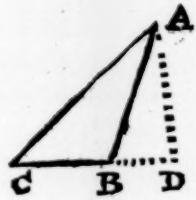
a 50. *dat.* Nam ad AB ſit pgr. Z ſimile ipſi Y. *a* Hujus
b 8. *dat.* ratio ad Y, & *b* proinde ad X datur. *c* ejuſque an-
c *hyp.* guli dantur. *d* ergo Z ſpecie datur. *e* proinde &
d 61. *dat.* Y. Q. E. D.
e 3. *def. d.*

PROP. LXIII.

Si triangulum ſpecie datum ſit, quod ab unoquoq; laterum deſcribitur quadratum, ad triangulum habebit rationem datam.

Sequitur ex 49. hujus.

PROP. LXIV.



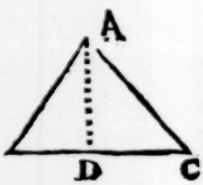
Si triangulum ABC *angulum obtuſum* ABC *datum habeat; illud ſpatium, quo latus* AC *obtuſum angulum ſubtendens magis poteſt quam latera* AB, CB *obtuſum angulum* ABC *ambientia, ad triangulum* ABC *habebit rationem datam.*

Nam demittatur AD perpendicularis produ-
a 4. *dat.* ctæ CBD. atque ob angulos *a* ABD, & D da-
b 40. *dat.* tos, *b* datur BD, *c* hoc eſt $\frac{BD \times CB}{AD}$. *d* ergo
c 1. 6. $\frac{AD \times CB}{AD}$
d 8. *dat.*

2 BD

2 BD × CB, hoc est, e ACq—ABq—CBq datur. e 12. 2.
½ AD×CB f triang. ABC f 41. 1.
Q. E. D.

PROP. LXV.



Si triangulum ACB *angulum acutum* C *datum habeat; illud ſpatium, quo latus* AB *angulum* C *ſubtendens minus poteſt, quam latera* AC, CB *angulum acutum* C *ambientia, habebit ad triangulum* ACB *rationem datam.*

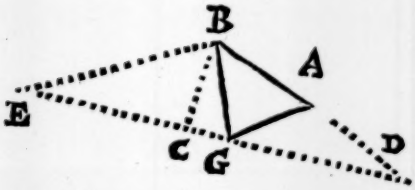
Nam duc perpendicularem AD. Datur a $\frac{CD}{AD}$, a 40. d.
b hoc eſt CD×BC. c ergo 2 CD×BC, hoc b 1. 6.
AD×BC ½ AD×BC c 8. d.
eſt d ACq+BCq—ABq datur. Q. E. D. d 13. 1.
e triang. ACB e 41. 1.

PROP. LXVI.

Si triangulum ACB *habuerit angulum* C *datum; quod ſub rectis* AC, CB *datum angulum* C *comprehendentibus, continetur rectangulum, habebit ad triangulum* ACB *rationem datam.*

Nam in figura præcedentis, eſt a $\frac{AC}{AD}$, b hoc a 40. d.
eſt, AC×BC, c hoc eſt AC×BC data. d ergo b 1. 6.
AD×BC 2 triang. ACB c 41. 1.
AC×BC datur. Q. E. D. d 8. d.
triang. ACB.

PROP. LXVII.



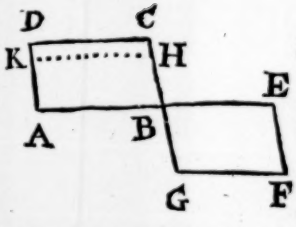
Si triangulum ABG *habuerit datum angulum* BAG; *illud ſpatium, quo duo datum angulum* BAG *comprehendentia latera tanquam una recta* BA+AG, *plus poſſunt, quam quadratum à reliquo latere* BG, *ad triangulum* ABG *habebit rationem datam.*

Produc BA ita ut AD=AG. per B duc BE parall. AG; cui occurrat DGE. denique duc normalem BC.

a 5. 1. Liquet ang. D *a*=AGD *b*=E. *c* quare BE=
b 29. 1. BD, ideoque EC=CD. *c* ergo EG×GD+
c 6. 1. CGq=CDq. proinde BDq *f* (CDq+BCq)
d *cor.* 3. 3. *g*=EG×GD+CGq+BCq=EG×GD *+
e 5. 2. BGq. Jam ob angulos AGD, & D *h* ſubduplos
f 47. 1. dati BAG, liquet *k* AD, ideoq; ADq dari. Cum
g 2. *ax.* 1. $\overline{DG}$ $\overline{DGq}$
* 47. 1. igitur BA×AD. ADq *l* :: BA. AD *m* :: EG.
h 32. 1. GD :: *l* EG×GD. GDq, & permutando BA×AD.
k 40. *d.* EG×GD :: ADq. GDq; *n* erit BA×AD; *o* hoc
l 1. 6. $\overline{EG\times GD}$
m 2. 6. eſt BA×AG data. *p* Atqui BA×AG datur; *q* er-
n 2. *def. d.* $\overline{EG\times GD}$ triang. AGB
o *conſtr.* go EG×GD datur. Q. E. D.
p 66. *d.* triang. AGB
q 8. *d.*

PROP.

PROP. LXVIII.

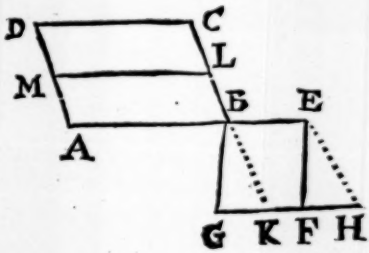


Si duo parallelogramma æquiangula AC, BF *habeant ad invicem rationem datam, & unum latus* AB *ad unum latus* BE *habeat rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

Nam fit AB. BE :: BG. BH. *a* ergo $\frac{BG}{BH}$ datur. *b* item $\frac{BC}{BH}$ datur. *c* ergo $\frac{BC}{BG}$ datur.

a 2. *def.* *d.*
b 56. *d.*
c 8. *d.*

PROP. LXIX.



Si duo parallelogramma AC, BF *datos angulos habeant, & ad invicem rationem datam; habeat autem & unum latus* AB *ad unum latus* BE *rationem datam; & reliquum latus* BC *ad reliquum latus* BG *habebit rationem datam.*

Latera AB, BE jaceant in directum. produc CBK, ac GFH ad occursum cum EH parall. CK.

Ob *a* ang. KBE (ABC) & pgr. *a* $\frac{AC}{BF}$, vel *a hyp.* AC

b 35. 1. c 68. d. d hyp. & 4. d. e 40. d. f 8. d.

$\frac{AC}{BL}$ & a $\frac{AB}{BE}$ datas, c liquet $\frac{KB}{BC}$ dari. item ob ang. G, & GBK d datos, e datur $\frac{KB}{BG}$, f quare $\frac{BC}{BG}$ datur. Q. E. D.

PROP. LXX.

Si duorum parallelogrammorum (AC, BH, *vel* BF) *circa æquales angulos* (ABC, KBE) *aut circa inæquales quidem* (ABC, GBE) *datos tamen, latera* (AB, BE, & BC, BK, & BC, BG) *ad invicem habeant rationem datam; & ipsa parallelogramma* (AC, BH, & AC, BF) *habebunt ad invicem rationem datam.*

Nam (in fig. præced.) sit AB. BE :: KB. BL. & duc LM parall. BA.

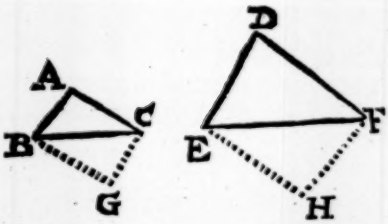
a hyp. b constr. c 8. d. d 1. 6. e 14. 6. f hyp. & 4. d. g 40. d. h 35. 1.

Primo, Quia a AB b id est KB a ac KB datæ $\overline{BE}$, $\overline{BL}$, $\overline{CB}$ sunt, c erit $\overline{CB}$, d hoc est $\overline{AC}$ e vel pgr. AC data. $\overline{BL}$ $\overline{AL}$, $\overline{BH}$ Q. E. D.

Secundo, Ob angulos G, & GBK f datos, g datur BK; item h CB data est. c ergo CB da- $\overline{BG}$ $\overline{BG}$ $\overline{BK}$ tur. proinde, ut prius, $\frac{AC}{BH}$, hoc est pgr. $\frac{AC}{BF}$ datur. Q. E. D.

PROP.

PROP. LXXI.



Si duorum triangulorum ABC, DEF, *circa æquales angulos, aut circa inæquales quidem, datos tamen* (A, *&* D) *latera* AB, DE, *&* AC, DF *ad invicem habeant rationem datam; & ipſa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

Nam compleantur pgra. AG, DH. *a* hæc data habent rationem, *b* proinde & trigona ABC, DEF illorum *c* ſubdupla. Q. E. D.

a 70. *d.*
b 15. 5.
c 34. 1.

PROP. LXXII.

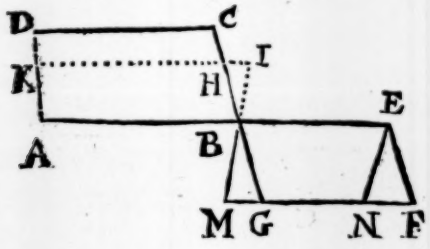
Si duorum triangulorum ABC, DEF *& baſes* BC, EF *fuerint in ratione data, & actæ ab angulis ad baſes* (AG, DH,) *quæ faciant ang.* AGC, DHF *æquales, aut inæquales quidem, ſed tamen datos, habeant ad invicem rationem datam; & ipſa triangula* ABC, DEF *habebunt ad invicem rationem datam.*

Nam duc BK ad AG, ac EM ad DH parallelas, & comple pgra. CK, FM. Hæc ſe habent juxta 70. hujus; quare triangula eorum * ſubdupla ABC, DEF rationem habent datam. Q. E. D.

* 34 1.

PROP.

PROP. LXXIII.



Si duorum parallelogrammorum (AC, BF, *vel* AC, BN) *circa æquales angulos, aut circa inæquales quidem, sed tamen datos, latera ad invicem ita se habeant, ut sit quemadmodum primi latus* AB *ad secundi latus* BE, *ita reliquum secundi latus* (BG, *vel* BM) *ad aliam aliquam rectam* (BH, *vel* BI;) *habeat autem & reliquum primi latus* BC *ad eandem rectam* (BH *vel* BI) *rationem datam; & ipsa parallelogramma* (AC, BF, *vel* AC, BN) *habebunt ad invicem rationem datam.*

Nam 1. Hyp. liquet *a* CB *b* id est AC da-
$\overline{BH}$ $\overline{AH}$ *c* ($\overline{BF}$)
ri. Q. E. D.

2. Hyp. Duc parallelam IHK. *a* Liquet angulos IBH (GBM) & BHI (ABH) dari. *b* ergo BH datur. item CB *a* data est. *c* proinde
$\overline{BI}$ $\overline{BI}$
CB, hoc est pgr. AC *d* vel AC datur. Q. E. D.
$\overline{BH}$ $\overline{BF}$ $\overline{BN}$,

a *hyp.*
b 2. 6.
c 14. 6.
a *hyp* & 4. *dat.*
b 40. *dat.*
c 8. *dat.*
q 35. 1.

PROP. LXXIV.

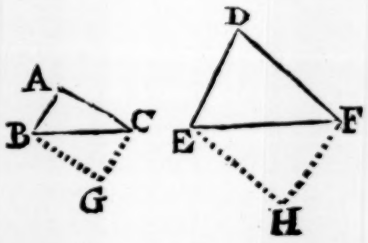
Si duo parallelogramma datam rationem habeant, aut in æqualibus angulis (*ut* AC, BF) *aut inæqualibus quidem, sed tamen datis* (*ut* AC, BN;) *erit ut primi latus* AB *ad secundi latus* BE, *ita alterum secundi latus* (BG, *vel* BM) *ad eam* (BH, *vel* BI) *ad quam reliquum primi latus* BC *rationem habet datam.*

Nam

Nam in fig. præcedentis. 1. Hyp. *a* Liquet $\frac{CB}{BI}$ dari. Q. E. D. [a 56. *dat.*]

2. Hyp. ut in præcedenti, datur BI, ac ex hyp. $\frac{\overline{BH}}{\overline{BF}(\overline{BN})}$ item AB.BE :: *a* *MB.BI *b* :: GB. BH. [* *hyp.* b 4 6.] *a* quare CB etiam datur. *e* ergo CB data eſt. [c 8. *dat.*] Q. E D.

PROP. LXXV.



Si duo triangula ABC, DEF *ad invicem habeant rationem datam, aut in angulis* (A, D) *æqualibus, aut inæqualibus quidem ſed tamen datis, erit ut primi latus* A B *ad ſecundi latus* D E, *ita alterum ſecundi latus* DF *ad eam rectam, ad quam reliquum primi latus* AC *habet rationem datam.*

Nam compleantur pgr. AG, DH. Ergo per præcedentem.

PROP. LXXVI.

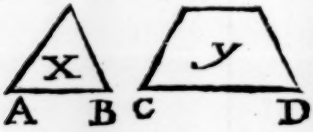
Si à trianguli ABC *ſpecie dati vertice* A *linea perpendicularis* A D *agatur ad baſim* B C, *acta linea* A D *ad baſim* B C *habebit rationum datam.*

Nam

* *hyp.* & 3. *def. d.* Nam ob angulos, *B, & ADB datos, *a* datur $\frac{AB}{AD}$; *a* item $\frac{AB}{BC}$ datur. *b* Ergo $\frac{AD}{BC}$ datur. Q. E. D.

a 40. *dat.*
b 8. *dat.*

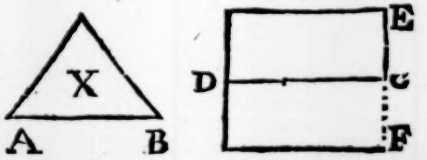
PROP. LXXVII.



Si datæ figuræ specie X, Y *ad invicem habeant rationem datam, & quodlibet latus unius* AB *ad quodlibet alterius latus* CD *habebit rationem datam.*

a 49. *dat.* Nam *a* ABq, & *b* $\frac{Y}{X}$, ac *c* proinde ABq datur;
b *hyp.*
c 8. *dat.* item CDq datur. *c* ergo $\frac{ABq}{CDq}$, ac ideo $\frac{AB}{CD}$ da-
$\frac{Y}{X}$
tur. Q. E. D.

PROP. LXXVIII.

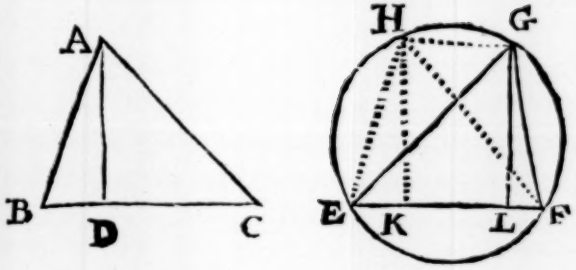


Si data figura specie X *ad aliquod rectangulum* DCE *habeat rationem datam; habeat autem & unum latus* AB *ad unum latus* DC *rationem datam; rectangulum* DCE *specie datum est.*

a 3. *dat.* 3. Sit DC. AB :: AB. CF. *a* ergo $\frac{DC}{CF}$ datur.
b 49. *dat.* Item ob *b* X, & *c* X datas, *a* erit $\frac{ABq}{DCE}$, *d* hoc est
c *hyp.*
d 17. 6. $\frac{DCE}{DC \times CF}$, vel *e* $\frac{DCE}{GF}$ data. proinde *e* DC datur.
e 1. 6.
f 3. *def. d.* *f* quare rectang. DCE specie datur. Q. E. D.

PROP.

PROP. LXXIX.



Si duo triangula ABC, GEF *unum angulum* BAC *uni angulo* EGF *æqualem habeant; ab æqualibus autem angulis* BAC, EGF *ad baſes* BC, EF *perpendiculares agantur* AD, GL; *ſitque ut primi trianguli baſis ad perpendicularem, ita & alterius trianguli baſis ad perpendicularem* (BC. AD :: EF. GL;) *illa triangula* ABC, EGF *æquiangula ſunt.*

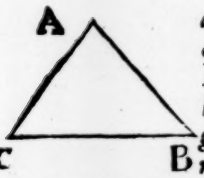
Circa triang. GEF deſcribe circulum. Fac ang. FEH = B. Connecte HF, HG; & demitte perpendicularem HK.

Liquet triangula ABC, HEF, & ABD, HEK, ac ACD, HFK æquiangula fore. Proinde EK. KH :: BD. DA. *a* & FK. KH :: CD. DA. *b* quare EF. KH :: BC. DA :: *c* EF. LG. *d* quare KH = LG. *e* ergo HG parall. KL. *f* unde ang. EGH = GEF. *g* ergo arcus EH, FG, *h* ideoque anguli EFH, GEF æquantur. *k* Item ang. EHF = EGF. *l* ergo trigona EHF, EGF; *m* proinde & trigona EGF, ABC ſibi mutuo æquiangula ſunt. Q. E. D.

a 4. 6.
b 24. 5.
c *hyp.*
d 9. 5.
e 33. 1.
f 29. 1.
g 26. 3.
h 27. 3.
k 21. 3.
l 32. 2.
m 21. 6.

PROP.

PROP. LXXX.



Si triangulum ABC *unum angulum* A *datum habuerit; quod autem sub lateribus* AB, AC *datum angulum comprehendentibus continetur rectangulum, habeat ad quadratum reliqui lateris* BC *rationem datam; triangulum* ABC *specie datum est.*

Nam Q: AC + AB: ⊢ CBq vocetur X.
a 67. d. *a* ergo $\frac{X}{\text{triang. } ABC}$; *b* & $\frac{AC \times AB}{\text{triang. } ABC}$; & *c* propterea
b 66. d.
c 8 d. $\frac{X}{AC \times AB}$ data est. *d* item $\frac{AC \times AB}{CBq}$ datur. *e* ergo
d hyp.
e 6. d. $\frac{X}{CBq}$ *e* ideoque $\frac{X + CBq}{CBq}$, *f* hoc est Q: $\frac{AC + AB}{CBq}$,
f hyp.
g 46. d. datur. *g* proinde triang. ABC specie datur. Q. E. D

PROP. LXXXI.

A. D.
B. E.
C. F.

Si tres rectæ proportionales A, B, C *tribus rectis proportionalibus* D, E, F *extremas* A, D, *&* C, F *habuerint in ratione data; medias quoque* B, E *habebunt in ratione data. Et si extrema* A *ad extremam* D, *& media* B *ad mediam* E *habeat rationem datam; & reliqua* C *ad reliquam* F *habebit rationem datam.*

a 70. d. Nam primo, ob $\frac{A}{D}$ & $\frac{C}{F}$ datas, *a* datur $\frac{AC}{DF}$,
b 17. 6. *b* hoc est, $\frac{Bq}{Eq}$. ergo $\frac{B}{E}$ datur. Q. E. D.

c hyp. Secundo, ob *c* $\frac{Bq}{Eq}$, *b* hoc est $\frac{AC}{DF}$ datam, & *c* $\frac{A}{D}$
d 68. d. datam, *d* datur $\frac{C}{F}$. Q. E. D.

PROP.

PROP. LXXXII.

A. B :: D. E.
B. C :: E. F.

Si quatuor rectæ proportionales fuerint (A, B :: D, E) *erit ut prima* A *ad eam* C, *ad quam ſecunda* B *rationem habet datam, ita tertia* D *ad eam* F, *ad quam quarta* E *rationem habet datam.*

Nam quia B. C :: *a* E. F. & *a* $\frac{B}{C}$ data eſt; *berit* $\frac{E}{F}$ data. atqui ex æquali A. C :: D. F. ergo, &c.

a *hyp.*
b 2. *def. d.*

PROP. LXXXIII.

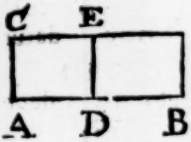
A. B. C. D.
F. E.

Si quatuor rectæ A, B, C, D *ita ad invicem ſe habeant, ut tribus ex iis, quibuſcunque ſumptis* A, B, C, *& quarta ipſis proportionali accepta* E, *ad quam reliqua* D *ex quatuor rectis proportionem habet datam; erit ut quarta* D *ad tertiam* C, *ita ſecunda* B *ad eam* F, *ad quam habet prima* A *rationem datam.*

Nam A E *a* = B C *b* = D F. & datur *b* $\frac{D}{E}$ *c* hoc eſt $\frac{AD}{AE}$, *d* vel $\frac{AD}{DF}$, *e* vel $\frac{A}{F}$. ergo, &c.

a 16. 6.
b *hyp.*
c 1. 6.
d 7. 5.

PROP. LXXXIV.



Si duæ rectæ A B, A C *datum ſpatium comprehendant in angulo* A *dato; ſit autem altera* A B *altera* A C *major data* D B; *etiam unaquæque ipſarum* A B, A C *data erit.*

Nam comple quadratum A E. *a* Hoc ſpecie datum eſt. *b* item pgr. C B, & recta D B dantur. *c* ergo A C, vel A D, & tota *d* proinde A B datur. Q. E D.

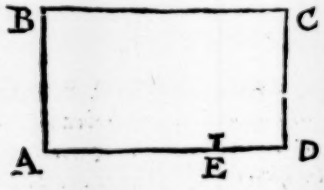
a 3. *def. d.*
b *hyp.*
c 59. *dat.*
q 3. *dat.*

PROP. LXXXV.

Si duæ rectæ BD, DE *datum ſpatium comprehendant in angulo* BDE *dato, ſit autem ſimul utraq;* (BD+DE) *data; & earum quoque unaquæque* BD, *&* DE *data erit.*

Nam ſume DA=DE, & comple quad. DC. Hoc ſpecie datur; item pgr. BE, & recta BA
a *hyp.* *a* dantur. *b* ergo AD (DE) & *c* reliqua DB dan-
b 58. *d.* tur. Q. E. D.
c 4. *d.*

PROP. LXXXVI.



Si duæ rectæ AB, AD *datum ſpatium* BD *comprehendant in angulo dato; quadratum autem unius* AD *quadrato alterius* AB *majus ſit dato quam in ratione* (nempe ut ſit
* 2. 2. ADxAE datum, & * reliqui ADxED ad ABq ratio data;) *& utraque ipſarum* AB, AD *data erit.*

a *hyp.* Nam ob BD, & DA x AE *a* data, *b* datur
b 1. *d.* BD. *c* ergo AB *d* ideoque ABq datur. *e* item
c 69. *d.* $\overline{DA \times AE}$ $\overline{AE}$ $\overline{AEq}$
d 51. *d.* ABq datur. *f* ergo AEq ideoque AEq
e *hyp.* $\overline{AD \times ED}$, $\overline{AD \times ED}$, $4\overline{AD \times ED}$,
f 8. *d.* *g* & AEq *b* hoc eſt AEq datur.
g 6. *d.* $4\overline{AD \times ED} + \overline{AEq}$ $\overline{Q:AD+ED}$
h 8. 2. *k* ergo AE & *l* componendo AE *ideoq;
k 54. *d.* $\overline{AD \times ED}$; $2\overline{AD}$,
l 6. *d.* AE *m* hoc eſt AEq datur. denique igitur ob
* 8. *d.* $\overline{AD}$, $\overline{AD \times AE}$
m 1. 6. *e* datum ADxAE, *n* erit AEq data. *o* ergo AE,
n 2. *d.* & *p* proinde AD, ac AB datæ ſunt. Q. E. D.
o 55. *d.*
p 57. *d.*

PROP.

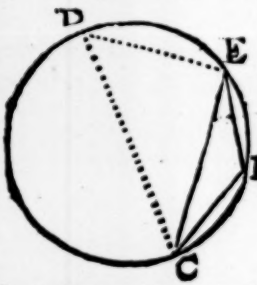
PROP LXXXVII.

Si duæ rectæ AB, AD *datum ſpatium comprehendant in angulo dato, quadratum autem unius* AD *quadrato alterius* AB *majus ſit dato* (AD× AE;) *earum utraque* AB, AD *data erit.*

Nam ob $\overline{BA}\times\overline{AE}$ *a* datum, *b* erit AE ideoque $\overline{BD}$, $\overline{AB}$, AEq *c* hoc eſt AEq. *d* ac idcirco AEq $\overline{ABq}$, $\overline{AD}\times\overline{ED}$, $\overline{AEq}+4\overline{AD}\times\overline{ED}$, *e* hoc eſt AEq ac proinde AE & *d* componendo AE *e* ac ideo AE *e* hoc eſt AEq $Q:\overline{AD}+\overline{ED}$, $\overline{AD}+\overline{ED}$, $2\overline{AD}$, $\overline{AD}$, $\overline{AD}\times\overline{AE}$ data. ergo ob ADxAE *f* datum, dantur *g* AEq, & *h* AE, ac *k* ideo AD, ac AB. Q. E. D.

a 2. *d.*
b 69. *d.*
c *hyp.* & 1. 2.
d 8. & 6. *d.*
e 8. 2.
d 6. *d.*
e 1. 6.
f *hyp.*
g 2. *d.*
h 55. *d.*
k 57. *d.*

PROP. LXXXVIII.



Si in circulum CFED *magnitudine datum acta ſit recta linea* CE, *quæ ſegmentum auferat, quod datum angulum* F *comprehendat; acta recta linea* CE *magnitudine data eſt.*

Nam ducatur diameter CD; & connectatur ED. Acob ang. F *a* datum, *b* erit ang. D (reliquus è 2 rectis) datus. item rectus CED datur. *c* quare $\frac{CE}{CD}$ datur. ergo ob *d* datam CD, *e* erit CE data, Q. E. D.

a *hyp.*
b 4 *d.*
c 40. *d.*
d *hyp.* & 5. *def. d.*
e 2. *d.*

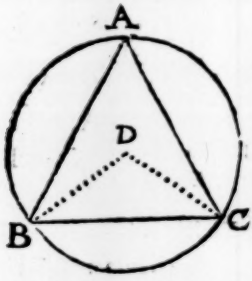
PROP. LXXXIX.

Si in datum magnitudine circulum CFED *data magnitudine recta* CE *acta fuerit, auferet segmentum quod angulum* (CFE) *datum comprehendet.*

Nam (in fig. præcedentis) quia $\frac{CE}{CD}$, & ang. CED dantur, *a* erit ang. D datus. *b* ergo ang. F *c* (1 Rect. — D) datus erit. Q. E. D.

a 43. *dat.*
b 4. *dat.*
c 22. 3.

PROP. XC.



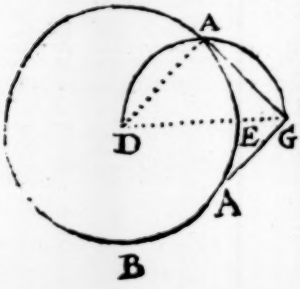
Si in circuli positione dati circumferentia BAC *datum fuerit punctum* B, *ab eo autem puncto* B *ad circumferentiam circuli inflexa fuerit recta* BAC *quæ datum angulum* A *efficiat; inflexa rectæ altera extremitas* C *data erit*

Ad *a* centrum D duc BD, & CD; *b* datusque est ang. D dati A *c* duplus. quare ob B D *d* datam, *e* erit DC data. *f* ergo punctum C datum est. Q. E D.

Si ang. A obtusus fuerit; sume reliquum è 2 rectis acutum; ejus subsidio punctum C invenies, juxta dicta.

a 1. 3.
b 2. *dat.*
c 20. 3.
d 26. *dat.*
e 29. *dat*
f *ſch.* 25. *d.*

PROP. XCI.



Si à dato puncto G *acta fuerit recta* G A, *quæ datum positione circulum* B E A *contingat; acta linea* G A *positione & magnitudine data est.*

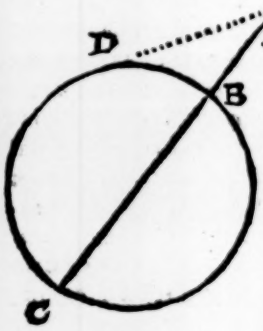
Nam centrum D & punctum G

G connectat recta D G. ſuper qua deſcriptus ſit ſemicirculus D A G circulo priori occurrens in A. Ob ang. DAG *a* rectum, GA circulum *b* tangit. *c* ergo G A ſitu & magnitudine datur. Q. E. D.

a 31. 3.
b *cor.* 16. 3.
c 26. *dat.*

Hinc modus dicitur à dato puncto tangentem ducendi, eo nonnunquam expeditior qui habetur ad 17. 3.

PROP. XCII.

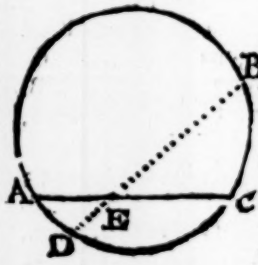


Si extra circulum poſitione datum BCD *accipiatur aliquod punctum* A, *à dato autem puncto* A *in circulum producatur quædam recta* A C; *datum eſt id quod ſub acta linea* AC, *& ea* AB, *quæ inter punctum* A *& convexam peripheriam* B *comprehenditur rectangulum* CAB.

a Nam duc tangentem A D, *b* eritque ADq (hoc eſt C A × A B) datum. Q. E. D.

a 91. *dat.*
b 36. 3.

PROP. XCIII.

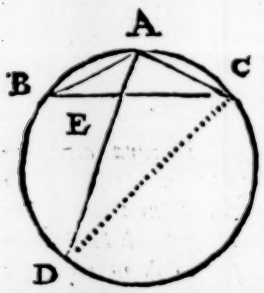


Si intra datum poſitione circulum A B C D *ſumatur aliquod punctum* E; *per punctum autem* E *agatur in circulum aliqua recta* A E C; *quod ſub ſegmentis* A E, E C *acta recta linea comprehenditur rectangulum, datum eſt.*

Nam

Nam per E duc rectam DEB utcunque occur-
a 35. 1. rentem circulo in B, & D. eſtque rectang. DEB
b 1. def. d. = a AEC. b ergo AEC datur. Q. E. D.

PROP. XCIV.



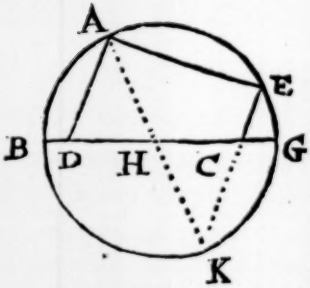
Si in circulum BA. C D *magnitudine datum agatur recta linea* B C, *quæ ſegmentum auferat, quod angulum* B A C *datum comprehendat; angulus autem* BAC, *qui in ſegmento conſiſtit, bifariam ſecetur; ſimul utraque rectarum* B A, A C *quæ angulum datum* B A C *comprehendunt, ad lineam* A D, *quæ angulum bifariam ſecat, habebit rationem datam & quod ſub ſimul utriſque* B A, A C, *quæ datum angulum* B A C *comprehendunt, rectis; & inferne abſciſſa* (E D) *ab ea* A D, *quæ angulum* B A C *in circumferentia datum bifariam ſecat, rectangulum datum erit.*

a 88 dat. Duc CD; & primo ob angulos BAC, CAD
* 1. dat. datos, a dantur ſubtenſæ BC, CD, * ideoque $\frac{CB}{DC}$
b 3. 6. datur. Cum igitur CA. AB :: b CE. EB, & per-
c 12. 5. mutando C A. C E :: AB. E B :: (CA + AB.
* 4. 6. CB ::) * AD. DC. (Nam * ob ang. BAE
d 2. def. d. = CAD; & D = B; trigona ABE, ADC ſimilia ſunt) ac rurſus permutando C A + A B. A D :: C B. D C. d erit $\frac{CA + AB}{AD}$ data.
Q. E. D.

e 21. 3. Secundo, ob triangula AEB, DEC e ſimilia;
1 4. 6. b erit C D, DE :: A B. BE c :: CA + AB. CB.
e prius. d ergo CA + AB in DE = CD in CB. atqui
d 16. 6. CD × CB e datur. f ergo CA + AB in DE da-
c 52. dat. tum eſt. Q. E. D.
f 1. def. d.

PROP.

PROP. XCV.



Si in circuli BAG *positione dati diametro* BG *sumatur datum punctum* D; *à puncto autem* D *in circulum producatur quædam recta* DA, *& agatur à sectione* A *ad rectos angulos in productam rectam* DA *linea* AE; *per punctum autem* E, *in quo linea* AE, *quæ ad rectos angulos consistit, occurrit circumferentiæ circuli, agatur parallela* (ECK) *productæ rectæ* DA; *datum est illud punctum* C, *in quo parallela* EK *occurrit ipsi diametro* BG; *& quod sub parallelis lineis* AD, EC *comprehenditur rectangulum, datum est.*

Nam connectatur AK. *a* estque AK (ob angulum E, vel DAE rectum) diameter. ergo intersectio H est centrum. *b* ergo DH datur. Atqui ob KH. HA *c* :: CH. HD, *d* est CH=HD. *e* ergo CH datur. *f* ergo punctum C datur. Q. E. D. *g* ergo KC x CE, hoc est *d* AD x CE datur. Q. E. D.

a 31. 3.
b 26. d.
c 4. 6.
d 9. 5.
e 1. def. d.
f 27. d.
g 93. dat.

FINIS.

ac.